

the *risk situation* of company i is clearly defined. Hence it follows that the utility $U_i(S_i, F_i(x_i))$ attached to this risk situation is obtained as

$$U_i(S_i, F_i(x_i)) = \int_0^{\infty} u_i(S - x_i) dF_i(x_i),$$

where $u_i(x)$ is the "utility of money" to company i .

In the reinsurance market the companies can conclude treaties which may be defined by a set of functions represented by a vector y in the n -dimensional x -space. If there exists no vector \bar{y} such that

$$U_i(y) = U_i(\bar{y}) \text{ for all } i,$$

the set of treaties represented by y will be referred to as *Pareto optimal*.

The author shows that the only Pareto optimal arrangement is that the companies should cede their entire portfolio into a pool and then decide on how to divide the claims among the companies. This leads to the finding of a solution to a n -person game.

The principles of an equilibrium price in a market are set forth and these principles are applied by assuming the utility of money to be represented by second degree polynomials. To obtain Pareto optimality the only solution is that all reinsurance is made on a net premium basis. The result derived is thus in a sense completely negative, indicating that no market price exists which will lead to a Pareto optimal arrangement. However, the author states that it is possible to construct a model of a reinsurance market in which unrestricted competition will lead to an equilibrium which is Pareto optimal if the sacrosanct principle of equivalence is sacrificed.

Some Applications of Collective Risk Theory to Reinsurance and Group Experience Rating, by PAUL MARKHAM KAHN, University of Michigan, 1961.

This dissertation is divided into four parts. Part I gives an introduction to the theory of risk, part II deals with stop-loss reinsurance, part III contains an extension of a theorem of Borch, while part IV is concerned with certain problems of group experience rating.

In his introduction the author reveals that the collective risk theory, a branch of the theory of random processes, remains largely unknown in the United States. It is therefore the major purpose of this thesis to examine the feasibility of applying the collective risk theory to practical problems in the field of reinsurance and experience rating.

Part I gives a brief outline of the main properties of individual and collective risk theory. Assuming a simple Poisson model, the distribution function and the ruin probability function are discussed.

In part II the results of examples from life insurance and those produced by the collective risk theory, particularly by Esscher's method, in the field of stop-loss reinsurance are compared by following closely the formulae developed by Ammeter. Stop-loss premiums are calculated under four different assumptions. The first method was suggested by Feay in which the normal distribution is used as an approximation to the distribution of claims. The second method is an application of individual risk theory. The third uses tables of the incomplete Gamma function and the fourth applies Esscher approximations. The stop-loss premiums calculated under these assumptions

are practically the same for methods three and four, while for the other two methods the results diverge with increasing retention limits.

In part III the author gives a wider set of transformations than the set considered by Borch. He asserts the theorem of Borch that stop-loss reinsurance minimizes the variance if a fixed amount is available for reinsurance premiums. In part IV Jackson's and Ammeter's methods for group experience rating are mentioned. It is shown that this process is intimately connected with stop-loss reinsurance and that there exist interrelationships between the different methods, the formulae of Jackson being special cases of Ammeter's formulae.

A valuable feature of this thesis lies in the fact that it not only deals with the theoretical aspects but also enumerates practical applications.

An Introduction to Credibility Theory, by L. H. LONGLEY-COOK,
published by the Casualty Actuarial Society, 200 E. 42nd Street,
New York 17, N.Y. (price \$ 1.50).

Liability and property insurers are often faced with problems for which the data are incomplete or usable only in a very indirect way. The determination of the statistical reliability of rates derived from such incomplete data and the relative weight to be given to indications of such experience are therefore matters of considerable importance.

In 1914 Professor Mowbray presented one of the first discussions of the reliability of exposure and his theory has been followed by almost every subsequent writer. Today, although a rather extensive literature exists on reliability of experience or "credibility", there is no elementary introduction available. Without a good background knowledge of the subject it is sometimes very difficult to fully comprehend some of the numerous papers.

This gap has been filled by an excellent paper by Mr. L. H. Longley-Cook. At the request of the Educational Committee of the Casualty Actuarial Society the author has prepared this introduction to provide actuaries and others interested in credibility theory with a framework into which they can fit these papers.

The purpose of the paper was to give an introduction to the subject and to avoid complicated mathematics. The author has therefore concentrated on principles rather than details and referred in an appendix to the extensive literature existing in this theory. The meaning of credibility and the need for a mathematical model are clearly explained, and, with a minimum of mathematics, the reader is acquainted with the formula first derived by Mowbray:

$$P = \frac{1}{\sqrt{2\pi}} \int_0^{k/\sqrt{ng}} \frac{e^{-t^2/2}}{e} dt$$

n = number of exposures
in one year
 q = average number of
accidents
 k = maximum departure
from expected

A few examples of the use of this formula are demonstrated, e.g. an accepted standard of credibility is 1082 claims corresponding to P equal 90 % and k equal 5 %.