Study of the electron–atom collisions in dense semiclassical plasma of noble gases

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We present the effective optical potential of the interaction of an electron with an atom in
dense semiclassical plasma of noble gases. This potential takes into account the collective
screening effect and the quantum mechanical effect of diffraction. The influence of
diffraction and screening effects on the characteristics of electron–atom collisions was
investigated. Scattering phase shifts decrease with increase of the de Broglie wave. The
electron–atom momentum-transfer cross-section at $\lambda_B \rightarrow 0$ tends to the data obtained
earlier with a neglecting of the diffraction effect.

Key words: plasma simulation, strongly coupled plasmas

1. Introduction

The study of electron scattering by noble gas atoms has always been a hot topic.
Suffice it to recall that the discovery of the Ramsauer–Townsend effect (Ramsauer 1921;
Townsend & Bailey 1922) at one time caused great amazement by its unexpectedness and
incomprehensible origin, but which was later explained as a consequence of the effect of
diffraction of de Broglie waves using the theory of quantum mechanics, which was making
its first successes at that time. Currently, these studies have not lost their relevance both in
experimental research and in theoretical study. Most of the experimental measurements
of the scattering cross-sections refer to the region of intermediate scattering angles,
approximately from $10°$ to $150°$. At smaller and larger angles, the existing experimental
data are often qualitative assessments, and here their greatest deviation from theoretical
estimates is observed. In this regard, the implementation of accurate measurements in
the regions of small and large scattering angles is still a big challenge in the problem of
obtaining reliable experimental scattering cross-sections over the entire range of scattering
angles (Adibzadeh & Theodosiou 2005; Cho *et al.* 2008). Moreover, this problem is
especially relevant for heavier atoms of noble gases.

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Recently, in a number of experimental works devoted to the investigation of the properties of partially ionized plasma of noble gases, unexpected effects have been noted, associated, as it turned out later, with electron–atom interactions in the system. For example, in Kuhlbrodt *et al.* (2005) it was shown that the presence of the temperature-dependent minimum on the direct current conductivity of noble gases and this minimum has been explained by electron–atom collisions (see Adams *et al.* (2007) and Rosmej, Reinholz & Röpke (2017)). In Adams *et al.* (2007), on the basis of the experimentally obtained cross-sections for isolated electron–atomic scattering and data on the composition calculated by COMPTRA04, the electrical conductivity of the plasma of noble gases was estimated and good agreement with the experimental data was shown. Rosmej *et al.* (2017) studied the momentum-transfer cross-sections (MTCS) based on the developed optical potential over a wide range of parameters, thus avoiding already marked drawbacks of the scattering cross-sections obtained experimentally. Along with the study of electron scattering by an isolated atom, in Rosmej *et al.* (2017) the effect of the plasma environment was also taken into account, in particular, the screening of the interaction potentials that contribute to the optical potential. Thus, they managed to obtain better agreement with experimental data on electrical conductivity in the studied ranges of temperatures and densities. It is known that screening is able to cause the significant changes in the properties of the systems under study, such as Coulomb liquids (March & Tosi 2002; Hansen & McDonald 2013) or dipole systems (Aldakul, Moldabekov & Ramazanov 2020). When screening depends on the velocities of the interacting particles, it is considered as dynamic screening. As was shown in Kremp, Ropke & Schlanges (1984), the dynamic screening can be taken into account in the effective interaction potentials by replacing the static Debye length with the dynamic one, which increases with the growth in the impact energy. So, when the impact energy is significantly larger than the thermal energy of the system the screening effect can be strongly weakened. In dense semiclassical plasma, in addition to collective effects, the interaction of particles is also affected by the wave properties of these particles, leading to some quantum mechanical effects (see Deutsch (1977), Deutsch, Furutani & Gombert (1981) and Ramazanov, Amirov & Moldabekov (2018)). The study of the properties of dense semiclassical plasma with simultaneous consideration of the effects of screening and diffraction in the interaction of charged particles as well as in the polarization interaction of electron with an atom has been carried out in many works (see, for e.g. Jung (2003), Ramazanov, Dzhumagulova & Akbarov (2006*a*), Ramazanov, Dzhumagulova & Gabdullin (2006*b*), Shalenov *et al.* (2018*b*), Shalenov *et al.* (2019*b*), Shalenov *et al.* (2019*a*), Jumagulov *et al.* (2020) and Karmakar, Das & Ghoshal (2020)). It has been shown that in dense semiclassical plasma the diffraction effect arising out of the uncertainty principle can play a significant role.

It has been shown, that in dense semiclassical plasma there are the bound states within the temperature and density ranges 10^5–10^6 K and 10^{20}–10^{22} cm^{-3} (Ramazanov *et al.* 2006*b*; Rosmej *et al.* 2017; Karmakar *et al.* 2020). In this regard, it is important to reveal the influence of the diffraction effect on the collisional characteristics and then on the transport properties of such plasma. We will determine the composition of both plasma and transport properties in the next work, and in this paper we present a study of the influence of screening and diffraction effects on electron–atom scattering in dense plasma of noble gases. In order to investigate the scattering problems we constructed the optical potential of electron–atom interaction on the basis of the conclusions of Rosmej *et al.* (2017). This potential along with the screening effect also takes into account the diffraction effect. The calculations were carried out on the basis of the method of phase functions. The obtained results were compared with the theoretical data of other authors.

Moreover, to check the reliability of the chosen method of phase functions, we performed calculations of the scattering phase shifts on the basis of the optical potential proposed in Rosmej *et al.* (2017), where calculations had been done by the Numerov method. An agreement between results of both works is shown.

This article is organized as follows. In §2, we describe the research method for the scattering problems. Optical potential for electron–atom interaction in dense plasma, highlighting the taking into account of screening and diffraction effects, is presented in §3. Results, including a comparison with data of Rosmej *et al.* (2017), are shown in §4.

2. Phase function method

In the method of partial waves the incident particle flux is represented as a set of partial waves, which are scattered on a spherically symmetric force centre independently of each other. The contribution of each partial wave to the scattering amplitude (and hence to the differential and full scattering cross-sections) is determined by the scattering phase, the so-called phase shift δ_l.

The scattering phase shifts can be determined from solving the radial Schrödinger equation for a particle scattered by some central field $V(r)$. In Rosmej *et al.* (2017), the radial Schrödinger equation was solved with the Numerov method.

In this work we used another way to calculate scattering phase shifts of an incident electron. In the method of phase functions, one can pass from the Schrödinger equation to the equation directly for the scattering phases (Drukarev 1965; Babikov 1988),

$$
\begin{cases}
\dfrac{\mathrm{d}\delta_l(r)}{\mathrm{d}r} = -\dfrac{1}{k}\dfrac{2m}{\hbar^2}V(r)\big[\cos\delta_l(r)\cdot j_l(kr) - \sin\delta_l(r)\cdot n_l(kr)\big]^2, \\
\delta_l(0) = 0,
\end{cases}
\tag{2.1}
$$

where $\delta_l(r)$ is the phase function, $V(r)$ is the interaction potential, k is the wavenumber of the incident electron, $j_l(kr)$ and $n_l(kr)$ are regular and irregular solutions of the Schrödinger equation. The phase shift is the asymptotical value of the phase function at large distances,

$$
\delta_l = \lim_{r\to\infty}\delta_l(r).
\tag{2.2}
$$

Note that the equation for the scattering phases has some advantages: it is much easier to solve this one than the Schrödinger equation; and the number of operations, and consequently the computation time, is also noticeably reduced.

Partial cross-sections for scattering in the quantum mechanical approximation are calculated on the basis of the phase shifts,

$$
Q_l^P(k) = \frac{4\pi}{k^2}(2l+1)\sin^2\delta_l.
\tag{2.3}
$$

Full cross-section is defined as the sum of the partial ones, i.e.

$$
Q^F(k) = \sum_{l=0}^{n} Q_l^P(k).
\tag{2.4}
$$

Momentum-transfer cross-sections, used for calculation of the transport coefficients, are also determined by the phase shifts. Thus, the transport cross-section of the first order was calculated by the following formula:

$$
Q^T(k) = \frac{4\pi}{k^2}\sum_{l=0}^{\infty}(l+1)\sin^2(\delta_{l+1} - \delta_l).
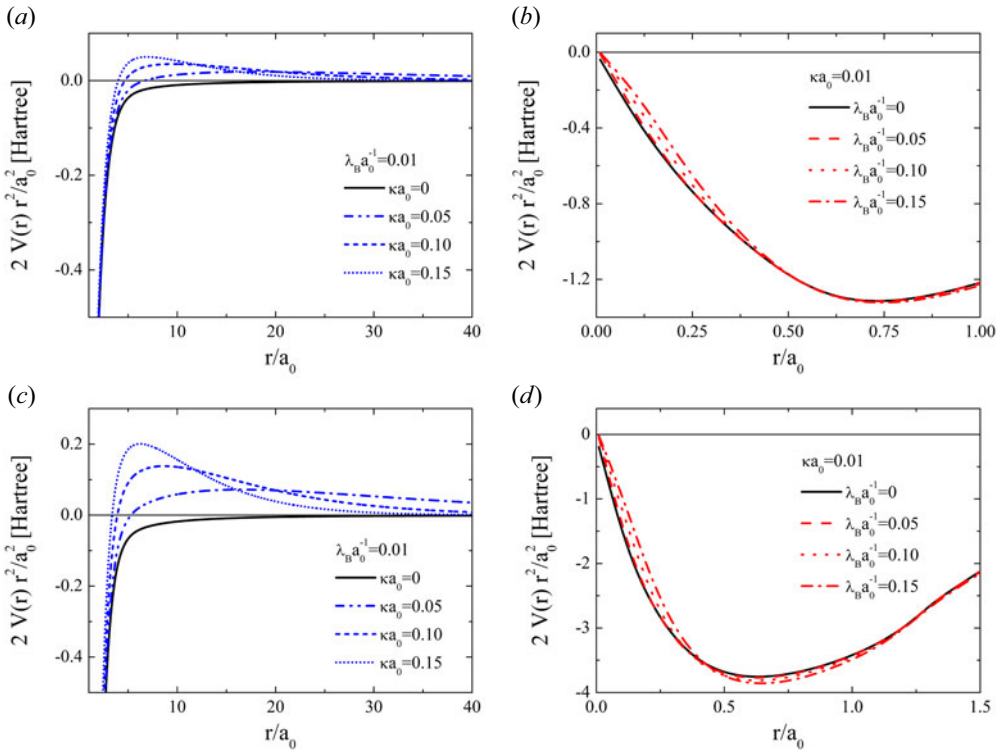\tag{2.5}
$$

FIGURE 1. Effective optical potential for electron–helium (a,b) and electron–neon (c,d) interaction at zero wavenumber $(k = 0)$ for different screening parameters κa_0 (a,c) or different de Broglie wavelengths $\lambda_B a_0^{-1}$ (b,d).

3. Effective electron–atom interaction potential in dense semiclassical plasma

3.1. *Optical potential for isolated systems*

In Rosmej *et al.* (2017), in order to determine the electron–atom MTCS the following optical potential was proposed and described in detail as the interaction potential of the isolated system:

$$V_{\text{opt}}(r) = V_{\text{HF}}(r) + V_P(r) + V_{\text{ex}}(r). \tag{3.1}$$

The first term in this potential is called the Hartree–Fock potential $V_{\text{HF}}(r)$, it describes the Coulomb interaction of the free electron with the nucleus as well as with electrons of the atomic shells,

$$V_{\text{HF}}(r) = \frac{e^2}{4\pi\varepsilon_0}\left[-\frac{Z}{r} + \int \frac{1}{|\mathbf{r}-\mathbf{r_1}|}\rho(r_1)\,\mathrm{d}^3\mathbf{r_1}\right], \tag{3.2}$$

where Z is the charge number of the nucleus and $\rho(r)$ is the density of the shell electrons.

The second term $V_P(r)$ describes the interaction of the free electron with the polarized atom, which prevails at large distances,

$$V_P(r) = -\frac{e^2\alpha_P}{8\pi\varepsilon_0(r+r_0)^4}, \tag{3.3}$$

	Helium				Neon			
	ka_0	(1)	(2)	(3)	ka_0	(1)	(2)	(3)
δ_0	0.10	2.994	2.993	2.993	0.20	6.072	6.104	6.097
	0.25	2.776	2.770	2.770	0.30	5.965	6.004	5.994
	0.50	2.436	2.412	2.410	0.40	5.857	5.899	5.885
	0.75	2.139	2.093	2.092	0.50	5.748	5.789	5.773
	1.00	1.890	1.835	1.835	0.90	5.321	5.349	5.327
	1.50	1.522	1.473	1.474	1.00	5.219	5.243	5.220
δ_1	0.50	0.043	0.045	0.048	0.50	3.040	3.052	3.044
	0.75	0.110	0.101	0.106	0.70	2.933	2.949	2.936
	1.00	0.183	0.159	0.165	0.80	2.873	2.890	2.875
	1.50	0.284	0.247	0.253	0.90	2.812	2.828	2.813
	—	—	—	—	1.00	2.751	2.766	2.750
δ_2	0.75	0.005	0.006	0.007	0.50	0.004	0.005	0.005
	1.00	0.014	0.015	0.017	1.00	0.065	0.059	0.057
	1.50	0.042	0.041	0.044	—	—	—	—

TABLE 1. Electron–helium, electron–neon partial wave phase shifts δ_l (in rad) for calculated without taking into consideration the polarization term in the optical potential, (3.1). (1) Numerical results in SEA, Duxler *et al.* (1971) and Pindzola & Kelly (1974); (2) Rosmej *et al.* (2017); and (3) present work.

where α_P is the dipole polarizability, r_0 is the cutoff parameter. Its values for different atoms of noble gases are taken as the same as in Rosmej *et al.* (2017). The introduction of the exchange term into the optical potential as a separate term is possible within the framework of a local field approximation. The replacement of the exchange kernels by an equivalent local potential makes the scattering problems much easier to solve numerically. A large number of local exchange potentials have been proposed in the literature. Systematic comparisons of some of them have been made by Yau, McEachran & Stauffer (1978). In the free-electron–gas exchange approximation, Mittleman & Watson (1960) derived the local exchange potential,

$$V_{\text{ex}}(r) = V_{\text{ex}}^{M}[r, K(r)] = -\frac{e^2}{4\pi\varepsilon_0}\frac{2}{\pi}K_F(r)\,F\left[\frac{K(r)}{K_F(r)}\right], \qquad (3.4)$$

with the Fermi momentum $K_F(r) = [3\pi^2\rho(r)]^{1/3}$, function $F(\eta) = \frac{1}{2} + ((1-\eta^2)/4\eta) \ln|(\eta+1)/(\eta-1)|$, and a local-electron momentum $K(r)$, which is here taken in the version of Rosmej *et al.* (2017),

$$K_{\text{RRR}}^2(r) = k^2 + \frac{2m}{\hbar^2}\left[|V_{\text{HF}}(r)| + |V_P(r)| + \left|V_{\text{ex}}^{M}(r,0)\right|\right], \qquad (3.5)$$

where $V_{ex}^{M}(r,0) = -(e^2/4\pi\varepsilon_0)(2/\pi)K_F(r)$ is the momentum-free exchange term. Rosmej *et al.* (2017) have shown that the calculations based on (3.1) with exchange potential, (3.4), (3.5), are in better agreement with the results obtained on the basis of the static-exchange approximation (SEA), using a non-local exchange term (Duxler, Poe & Labahn 1971; Pindzola & Kelly 1974), than calculations based on other approximations for $K(r)$ (Hara 1967; Riley & Truhlar 1975).
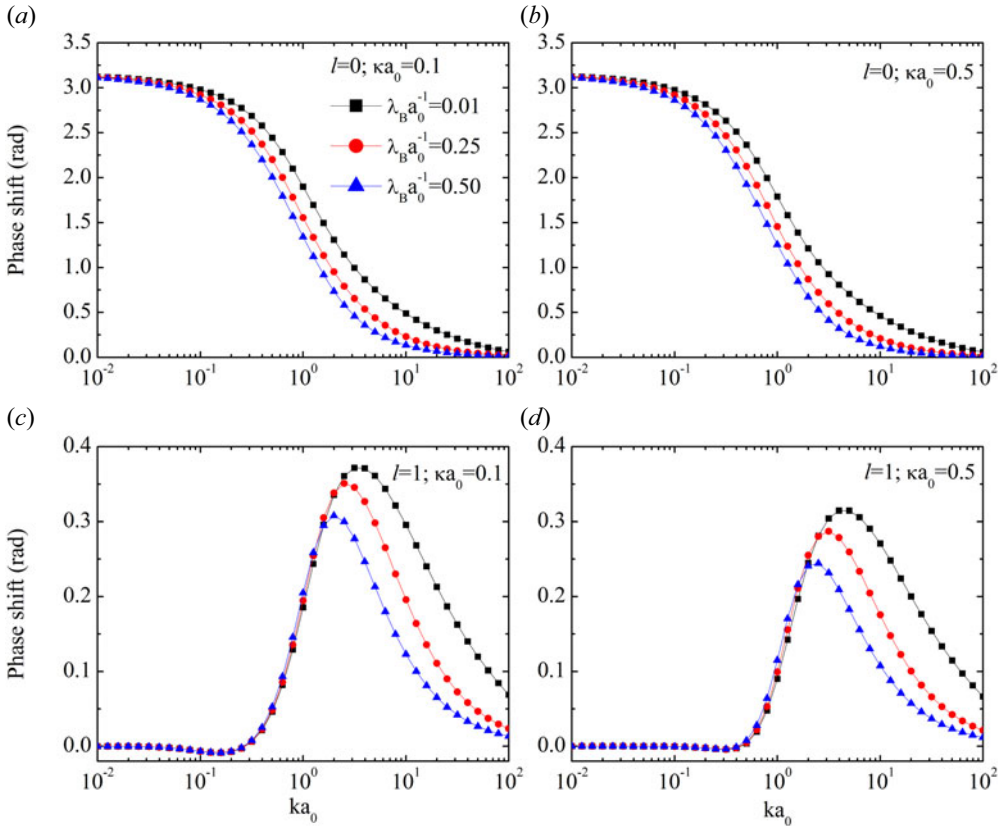
FIGURE 2. Phase shifts of the electron scattering on the helium atom for $l = 0$ (a,b) and $l = 1$ (c,d) with two values of the screening parameter κa_0 at different de Broglie wavelengths $\lambda_B a_0^{-1}$.

3.2. *Screening and diffraction effects*

It is well known that in dense non-ideal plasma the interaction of isolated particles is no longer relevant. At high densities, the average distance between particles becomes less than the Debye length,

$$r_D = \left(k_B T \left/ \left(4\pi e^2 \sum_a n_a Z_a^2 \right) \right. \right)^{1/2}, \tag{3.6}$$

where Z_a is the charge number of the particles of plasma component $a = e, i$. In this case, charged particles participate in collective interaction, which leads to screening of the field of individual particles. Rosmej *et al.* (2017) quite rightly have taken into account the screening effect of the plasma on each term of the optical potential, (3.1), and construct the screened optical potential,

$$V_{\text{opt}}^s (r) = V_{\text{HF}}^s (r) + V_P^s (r) + V_{\text{ex}}^s (r). \tag{3.7}$$

Here, the screened polarization potential has the following form:

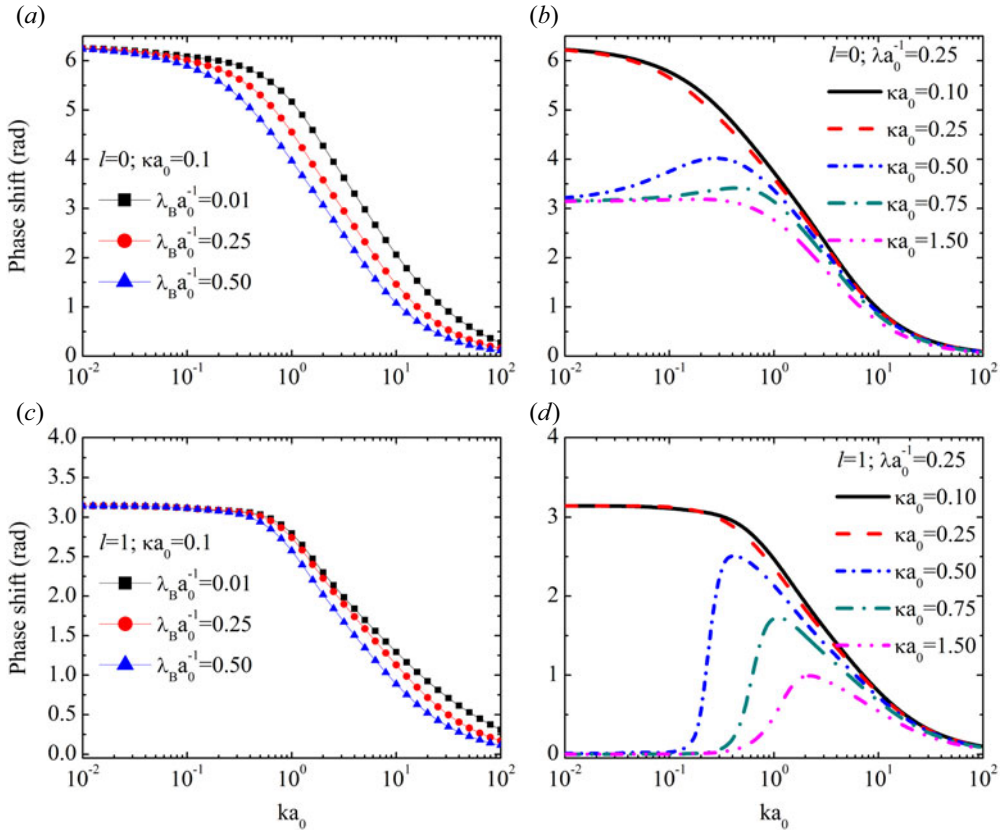$$V_P^s (r) = V_P (r) e^{-2\kappa r} (1 + \kappa r)^2, \tag{3.8}$$

FIGURE 3. Phase shifts of the electron scattering on the neon atom for $l = 0$ (*a,b*) and $l = 1$ (*c,d*) with screening parameter $\kappa a_0 = 0.1$ at different de Broglie wavelengths (*a,c*) and with de Broglie wavelength $\lambda_B a_0^{-1} = 0.25$ at different screening parameters (*b,d*).

where $\kappa = r_D^{-1}$ is the screening parameter. For the screened Hartree–Fock potential, the Coulomb interaction in (3.2) for an isolated system can be replaced by the Debye potential, and then the following expression is obtained:

$$V_{\mathrm{HF}}^{s}(r) = \frac{e^2}{4\pi\varepsilon_0}\left[-\frac{Ze^{-\kappa r}}{r} + I_1 + I_2 + I_3\right], \tag{3.9}$$

where

$$I_1 = \frac{e^{-\kappa r}}{\kappa r}\int_0^r 2\pi r_1\rho(r_1)\,e^{\kappa r_1}\,\mathrm{d}r_1, \tag{3.10}$$

$$I_2 = -\frac{e^{-\kappa r}}{\kappa r}\int_0^\infty 2\pi r_1\rho(r_1)\,e^{-\kappa r_1}\,\mathrm{d}r_1, \tag{3.11}$$

$$I_3 = \frac{e^{\kappa r}}{\kappa r}\int_r^\infty 2\pi r_1\rho(r_1)\,e^{-\kappa r_1}\,\mathrm{d}r_1. \tag{3.12}$$

To obtain the screened exchange term in (3.7), polarization and the Hartree–Fock terms in (3.5) were replaced by their screened versions, (3.8) and (3.9), respectively. On the
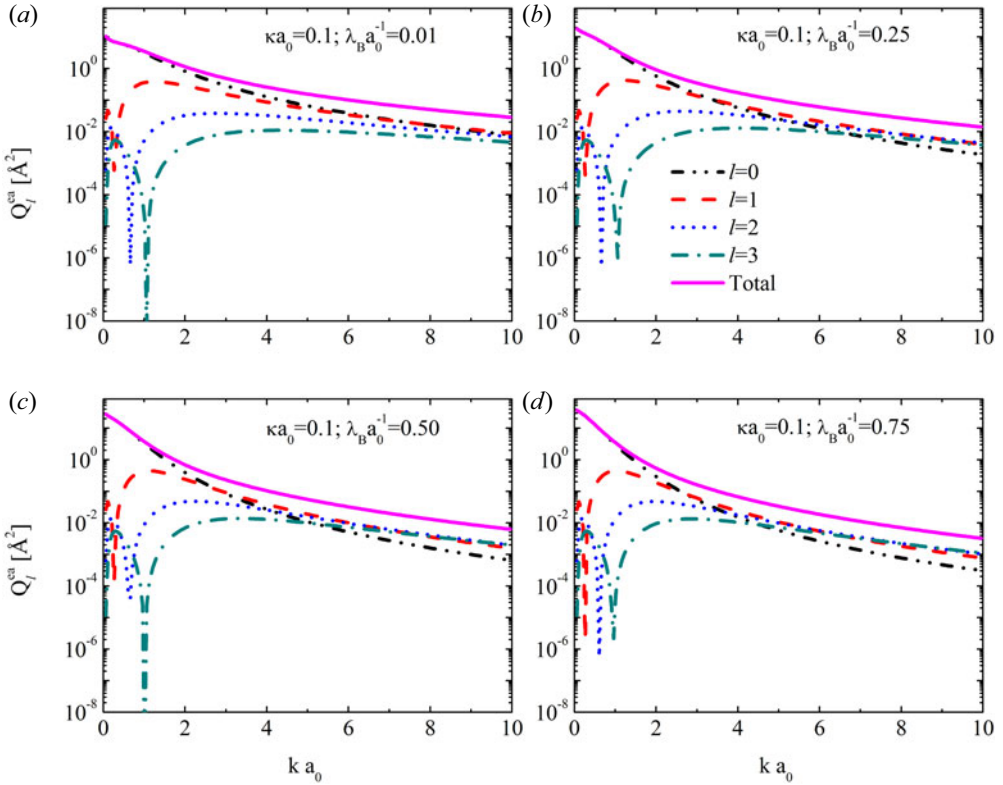
FIGURE 4. The PCS and FCS for electron scattering on the helium atom with screening parameter $\kappa a_0 = 0.1$ at different de Broglie wavelengths $\lambda_B a_0^{-1} = 0.01$ (a), $\lambda_B a_0^{-1} = 0.25$ (b), $\lambda_B a_0^{-1} = 0.50$ (c), $\lambda_B a_0^{-1} = 0.75$ (d).

basis of optical potential, (3.7), the influence of the screening effect on the collisional characteristics of particles in dense plasma of noble gases was investigated in Rosmej *et al.* (2017). However, in a dense plasma, the average interparticle distance can be also comparable with the de Broglie wavelength of the particles. So, it is necessary to take into account the quantum mechanical effects, such as diffraction and symmetry, along with screening effects, for the correct description of the properties of such plasma. But, it is worth mentioning that the effect of symmetry can be neglected at high temperatures and one can only consider the diffraction and screening effects. In Ramazanov & Dzhumagulova (2002) the effective potential, describing interaction of the charge carriers and taking into account both effects of screening and diffraction, was presented. For the interaction of the electron and the charged particle of sort α, it can be written as

$$V_{e\alpha}^{SD}(r) = -\frac{Z_\alpha e^2}{4\pi\varepsilon_0 C}\left(\frac{e^{-Br}}{r} - \frac{e^{-Ar}}{r}\right),\tag{3.13}$$

where $C^2 = 1 - 4\lambda_B^2\kappa^2$; $A^2 = 1/2\lambda_B^2(1 + C)$; $B^2 = 1/2\lambda_B^2(1 - C)$; and where $\lambda_B = \hbar/\sqrt{2\pi m_e k_B T}$ is the thermal de Broglie wavelength of electrons. We substituted this potential into (3.2) instead of the Coulomb interaction and obtained the effective
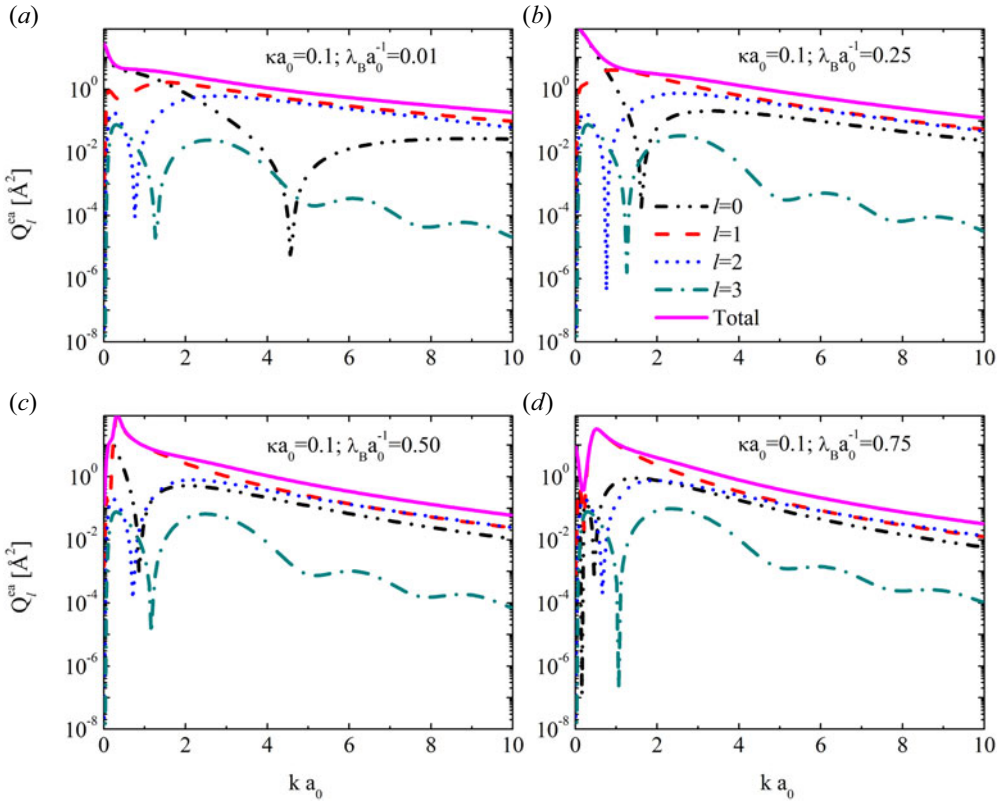
FIGURE 5. The PCS and FCS for electron scattering on the neon atom with screening parameter $\kappa a_0 = 0.1$ at different de Broglie wavelengths $\lambda_B a_0^{-1} = 0.01$ (figure 4a), $\lambda_B a_0^{-1} = 0.25$ (figure 4b), $\lambda_B a_0^{-1} = 0.50$ (figure 4c), $\lambda_B a_0^{-1} = 0.75$ (figure 4d).

Hartree–Fock potential,

$$V_{\mathrm{HF}}^{\mathrm{SD}}(r) = \frac{e^2}{4\pi\varepsilon_0\,C}\left[-\frac{Z\left(e^{-Ar} - e^{-Br}\right)}{r} + \xi_1 + \xi_2 + \xi_3\right], \tag{3.14}$$

where

$$\xi_1 = \frac{e^{-Ar}}{Ar}\int_0^r 2\pi r_1\rho\left(r_1\right)e^{Ar_1}\,\mathrm{d}r_1 - \frac{e^{-Br}}{Br}\int_0^r 2\pi r_1\rho\left(r_1\right)e^{Br_1}\,\mathrm{d}r_1, \tag{3.15}$$

$$\xi_2 = -\frac{e^{-Ar}}{Ar}\int_0^\infty 2\pi r_1\rho\left(r_1\right)e^{-Ar_1}\,\mathrm{d}r_1 + \frac{e^{-Br}}{Br}\int_0^\infty 2\pi r_1\rho\left(r_1\right)e^{-Br_1}\,\mathrm{d}r_1, \tag{3.16}$$

$$\xi_3 = \frac{e^{-Ar}}{Ar}\int_r^\infty 2\pi r_1\rho\left(r_1\right)e^{-Ar_1}\,\mathrm{d}r_1 - \frac{e^{-Br}}{Br}\int_r^\infty 2\pi r_1\rho\left(r_1\right)e^{-Br_1}\,\mathrm{d}r. \tag{3.17}$$

The polarization potential of the interaction between an electron and an atom, taking into account the static Debye screening and the diffraction effect, was obtained in Ramazanov,

Dzhumagulova & Omarbakiyeva (2005), Shalenov *et al.* (2018*a*) and Dzhumagulova *et al.* (2022) as

$$V_P^{\mathrm{SD}}(r) = -\frac{e^2 \alpha_p}{8\pi\varepsilon_0 (r+r_0)^4 C^2} \left(e^{-Br}(1+Br) - e^{-Ar}(1+Ar) \right)^2, \qquad (3.18)$$

where A, B and C are the same quantities as in (3.13). The effective term of the exchange interaction was obtained by the already described method, when in (3.5) the Hartree–Fock and polarization contributions change to their versions, taking into account the effect of diffraction and screening, (3.14) and (3.18), respectively. Thus, after substituting all effective contributions, we have the following effective optical potential, which takes into account, along with the screening effect, the quantum mechanical effect of diffraction:

$$V_{\mathrm{opt}}^{\mathrm{SD}}(r) = V_{\mathrm{HF}}^{\mathrm{SD}}(r) + V_P^{\mathrm{SD}}(r) + V_{\mathrm{ex}}^{\mathrm{SD}}(r). \qquad (3.19)$$

It should be noted that optical potential, (3.19), transforms into potential, (3.7), at $\lambda_B \to 0$. Furthermore, if one neglects the effects of both screening and diffraction ($\kappa \to 0$ and $\lambda_B \to 0$), the optical potential (3.19) reduces to the potential of the isolated system, (3.1). Figure 1 shows the effective optical potentials, (3.19), for electron–helium (figures 1*a* and 1*b*) and electron–neon (figures 1*c* and 1*d*) interactions. The role of the screening effect in the appearance of repulsion at large distances (figures 1*a* and 1*c*) was already noted in Rosmej *et al.* (2017). At short range, screening effects are not relevant, while the diffraction effect leads to the weakening of the attraction between electron and atom (figures 1*b* and 1*d*).

4. Results

First of all, we tested the method of phase functions using the optical potential, (3.1), for agreement with the results of Rosmej *et al.* (2017). For this purpose, the phase shifts for partial waves were calculated and presented in table 1. Here, the phase shifts obtained in Rosmej *et al.* (2017), in this work and the data calculated on the basis of a SEA (see Duxler *et al.* (1971) and Pindzola & Kelly (1974)) were obtained using a non-local exchange term without taking into consideration the polarization potential. High agreement with the data of Rosmej *et al.* (2017) made it possible to conclude that the use of the chosen method of phase functions has a minimal effect on the final results.

All further presented results were obtained on the basis of the optical potential, (3.19), in order to study the simultaneous influence of the effects of screening and diffraction on the collisional characteristics in dense plasma of noble gases using the example of helium and neon. We consider the influence of the de Broglie wavelength on the results obtained; an increase in the latter is equivalent to an increase in the influence of the diffraction effect.

Figure 2 shows the phase shifts for the partial waves $l = 0$ (figures 2*a* and 2*b*) and $l = 1$ (figures 2*c* and 2*d*) arising from the collision of the electron with a helium atom. As well known, an increase in screening leads to a decrease in phase shifts. This trend can be especially seen in figure 2. The influence of the diffraction effect can be observed on all figures 2(*a*–*d*): with increasing λ_B phase shifts decrease. A similar picture of the behaviour of phase shifts in the collision of the electron with a neon atom can be observed in figure 3 with the only difference that figures 3(*b*) and 3(*d*) show phase shifts, calculated at a fixed value of the de Broglie wavelength $\lambda_B a_0^{-1} = 0.25$ and different values of the screening parameter. They illustrate the fact that rising screening leads to the fall in the number of bound states following from Levinson's theorem (Wellner 1964; Sassoli De Bianchi 1994; Shalenov, Dzhumagulova & Ramazanov 2017).
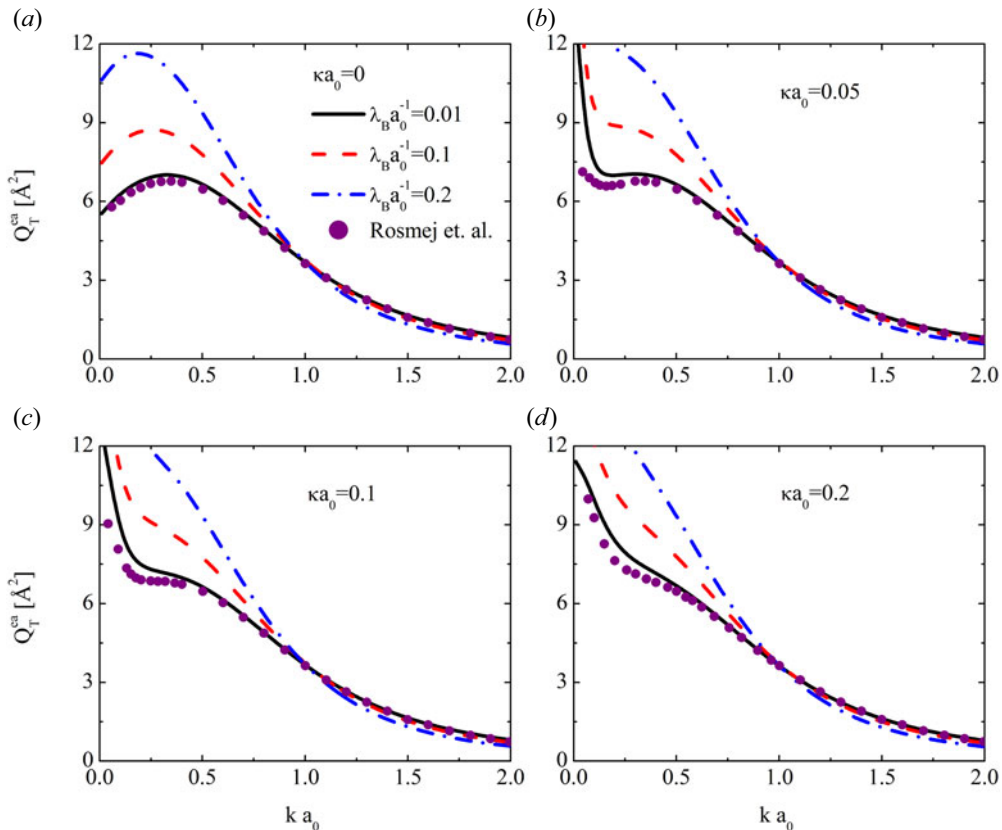
FIGURE 6. The MTCS for electron scattering on the helium atom for four values of screening parameters $\kappa a_0 = 0$ (figure 4a), $\kappa a_0 = 0.05$ (figure 4b), $\kappa a_0 = 0.1$ (figure 4c), $\kappa a_0 = 0.2$ (figure 4d) at different de Broglie wavelengths.

Full and partial collision cross-sections (FCS and PCS) for electron–helium and electron–neon scattering as a function of incident energy for different values of de Broglie wavelength are presented in figures 4 and 5, respectively. For electron–helium scattering, we notice that FCS is peaked when $ka_0 \rightarrow 0$ for a given screening parameter, the height of the peak increases with rise of λ_B. With increasing incident energy, FCS and PCS fall. Furthermore, FCS decreases more rapidly with growing λ_B. In the case of electron–helium scattering, besides mentioned the above feature we can notice that in PCS for $l = 0$ the minimum's position is gradually shifted to lower energies.

In figures 6 and 7 another important scattering quantity of interest, the MTCS, is presented for helium and neon cases, respectively. It can be seen that an increase in the screening parameter, as well as in the de Broglie length, leads to an increase in MTCS at low energies. For such behaviour the diffraction effect is responsible. It is also shown that with a decrease in de Broglie wavelength, the results tend to the data obtained in Rosmej *et al.* (2017), which do not take into account the influence of the diffraction effect.

5. Conclusion

Based on the optical potential, which takes into account the screening effect of the plasma environment in dense plasma of noble gases, we have constructed a new version
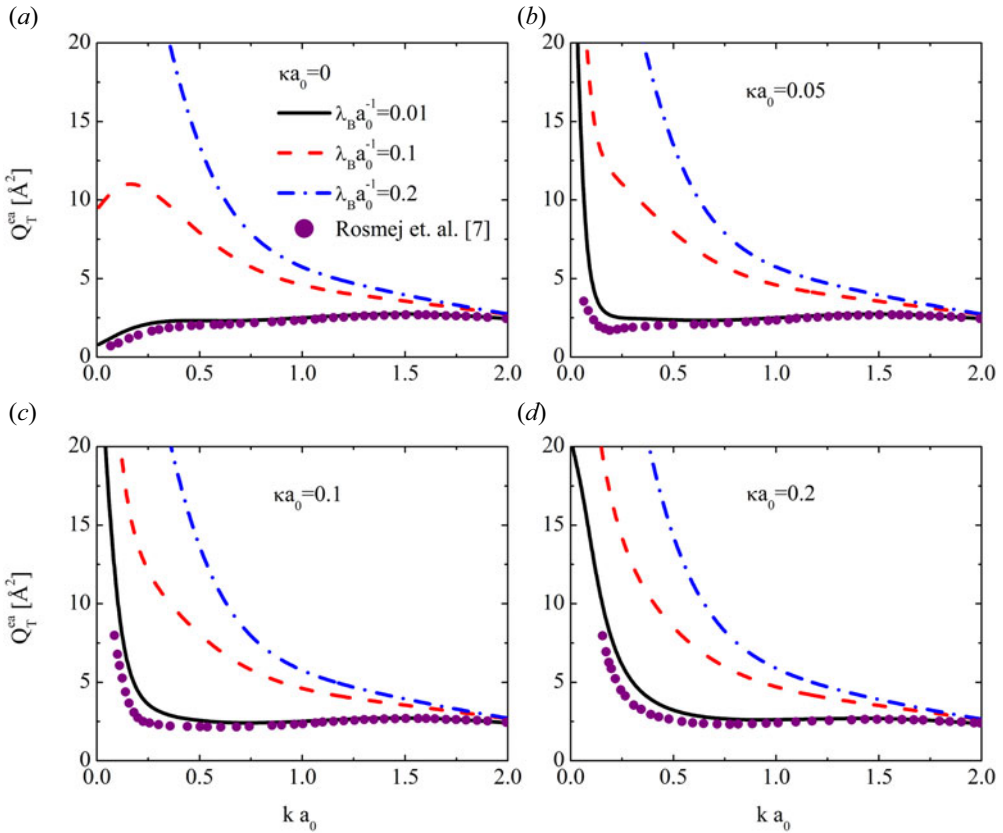
FIGURE 7. The MTCS for electron scattering on the neon atom for four values of screening parameters $\kappa a_0 = 0$ (figure 4a), $\kappa a_0 = 0.05$ (figure 4b), $\kappa a_0 = 0.1$ (figure 4c), $\kappa a_0 = 0.2$ (figure 4d) at different de Broglie wavelengths.

that also takes into account the quantum mechanical effect of diffraction. For this purpose, as the polarization potential of the electron–atom interaction, as well as to calculate the Hartree–Fock potential, we used the potentials obtained by us earlier, and took into account both of these effects. The exchange potential was presented in the approximation of an effective local field. Within the framework of the presented model of the optical potential, the problems of electron–atom collisions were investigated using the example of helium and neon atoms. It is shown that with an increase in the de Broglie wavelength, a decrease in the phase shifts of scattering is observed at any values of the screening parameter. On the contrary, the MTCS increases with increasing de Broglie wavelength at low velocities. As the velocity increases, the screening and the quantum mechanical effect of diffraction cease to affect the MTCS. The results are shown to be in good agreement with the data of S. Rosmej in the limit when the de Broglie wavelength tends to zero.

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Declaration of interests

The authors report no conflict of interest.

Data availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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