Easy Geometrical Proof of a Theorem of Chasles.

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The subjoined is an easy geometrical proof of the following theorem (which derives its importance from being part of the method of Chasles of constructing geometrically the ninth point when eight points of an "associated system" are given).

THEOREM:—*The locus of the "points opposés" of four of the nine points common to a pencil of cubic curves is the conic through the other five.*

[*N.B.*—If we have a system of conics passing through four points on a cubic curve, then the chord joining the other two points where a conic of the system cuts the cubic always passes through a fixed point on the cubic. The fixed point is known as the "point opposé" of the other four.]

Now let $P_1 P_2 P_3 P_4$ be four of the nine points common to a pencil of cubic curves. Let $Q_1 Q_2 Q_3 Q_4 Q_5$ be the other five. Pass a conic through $P_1 P_2 P_3 P_4 Q_1$. Take any line through Q_1 cutting the conic in K. Draw the cubic of the system through K. Then this cubic is uniquely determined. Let $Q_1 K$ cut this cubic again in X. Then X is uniquely determined, and X is the "point opposé" of $P_1 P_2 P_3 P_4$ with respect to this cubic. Suppose this cubic cuts $Q_2 X$ in K'. Then by the theorem quoted, a conic passes through $P_1 P_2 P_3 P_4 Q_2 K$.

Therefore, given $Q_1 X$ we obtain $Q_2 X$ uniquely, and given $Q_2 X$ we obtain $Q_1 X$ uniquely.

Therefore $Q_1 X$ and $Q_2 X$ are corresponding numbers of a (1, 1) correspondence.

Therefore the locus of X is a conic through Q_1 and Q_2.

Similarly it passes through $Q_3 Q_4$ and Q_5.

Thus the required theorem is established.