DEAR EDITOR,

I was interested to read the note by R. J. Cook and G. V. Wood on Feynman's triangle. The general form of this result was brought to my attention about twenty years ago by Dr (now Professor) Peter Baptist of the University of Bayreuth, who was researching the teaching of geometry in the 19th century. He had found it in a footnote in E. J. Routh's *Analytical statics* [1], and asked if I could find out how Routh had proved it. I spent a morning trawling through Routh's collected papers (mostly undergraduate problems written on the back of college menus, lovingly bound in four thick volumes by Peterhouse library), but did not succeed in tracing it.

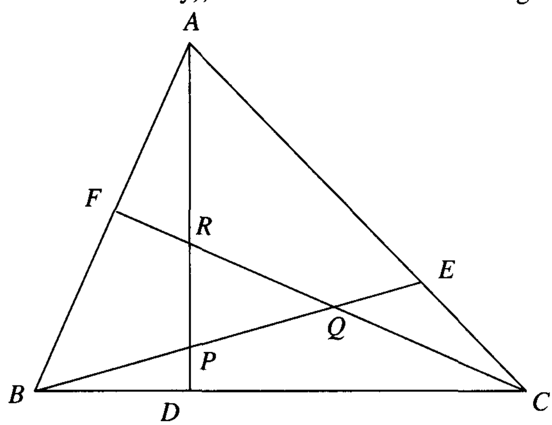

FIGURE 1

Routh's formulation, in the notation of Figure 1, states that if area $ABC = \Delta$, area $DEF = \Delta'$, and area $PQR = \Delta''$, then

$$\frac{\Delta'}{\Delta} = \frac{AF.BD.CE + AE.CD.BF}{abc}$$

and
$$\frac{\Delta''}{\Delta} = \frac{(AF.BD.CE - AE.CD.BF)^2}{(bc - AE.AF)(ca - BF.BD)(ab - CE.CD)}.$$

He used these results in solving problems on rigid frameworks, and notes that 'the author has not met with these expressions … which often occur'. It is the second of these which is relevant to the note by Drs Cook and Wood.

H. S. M. Coxeter, in *Introduction to geometry* [2] (to which Cook and Wood themselves refer), expresses the ratio more elegantly in the form

$$\frac{(\lambda\mu\nu - 1)^2}{(\lambda\mu + \lambda + 1)(\mu\nu + \mu + 1)(\nu\lambda + \nu + 1)},$$

where $\frac{BD}{DC} = \lambda$, $\frac{CE}{EA} = \mu$ and $\frac{AF}{FB} = \nu$. Coxeter calls this 'Routh's theorem', and gives the special case with $\lambda = \mu = \nu = 2$ (Feynman's triangle of Note 88.46) as an example. His formulation makes it clear, as Routh's does not, that the result belongs to affine geometry.

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Like Fermat, Routh clearly found the margin too small to offer a proof of these results. I myself constructed a rather pedestrian vector proof, which Routh would certainly have spurned. Coxeter gives a much neater proof using areal coordinates; Routh could have used this, but the form of his expression suggests that he more likely proved it by Euclidean methods. However, the third proof of Feynman's triangle given by Cook and Wood, based on Menelaus' theorem, does not obviously generalise. Can any reader suggest a reconstruction of Routh's argument?

A good account of Routh's life and times is provided by A. R. Forsyth's obituary notice for the London Mathematical Society [3]. Routh was Senior Wrangler in 1854 (beating Clerk Maxwell into second place) and stayed in Cambridge for the rest of his career. His claim to fame is as the great 'coach' for the Mathematical Tripos of his generation, with the remarkable record of having coached the Senior Wrangler in 24 consecutive years. As a teacher, he would have influenced many of those who were prominent in the Association for the Improvement of Geometrical Teaching (later the Mathematical Association) in its early years, though I doubt that he would have had much sympathy with the Association' s aims.

References

1. E. J. Routh, *Analytical statics* (2nd edn), Cambridge University Press (1896) p. 82 (footnote).
2. H. S. M. Coxeter, *Introduction to geometry*, Wiley (1961).
3. A. R. Forsyth, Edward John Routh, *Proceedings of the London Mathematical Society*, Series 2 Vol. 5 (1907).

Yours sincerely,

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Readers will be saddened to learn of the passing of Murray Klamkin who died aged 83 in August 2004. He was universally admired as the doyen of problemists worldwide with an encyclopaedic knowledge which he was delighted to share. His problems and solutions graced the columns of every mathematics journal which has a problems section; in particular, he responded regularly to the problems in the *Gazette*. His last contribution (July 2004, pp. 324-325) typified his elegant, incisive style.

Nick Lord