STATISTICS APPLIED TO THE ESTIMATION OF THE INFLUENCE OF THE ENVIRONMENT ON RESULTS OF OBSERVATIONS

M.K. Szacherska and Z. Wiśniewski
Institute for Geodesy and Photogrammetry,
Agricultural and Technical Academy at Olsztyn
Poland

ABSTRACT

The assumption of a generalized model of the composition of random and systematic errors of measurements can constitute a basis for the estimation of the influence of the environment on the results of geodetic observations in the course of their numerical elaboration. The method of the estimation of systematic errors on the basis of the analysis of an empiric distribution of the results of observations has been established on the assumption of such a model, which permits also to estimate systematic errors in the course of the adjustment of observations, these errors being treated as additional unknowns in the system.

1. INTRODUCTION

Statistical methods are nowadays frequently used for the analysis of the precision of geodetic measurements and the estimation of random and systematic observation disturbances of different kind. The possibility of applying mathematical statistics to the estimation of the influence of the environment and atmospheric conditions on the results of observations is widely discussed (Tengström, E., 1975), (Proceedings, 1977). Investigations deal with the application of statistics to the elaboration of mathematical models describing the character and the development of the physical phenomena as well as their influence on the results of geodetic observations. In other researches statistical methods are utilized for the numerical elaboration of the results of observations and simultaneous estimation of systematic disturbances caused by the influence of the environment.

As basis for the numerical elaboration of observation results we can adopt a generalized model of the composition of random and systematic errors of measurements (Szacherska, M.K., 1974), (Szacherska, M.K., 1977). Let us assume, that a set of errors $\{Z\}_C$ has been obtined as the result of a number of measurements N. We can use then as theoretical model of the composition of errors (Szacherska, M.K., 1977) the general formula

E. Tengström and G. Teleki (eds.), Refractional Influences in Astrometry and Geodesy, 179–189.

$$(z_c)_i = \sum_{l=1}^{n_1} (z_r)_{l,k} + \sum_{j=1}^{n_2} (z_s)_{j,i} \quad , \tag{1.1}$$

for $i = 1, 2 \ldots N$,

where
- k - is the random variable with uniform distribution (the interval being $0 < k \leq N$),
- n_1 - denotes the number of random disturbances,
- n_2 - the number of systematic disturbances,
- n - is the number of all the disturbances of the analysed measurements $(n = n_1 + n_2)$.

In formula (1.1) the symbol

$$(z_r)_l \in R_l(a_l, s_l) \ , \qquad l = 1, 2 \ldots n_1$$

denotes random errors characterized by the probability distribution $R_l(a_l, s_l)$ with the expected values a_l and the standard deviations s_l . If we assume, that the random components of the mixture are of elementary character and their number $n_1 \to \infty$, the first sum in formula (1.1) can be replaced by $(z_G)_k$, $0 < k \leq N$, that is a random error with the Gaussian distribution.

The systematic errors are expressed by the formula

$$(z_s)_{j,i} = \phi_j(t_i) \ , \qquad \begin{array}{l} j = 1, 2 \ldots n_2 \\ i = 1, 2 \ldots N \ , \end{array} \tag{1.2}$$

where
- t - denotes any measurement parameter,
- $\phi(t)$ - the relation between alterations of this parameter and the value of the occuring systematic errors.

The generalized model of the composition of errors - briefly recapitulated here - has been justified and discussed in previous publications (Szacherska, M.K., 1973), (Szacherska, M.K., 1974), particularly in a guest lecture (1977) at the Geodetic Institute of Uppsala University. These publications deal also with the method of estimating the precision on the basis of an analysis of the empiric distribution of observations, founded on the assumption of a generalized model of the composition of errors. This assumption is also the basis for the method of the estimation of systematic errors in the course of the adjustment of observations (Szacherska, M.K., Wiśniewski, Z., 1977).

2. ADJUSTMENT OF OBSERVATIONS WITH ESTIMATION OF SYSTEMATIC ERRORS

The traditional methods of adjustment are based on the assumption that systematic errors have been previously eliminated. Practice shows however, that this assumption is not always realised, particularly when disturbances caused by the influence of the environment and atmospheric conditions have occured. The method of the adjustment of observations with simultaneous estimation of the influence of systematic disturbances is founded on the assumption of a generalized model of the composition of errors described by formula (1.1).

If we assume, that observations u_i of a certain parameter of the geodetic network have been performed, we can express the corresponding true value by the equation

$$U = u_i + (z_G)_i + \sum_{j=1}^{n_2} \phi_j(t_i) \ , \qquad i = 1, 2 \ldots N \ . \tag{2.1}$$

Equation (2.1) shows, that the results of observations u_i and their environment described by the functions $\phi_j(t_i)$ are taken into consideration when determining the most probable value of a parameter. We can adopt the principle described by equation (2.1) for the adjustment of a geodetic network, thus making use of the geometric conditions of the network and the physical conditions of the observations.

With regard to the chosen method of adjustment, formula (2.1) can be utilized in order to establish a system of observation equations or conditional equations. The principle of establishing observation equations can be expressed by the formula

$$U_h = u_{h,i_h} + (z_G)_{h,i_h} + \sum_{j=1}^{n_{2_h}} \phi_{h_j}(t_{i_h}) = f_h(x_1, x_2 \ldots x_p) \ , \tag{2.2}$$

$$i_h = 1, 2 \ldots N_h$$

$$h = 1, 2 \ldots M$$

where M - denotes the number of observed parameters,

U_h, $u_{h,i_h} \in \{u\}_h$, N_h - the true value, an observation result and the number of measurements of one of the observed parameters,

$\{x\}$, p - the set of determined parameters and their number.

The number of observation equations, which can be established in conformity with the proposed principle is

https://doi.org/10.1017/S0074180900065980 Published online by Cambridge University Press

$$r_o = \sum_{h=1}^{M} N_h$$

Systematic components of the mixture of errors are additional unknowns in the system of equations; the total number of necessary observations will be consequently determined by the sum $(p + q)$. The symbol q denotes the number of additional unknowns, which depends from the type of the observations and the kind of disturbances.

Applying formula (2.1) we can establish also a system of conditional equations, the construction of which is described by the relation

$$F_d[u_{1,i_1} + (z_G)_{1,i_1} + \sum_{j=1}^{n_{2_1}} \phi_{1_j}(t_{i_1}) \; , \; \ldots \; , \; u_{M,i_M} + (z_G)_{M,i_M} + \\ + \sum_{j=1}^{n_{2_M}} \phi_{M_j}(t_{i_M})] = c_d \tag{2.3}$$

$$d = 1, 2 \ldots r_c \; .$$

The true value of a parameter U_h can be defined on the basis of a chosen observation of this parameter $u_{h,i_h} \in \{u\}_h$. Taking into account the geometric scheme of the network and different combinations of observations $u_{h,i_h} \in \{u\}_h$ in an equation of the type (2.3), we can form a system of conditional equations, the number of which

$$r_c = \sum_{h=1}^{M} N_h - (p + q)$$

will depend from the number of all the observations and the number of fundamental and additional unknowns of the system.

The proposed scheme of the adjustment of observations allows the joined determination of the parameters of the network and of the systematic errors treated as additional unknowns in the system. A fundamental problem is the correct determination of the character of disturbances of the measurement process and the choice of the function $\phi_j(t_i)$.

The choice of the function $\phi_j(t_i)$ ought to be based on a thorough knowledge of the measurement process and of the disturbances of this process. Informations can be obtained also by means of an analysis of the empiric distribution of observation errors (Szacherska, M.K., 1973), (Szacherska, M.K., 1977). The computations are generally carried out in several variants with different assumptions of the function $\phi_j(t_i)$, that is for different models of the disturbances of the measurement process. The conformity of the distribution of corrections after the adjustment with the theoretical distribution is decisive for the choice of the optimal variant of the solution.

3. APPLICATION OF THE METHOD TO THE ESTIMATION OF THE INFLUENCE OF THE ENVIRONMENT ON THE RESULTS OF OBSERVATIONS

The method of estimating systematic errors in the course of the adjustment of the network is completed by the algorithm of the computations and an Algol-programme. With their help control tests have been made at the Institute for Geodesy and Photogrammetry of the Agricultural and Technical Academy at Olsztyn. The analysis of the precision levelling network of the Silesian Coal-Basin as well as of the trigonometric levelling network in the region of Cracow shows the considerable influence exerted by the environment and the atmospheric conditions on the results of the observations.

At the elaboration of the precision levelling network of the Silesian Coal-Basin (1519 sections forming 79 lines of I and II order levelling) a model of the composition of errors has been applied, in which the characteristic errors of observers (6 observers), errors connected with the type of the instruments (Wild N 3, Opton Ni 1) and the influence of the refraction were treated as systematic components. The separation of the systematic errors contributed to increase the real precision of the determinations, proved by a reduction of the mean square error amounting to 30% of its value (Szacherska, M.K., Wiśniewski, Z., 1977).

Another interesting example is furnished by the analysis of the triangulation network situated in the submontane region east of Cracow. We have chosen for the analysis a section of the network consisting of 110 triangles, for all sides of which reciprocal observations of the vertical angles have been performed. The measurements have been carried out in 1973. The vertical angles have been measured in three series by seven observers using a Wild T 3 theodolite, the length of the sides - with a tellurometre CA 100.

Since in the chosen section of the network we had for all the sides of the triangles the results of reciprocal measurements of the vertical angles, we could in the analysis make use of conditions furnished by the differences of the results of reciprocal measurements of the altitude difference and the misclosure of altitude in triangles of the network. In the analysis we utilized all the informations concerning the conditions of the measurements noted in the reports of the observers, that is date and time, weather and temperature.

According to theory (Levallois, J.J., 1969), refraction and deflection of the vertical exert an influence on the results of measurements of the vertical angles. Examples of the determination of adequate values in the course of the adjustment of observations have been discussed in literature (Hradilek, L., 1972), (Makowska, A., Zorski, Z., 1976). The precision of any measurements depends of course also from the characteristic features of the observer and the qualities of the instrument.

Considering the principles of the determination of altitude differences by the method of trigonometric levelling, we ought to include in the composition of errors, apart from the random component, systematic errors connected with the influence of changeable atmospheric conditions, the position of the stations, as well as errors of the observer and the instrument. In conformity with formula (1.1) we can express

$$z_{c_i} = z_{G_i} + \phi_1(\{A\}_i) + \phi_2(\{P\}_i) + z_r, \tag{3.1}$$

where $\{A\}_i$ and $\{P\}_i$ - denote sets of informations concerning the measurement conditions and the position of the stations for the i-observation,. z_r is the error of the r-observer.

We have assumed such a general model of the composition of errors for the analysis of the discussed triangulation network. According to the adopted scheme of elaboration (Szacherska, M.K., Wiśniewski, Z., 1977) the fundamental task consisted in the choice of the functions $\phi_1(\{A\}_i)$ and $\phi_2(\{P\}_i)$.

When establishing the function $\phi_2(\{P\}_i)$ we have used well known relations (Levallois, J.J., 1969). The analysis of the influence of the environment has been made in several variants based on different assumptions of the function $\phi_1(\{A\}_i)$. The choice of the variant was limited by the informations contained in the set $\{A\}_i$, for we have not utilized specially planned experimental observations, only the results of typical field works.

We have carried out the computations for each of the variants in two stages. At the first stage unknown systematic disturbances have been computed on the basis of a system of conditional equations (type I) established for each side, the differences between the results of reciprocal observations being the free terms. At the second stage conditional equations (type II) of geometric figures and altitude differences between constant points have been introduced into the system. The differences between the results of these two stages were insignificant when compared with the differences between the results obtained in the particular variants.

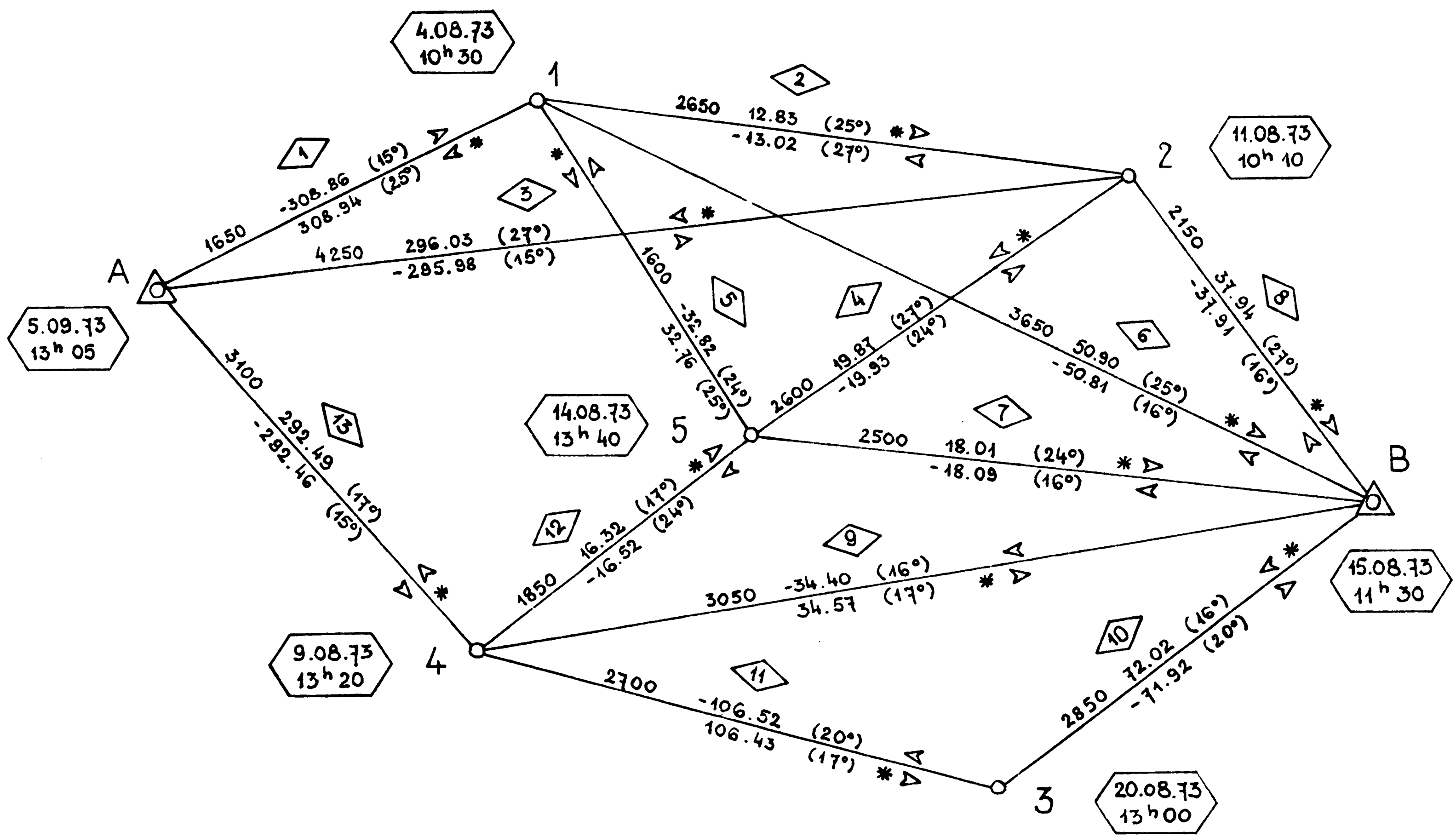

Fig. 1. Scheme of a section of the network with results of reciprocal observations.

https://doi.org/10.1017/S0074180900065980 Published online by Cambridge University Press

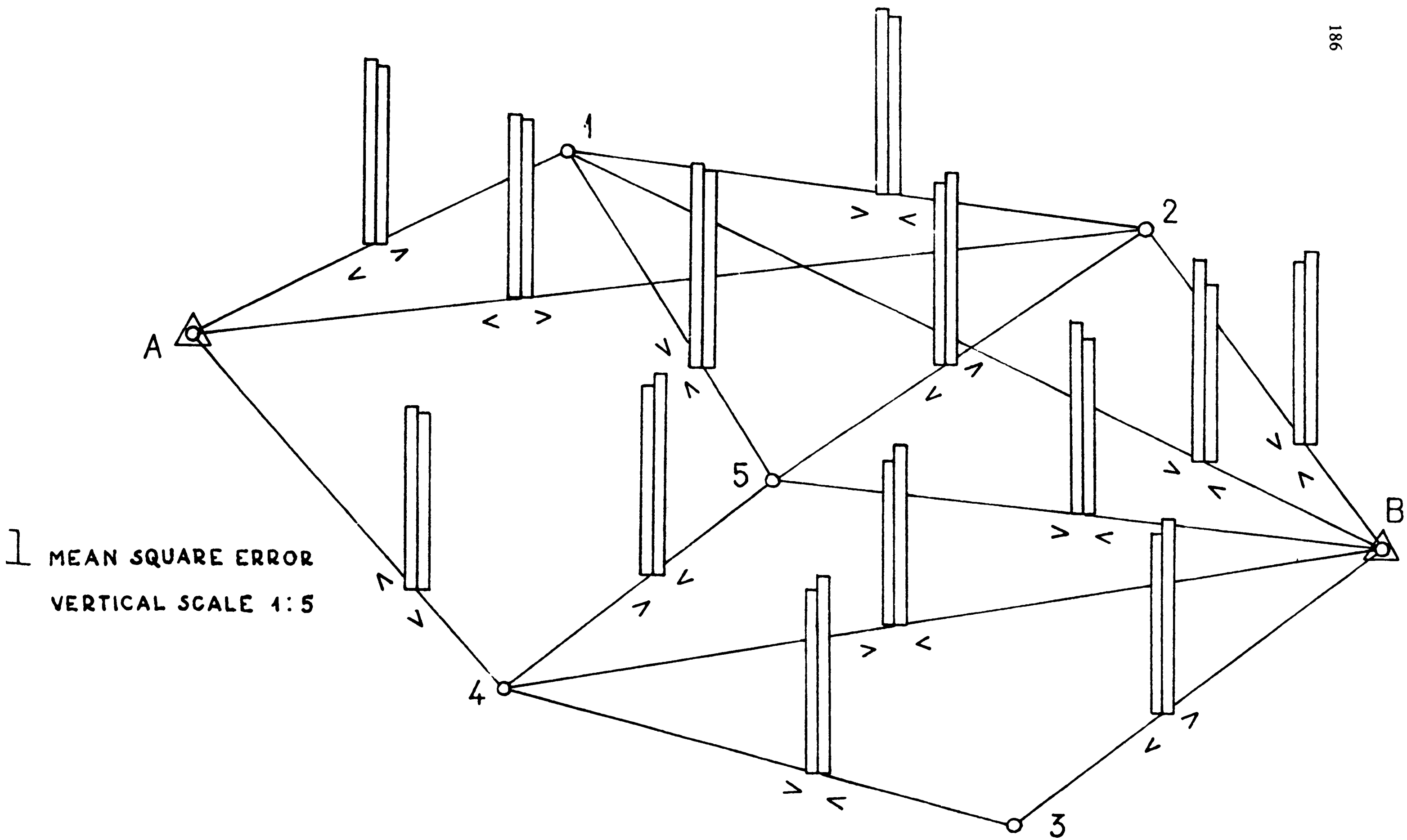

Fig. 2. Systematic components of mixture of errors connected with the influence of observation conditions.

The conformity of the distribution of the random components of the mixture of errors with the model of Gauss and the minimum standard deviation was the criterion of the choice of the variant. We have chosen as optimal the variant of the solution, in which systematic errors resulting from the influence of the environment are described by the linear function of the temperature and the circular function of the time of the observations.

The presented scheme of the analysis is illustrated by an example of a section of a network (fig. 1). In this system 13 conditional equations of type I and 8 independent conditional equations of type II can be formed. The directions of the observations, the results of which have been used for the computation of the free terms of the equations of type II, are indicated on fig. 1.

Systematic errors caused by the influence of the observation conditions, determined for the optimal variant by the adjustment of the system, are shown by fig. 2. The above mentioned components of the mixture of errors of altitude differences, determined on the basis of single measurements, are indicated on this figure. The differences of these values illustrate the corresponding components of the mixture of errors in reciprocal measurements. The reliability of the obtained results is confirmed by the value of the mean square error given at the bottom of the figure.

4. CONCLUSIONS

The method of adjustment presented here is a proposition to apply statistics to the estimation of the influence of the environment, of the atmospheric conditions and refraction on the results of observations. The method has been conceived in the first place as a means of elaborating typical field observations and has been tested in practice on such examples.

It is not necessary, that the observations destined for the analysis should follow a special programme and the observations of each of the parameters need not be more numerous than recommended for typical field works. It is however very important, to register as detailed informations as possible about the conditions in which each measurement was carried out. It results from the assumptions of the method, that each of the parameters ought to be measured at least twice in different atmospheric conditions; this principle is however always observed in practice. It is advisable, that each of the observations should be carried out in atmospheric conditions differing from each other as much as possible. A considerable density of sights in the network is advantageous.

Since the systematic errors are treated as additional unknowns, the total number of unknowns in the system increases, thus weakening the system. This can be prevented by diminishing the number of unknowns by means of a joined determination of the influence of different disturbances. A strengthening of the system can be achieved also by carrying out additional observations, which should be double.

A fundamental question is the correct choice of the functions describing the influence of the systematic disturbances of observations on their results. It seems advisable to elaborate some scientifically justified variants together with recommendations for the practical realization, concerning the indispensable additional informations about the conditions of the measurements. Utilizing the results of typical field works, it will thus be possible to complete the effects of scientific experiments.

REFERENCES

Hradilek, L.: 1972, "Refraction in trigonometric and three-dimensional terrestrial networks", Canadian Surveyor, 26, pp. 59-70.

Levallois, J.J.: 1969. Géodésie Générale, I, Paris.

Makowska, A., Zorski, Z.: 1976, "Analiza wyrównania trygonometrycznej sieci wysokościowej z uwzględnieniem odchyleń pionów i refrakcji" (Analyse de la compensation d'un réseau trigonométrique des altitudes en tenant compte des déviations de la verticale et de la réfraction), Géodezja i Kartografia, 25, pp. 3-16.

Proceedings of the International Symposium on Electromagnetic Distance Measurement and the Influence of Refraction: 1977, Wageningen.

Szacherska, M.K.: 1973, "Metoda analizy rozkładu błędów obserwacji geodezyjnych" (Méthode de l'analyse de la répartition des erreurs d'observations géodésiques), Geodezja i Kartografia, 22, pp. 255-275.

Szacherska, M.K.: 1974, "Model kompozycji błędów pomiarów geodezyjnych" (Modèle de la composition des erreurs de mesures géodésiques), Geodezja i Kartografia, 23, pp. 21-51.

Szacherska, M.K.: 1977, "Method of analysis of the composition of random and systematic observation errors", Guest-lecture at the Geodetic Institute of Uppsala University, Uppsala, Sweden.

Szacherska, M.K., Wiśniewski, Z.: 1977, "Evaluation des erreurs systématiques au cours de la compensation des observations", Invited paper for XV International Congress of Surveyors, Proceedings of FIG Congress, Stockholm, vol. 5, pp. 229-238.

Tengström, E.: 1975, Report of the work of IAG Special Study Group No. I.23, Atmospheric effects on angular measurements, 1971-1975, 16th Assembly of IUGG, Grenoble.

DISCUSSION

K. Poder: expressed his high appreciation of the research done by Szacherska et al, which has started a new theoretical cooperation of great interest to all scientists in Europe, not least in Scandinavia, where also Uppsala University plays a role of unifying all efforts of statistical approaches in geodesy.

https://doi.org/10.1017/S0074180900065980 Published online by Cambridge University Press