1

Introduction

This introductory chapter does not study any particular origami techniques or models, but establishes some conventions and terminology necessary for the remainder of the book.

1.1 Mathematics Boxes

As mentioned in the Preface, mathematics terminology and material that may not be familiar is boxed to make it easy to skip if not needed. An example is Box 1.1.

Box 1.1 Real Numbers

Mathematicians use the symbol $\mathbb{R}$ to designate the set of ***real numbers***, numbers that can be represented by a (possibly infinite) decimal expansion. For example, $\pi = 3.1415926535\ldots$. Reals are distinguished from "natural numbers" or "whole numbers" $0, 1, 2, 3, \ldots$, and distinguished from ***integers***, natural numbers plus their negatives. $\mathbb{R}^2$ represents a 2-dimensional plane, say, the xy-plane: Two real coordinates specify a point p on the plane. Similarly, $\mathbb{R}^3$ represents 3-dimensional space, often called ***3-space***.

Sometimes just a footnote instead of a box suffices to explain terminology. Rather than filling this chapter with all the preliminary mathematics boxes, the boxes instead will be located where first needed, with back referencing for reminders in later chapters.

1.2 Creases, M/V

All origami in this book is created by folding a single piece of paper. No ***modular origami*** is included. A ***crease*** on a piece of paper is a line segment,

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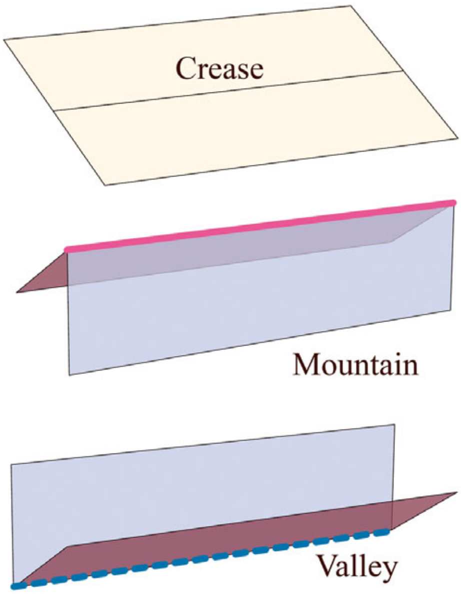

Figure 1.1 Crease and M/V folds.

which might not cross from boundary-to-boundary, but rather start and end in the interior of the paper. These internal creases are one way in which our emphasis in this book differs from the practice of origami folding, which almost always includes preparatory creases that are ultimately flattened in the final model. A ***crease pattern*** is the collection of creases on the paper that become specific folds in the final model.

The insistence that crease segments are straight is relaxed in Chapter 8, where we investigate curved creases.

A crease is flat. An ***M/V assignment*** to a crease is an indication that the crease is to be folded either as a mountain (M) or as a valley (V); see Figure 1.1. Of course a mountain fold is a valley fold viewed from the other side. Although we are defining a crease as flat, it will often be convenient to use "mountain crease" as shorthand for an M-fold crease, and similarly for V-folds. Note that an M- or V-fold does not specify the fold angle, just that the angle is either plus or minus.

How to mark figures with M/V creases in the age of color printing has not been entirely settled in the community. I use the convention that solid red = M, whereas dashed blue = V. When it seems infeasible to dash, solid blue = V.

The next section addresses a bit of a terminological conundrum.

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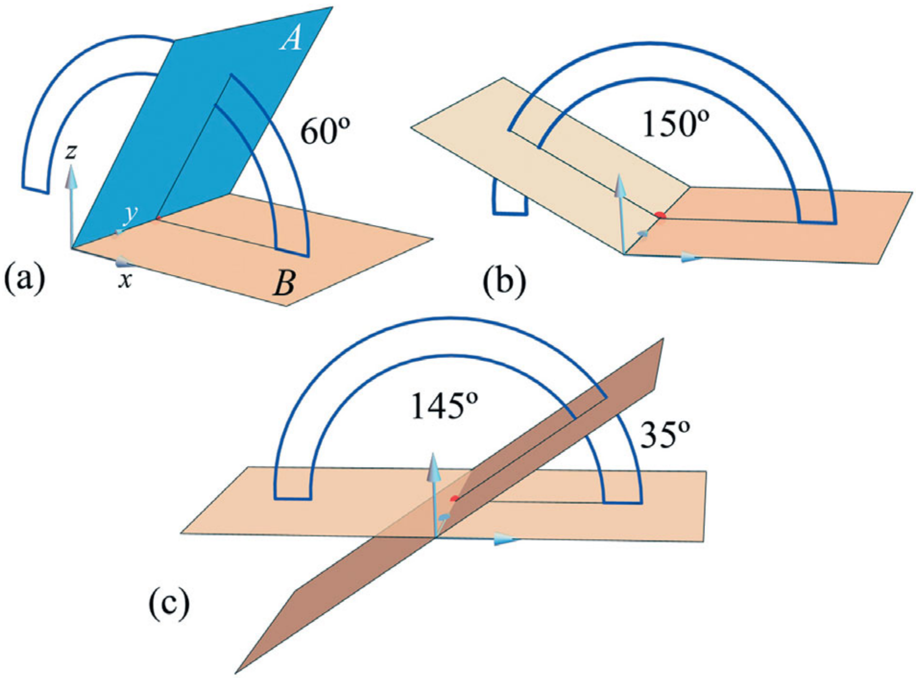

Figure 1.2 (a,b) Dihedral angles as measured by a protractor orthogonal to L, which is here the y-axis. (c) Two intersecting planes determine two supplementary angles: $35° + 145° = 180°$.

1.3 Dihedral vs. Fold Angles

Angles play a central role in the mathematics of origami. Two-dimensional angles between creases need no further explanation, but less familiar are angles between planes in 3-space, to which we now turn.

The ***dihedral angle*** δ is the angle between two planes: dihedral = "two sides." Any pair of nonparallel planes A and B intersect in a line $L = A \cap B$.[1] Placing a protractor in a plane orthogonal to L measures the dihedral angle δ between A and B.

Note the convention, extrapolated from Figure 1.2(a,b), that $\delta = 180°$ when A makes a flat angle with B along L (the y-axis in the figure), and $\delta = 0°$ when they make a sharp crease along L. Of course a pair of planes determine two supplementary angles at L, as indicated in Figure 1.2(c).

[1] $\cap$ is the symbol meaning "intersect." $\cup$ means "union."

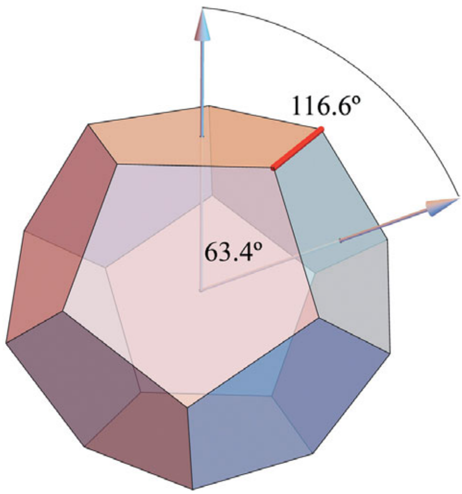

Figure 1.3 $\delta = \arccos(-1/\sqrt{5}) \approx 116.565°$. The supplementary angle ϕ is equal to the spread of the depicted normal vectors.

It is often more convenient in origami to use the supplementary ***fold angle*** ϕ, the angle needed to fold two planar faces to obtain a desired angle along a crease line L. Then, because $\phi + \delta = 180°$, $\phi = 0°$ when there is no fold at L, and $\phi = 180°$ when the faces are folded into a sharp crease, with the two faces overlapping to the same side of L.

Box 1.2 Normal Vectors

A ***vector*** is an oriented line segment, an arrow pointing from tail to head. A vector orthogonal (perpendicular) to a plane is called a ***normal vector***. "Orthogonal" and "perpendicular" are synonyms, but we'll reserve "normal" for vectors.

The term "dihedral" originates in the ancient study of polyhedra. For example, the dihedral angle at each edge of a dodecahedron, the fourth of the five Platonic solids, is about 116.6°. The supplementary angle, 63.4°, can be viewed as the spread of the normal vectors (Box 1.2) to the faces sharing an edge: see Figure 1.3.

Both dihedral angles and fold angles can be positive or negative. For example, a positive fold angle is usually a mountain and a negative angle a valley. But in many contexts, only the magnitude of a dihedral or fold angle is relevant. Because "dihedral angle" is prominent in geometry and "fold angle" more common in the origami literature, it is best for readers to become comfortable with both. So we'll use whichever seems most appropriate in each particular context.

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We now turn to addressing two questions that might occur to readers.

1.4 Why Theoretical Computer Science?

Why is the infusion from computer science relevant to this book's focus on the mathematics of origami?

First, theoretical computer science is arguably a part of mathematics. After all, one of the most important unsolved problem in all of mathematics is the P =? NP Millennium Prize question (see Chapter 4), which falls squarely in theoretical computer science. But second, understanding the transition from "easy" (e.g., characterization of single-vertex flat-foldability) to NP-hard (multiple-vertex flat-foldability) both explains historical difficulties and suggests feasible future foci. Computer science has illuminated aspects of origami mathematics that were previously inaccessible.

Finally, the design of NP-hard proof origami "gadgets" is both structurally revealing and—Fun!

1.5 Why Theorems and Proofs?

Why is there so much emphasis in this book on theorems and proofs?

Theorems are timeless nuggets of truth that pin-down our advance in understanding. Kawasaki's Theorem 3.2 settled once and for all time the conditions for when a single vertex can fold flat. The Degree-4 Folding Theorem 5.2 more recently uncovered a beautiful understanding of flat-foldable degree-4 vertices. Each such theorem is a step on a route to the frontier of mathematical understanding.

Theorems are established through ***proofs***, watertight logical arguments that are convincing to anyone who has the background (the "mathematical maturity") to follow the arguments.

Recent advances suggest we are nowhere near resolving all the questions at the mathematical origami frontier, let alone those open problems possibly well beyond, several of which are highlighted in the following chapters. Anticipating advances, this book may need a second edition!

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