chapter on Gröbner bases and reconsidered in this context via a well known result due to Macaulay (that the Hilbert function of an ideal is the same as that of the ideal generated by the initial terms of the ideal, under an admissible order). As another example we quote again from the author (Chapter 8): 'So what? Well, this is exactly what proves that we can compute $\mathrm{Tor}_i(M,N)$ ...'. This also indicates the informal style of the writing: friendly but not overly so. There are many exercises throughout, of both a theoretical and computational nature. They also range from the straightforward to the hard (for which the author gives helpful hints or references where the solution can be found).

To sum up, a student considering this area will find the guided-tour aspect of this book very helpful. The computing aspect is a good pointer to what can be done, but the book is not intended to act as a manual. Unless such a student is already well informed, he or she will find the suggestions for further reading essential.

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STOPPLE, J. *A primer of analytic number theory* (Cambridge University Press, 2003), 0 521 01253 8 (paperback) £22.95, 0 521 81309 3 (hardback) £65.

The author makes it immediately clear in the introduction that this book is not intended to be anything like a conventional text on analytic number theory. The starting point is the observation that courses on number theory normally appear quite early in the curriculum, and consequently avoid all results that require any input from analysis. His basic thesis is that this avoidance is unnecessary. In this book, he seeks to demonstrate that one can introduce a wide range of topics within analytic number theory assuming no more than a basic exposure to 'calculus'. It is not even assumed that the reader knows what is meant by the sum of a series: this is the subject of a 30-page digression, starting with Taylor series, in which the actual definition of convergence only appears on about the twentieth page.

The general style is user-friendly and interactive. The simpler steps of proofs are often set as exercises, so that the reader is recruited as a partner in the process of discovery. Solutions to the exercises are provided at the end. There are also a large number of numerical exercises and illustrations, based on MAPLE or MATHEMATICA. Readers are repeatedly assured that they can leave out proofs if they find them hard. The text is liberally interspersed with biographies, historical information and quotations. A good deal of space is given to informal discussion of famous conjectures and open problems. The claim that nothing is required beyond basic calculus is partly sustained by a rather free policy of importing unproved results, both from analysis and from number theory, when wanted.

Despite all this, the range of topics attempted is distinctly ambitious for a reader who is really starting from the position described. In the reviewer's opinion, any such reader is likely to find large parts of the book extremely challenging!

The early material covers topics like perfect numbers, convolutions, the Möbius function and the estimates for the partial sums of the divisor function τ and the sum-of-divisors function σ. Readers of the type postulated should get this far without much trouble. With strong enough motivation, they may also succeed with the next few topics: the Chebyshev estimates, the Bernoulli numbers and the Euler product. Here there is a typical example of the consequences of the author's self-imposed task. The cotangent series is needed for the evaluation of $\zeta(2n)$. Fourier series are not allowed, so a direct proof is given (to the reviewer's mind, this is one of the author's less satisfactory proofs; it is based on a rather casual use of the O-notation, and would seem to require some non-trivial further work for its completion).

After that, the level rises steeply. Mertens's theorems are presented in some detail, with the application to the maximal order of σ. Three chapters then develop the theory of the gamma and zeta functions. The Bernoulli numbers are used to define the extended zeta function. The functional equation is derived using the Jacobi theta function (a fairly complicated 'elementary' calculation, attributed to Polya, replaces the customary use of the Poisson summation formula). Hadamard's product is then simply assumed, and Von Mangoldt's explicit formula for $\psi(x)$ is 'derived', taking injectivity of the Mellin transform as a further assumption. A very rough and ready argument is then given to indicate why one might believe Riemann's explicit formula for $\Pi(x)$. It is then stated that the prime number theorem is a consequence of Riemann's formula, and that the proof can be found in other books; no reference is made to the possible existence of simpler proofs not based on these formulae.

The remaining three chapters are devoted to topics related to Diophantine equations: first, Pell's equation and quadratic forms with positive discriminant, with quadratic reciprocity assumed—the connection with Dirichlet L-functions is illustrated by several particular cases (the reader is informed that this is a 'miracle'!). Next, a largely descriptive introduction to elliptic curves is given, culminating in the conjecture of Birch and Swinnerton-Dyer. The final chapter, by contrast, gives a fairly detailed account of the analytic class number formula for quadratic forms with negative discriminants.

In summary, the book is a well presented and stimulating informal introduction to a wide range of topics, but it is hardly realistic to claim that most of it is accessible to students who have only done basic calculus. Among mature mathematicians wanting to extend their knowledge of the subject, it will appeal to those who are more interested in 'flavour' and background than in full proofs.

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PISIER, G. *Introduction to operator space theory* (Cambridge University Press, 2003), 0 521 81165 1 (paperback), £40.

The starting point of operator space theory is the relatively recent viewpoint that operator spaces, that is, linear spaces whose elements are operators on a Hilbert space, should not in general be studied within the category of Banach spaces. Although they are Banach spaces, Banach-space methods often do not take one very far in the analysis of such spaces. This is witnessed, for example, by the lack of overlap one sees between Banach-space and C^*-algebra literature. A 'better' category for the study of subspaces of C^*-algebras began to emerge around 1970, in the work of Arveson [**1**]. Namely, in addition to the norm on such a space X, we consider for each positive integer n the natural norm $\|\cdot\|_n$ on the space of $n \times n$ matrices $M_n(X)$ inherited from the C^*-algebra $M_n(A)$. The 'morphisms' then consist of the *completely bounded maps*, namely the linear maps T for which the norm of the map $[x_{ij}] \mapsto [T(x_{ij})]$ on $M_n(X)$, is bounded independently of n. The least upper bound of this sequence of norms, is called the *completely bounded norm* of T. The subject started to become a field in its own right with the abstract characterization of operator spaces, in terms of the just-mentioned matrix norms, done in Ruan's thesis under the direction of Effros. They had begun the creation of a variant of functional analysis appropriate to the new category, a variant that bridged (a large part of) the gap between Banach space theory and operator algebras.

As evidenced by the length of time it took in coming, the importance of the matrix norms mentioned above, and of the completely bounded maps, is not obvious from the outside. The first hint of it comes by reflecting on the basic 'tensor product' of operator algebras. Suppose that X and Y are C^*-algebras (respectively, operator spaces) contained in $B(H)$ and $B(K)$, respectively. Here H, K are Hilbert spaces. If $H \otimes K$ is the Hilbert-space tensor product of H and K, then the symbol $x \otimes y$, for $x \in X$ and $y \in Y$, corresponds canonically to an operator on $H \otimes K$. The (closure of the) span of these operators in $B(H \otimes K)$ is called the spatial (or minimal) tensor product of X and Y. It is again a C^*-algebra (respectively, operator space).