algebraic topology. There is then a preliminary chapter on the geometry of simplicial complexes, amplifying the material already treated in Volume 1 on this topic. Simplicial maps, simplicial approximation and contiguous maps are discussed. Chapter 2 gives the construction of the oriented simplicial homology and cohomology groups of a simplicial complex over the integers. Chapter 3 describes the algebra of chain complexes with applications to simplicial complexes. At this stage the relative homology and cohomology groups are introduced. In Chapter 4 the basic theorem for finitely generated abelian groups is proved and applied to the construction of a canonical basis for a complex. The Euler characteristic theorem is proved. In Chapter 5, cell complexes are introduced, and it is shown that the homology groups of a cell subdivision of a simplicial complex are isomorphic to the original simplicial groups. In particular it is shown that homology groups are invariant under the operation of barycentric subdivision. The topological invariance of the homology and cohomology groups is proved in Chapter 6. The method is to set up, using barycentric subdivision and simplicial approximation, isomorphisms between the homology and cohomology groups of different subdivisions of a complex, and so to construct abstract groups, independent of the triangulations. The discussion is carried out without the introduction of the singular theory. This chapter concludes with a treatment of fixed point theorems and local homology groups. In Chapter 7 the cup and cap products are constructed and their topological invariance is proved. Chapter 8 gives an introduction to the theory of homology manifolds. Poincaré duality is proved and dual bases are set up for the Betti groups. The classification of surfaces is described without proof.

There are two points of terminology which may cause a little confusion. "Teilkomplex" is as usual to be translated into the English "subcomplex". However the author also introduces the term "Subkomplex" the object of which turns out to be to get relative chain groups on which the boundary operator d will satisfy $d^2 = 0$. Also the author uses the term "Reduzierte Homologiegruppe", which is homology modulo torsion and not reduced homology in the usual sense.

Andrew H. Wallace, Institute for Advanced Study

<u>An Introduction to Modern Calculus</u>, by Wilhelm Maak. Translated from the German. Holt, Rinehart and Winston Inc. Publ., New York, 1963. x + 390 pages. $7.00.

The purpose of this book is described by the author in his preface: "This book is intended for use together with lectures. It is designed to help the student in his efforts to understand what is fundamental in differential and integral calculus." He goes on to say that whereas in Mathematics there are many "complete theories which start with axioms and, with a series of definitions and theorems, march to a well defined goal". Calculus, however, "resists presentation as an ideal theory". The book tries to "give calculus approximately the form of an axiomatic theory. The student is to be offered a framework into which he can fit the complex material of lectures and exercises."

As in many modern books, Chapter I discusses (mainly without proofs) the real number system which is followed by the definition of a real function of a real variable. The following chapter is devoted to the limit concept for sequences with applications to the definition of logarithmic functions and ends with a discussion of infinite series. On this basis Chapter III develops limits of functions and continuity. Chapter IV introduces the "differential quotient" and the elementary rules and theorems with applications to special functions, Interpolation, Taylor's series, and uniform convergence. Chapter V, Integral calculus including the definition of the Riemann integral, integrability of continuous and of monotonic functions, Darboux's theorem for bounded functions, and evaluation of integrals; and Chapter VI making the connection with the indefinite integral and integration of rational functions, conclude the first part of the work.

Whereas for these first 154 pages practically no preliminary knowledge is required, for the second part the reader should be familiar with the elements of n-dimensional analytic geometry of which brief summaries are given wherever it is needed. This conforms with the practice in German universities where every first-year mathematics student follows an extensive course on this subject during the first semester. The main theme now is "Functions of several variables". Chapter VII introduces the notion of function as a mapping $X \to y$ of a set of points $X \in R^n$ into the set of real numbers y. Continuity and some other elementary notions of topology are discussed; curves and differentials on curves, and the line integral follow. Only in Chapter VIII one finds the general concept of a differentiable function and gradient. The total differential is introduced and generalized into a differential form of the first degree, higher derivatives and Taylor's formula. Vector notation is used. Chapter IX deals with differentiable mappings of R^n into itself which requires matrices and functional determinants. One section discusses solution of an operator equation in R^n by iteration; the following one (17 pages) deals with the theory of implicit functions. The chapter ends with the decomposition of a differentiable mapping into a product of "primitive" mappings, each factor affecting only one coordinate. Chapter X contains the calculus of alternating differential forms which in its emphasis shows that it is particularly important to the author. In analogy to the curves in R^n (cf. Chapter VII) we now learn about p-dimensional surfaces in R^n and of the differential of the p^{th} degree on a surface segment, generalizing the previously defined differential on a curve, followed by the general differential form of the p^{th} degree; their "alternating" property is established and the systematic calculus is developed, thus preparing the "Integral Calculus" of Chapter XI. Volume is introduced axiomati-

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cally, and a generalization of Cavalieri' s principle is proved; space integrals, introduced as linear functionals invariant under translation, monotonic and normalized, are expressed as iterated simple integrals and the transformation formula is proved. Finally surface integrals, as preparation for Chapter XII: Gauss' Integral theorem and the general formula of Stokes.

From this account it may be seen that the present work is not an ordinary text-book on calculus. It contains in a relatively small space an extraordinary amount of material and should be of interest to everybody teaching analysis on a higher level. It presents rigorous proofs for many theorems of fundamental nature which are often settled by referring to physical intuition or proved by using sloppy argument.

Although the text contains a number of illustrating examples which should help the reader to understand the preceding theory, there are no exercises on which he could test his ability. In return for this there are the introductory remarks at the beginnings of the chapters stating clearly the aims of the subsequent development.

The translation is in general satisfactory; occasional unidiomatic expressions (e.g. "differential quotient", "monotonical" etc.) should not constitute a difficulty for any reader.

Hanna and Hans Schwerdtfeger, McGill University

Generators and Relations for Discrete Groups, by H.S.M. Coxeter and W.O.J. Moser. Second Edition. Ergebnisse der Mathematik, Neue Folge, Band 14. Springer Verlag, Berlin, 1965. ix + 161 pages. Price D.M. 32.00.

The original purpose of this book was to provide a list of abstract definitions by relations between generators for all finitely generated groups which might occur as examples in group-theoretical investigations. Evidently the first edition proved to be extremely useful. So will the new edition continue to be helpful to all those who, like Hilbert expect progress from the insight into a large number of special cases.

Apart from minor corrections throughout the text, the new edition differs from the first one by a more complete description of the binary polyhedral group (§ 6.5) and of the recent progress on the Burnside problem (§ 6.8); a new presentation is given for the groups GL(2, p), PGL(2, p), M_{11}, M_{12}.

A detailed review has been published in the Bulletin of the American Mathematical Society, Vol. 64, No. 3, 1, pp. 106-108. For the newcomer to the work we list the chapter headings: 1. Caylic, Cicylic and Metacyclic Groups. 2. Systematic Enumeration of Cosets.