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Groups acting simply transitively on $\tilde{A}_n$ buildings

TIM SVENSON

We study groups acting simply transitively on the vertices of $\tilde{A}_n$ buildings. The presentations of these groups are completely characterized. In earlier work on this subject the greatly simplifying assumption was made that the groups induce a rotation of the building diagram. We do without that assumption here.

Prior to this thesis it was known that any group with a type-rotating "triangle presentation" acts simply transitively [**2**, **1**]. The main result presented here removes the type-rotating assumption from that result. Then follow the results, with discussion, of a complete enumeration of groups acting simply transitively on $\tilde{A}_2$ buildings of order 2 or 3, using the main result just proved. This extends the work of [**3**]. Embeddings into (semidirect extensions of) linear groups of those groups found which act on linear buildings are then computed.

A type-rotating group and a non-type-rotating group acting simply transitively on a 5-adic $\tilde{A}_3$ building are constructed (by adapting the method of [**4**, pp. 120–126] rather than using the main result above). A triangle presentation for the type-rotating group is then explicitly computed.

In the course of the above, the definition of a triangle presentation currently in the literature (see for example, [**2**]) is improved, methods are given for obtaining non-type-rotating (respectively type-rotating) groups from type-rotating (respectively non-type-rotating) groups with certain automorphism groups, and several auxiliary results are proved which are of use when working with these groups.

The main definitions and theorem are:

DEFINITION. Let Π be a projective space of dimension $n \geqslant 2$, with incidence (not including equality) denoted by $\sim$. Let λ be an involution of Π. A set $\mathcal{T} \subset \Pi^3$ of triples of elements of Π is said to be *compatible* with λ if

A: given $u, v \in \Pi$, then $(u, v, w) \in \mathcal{T}$ for some $w \in \Pi$ if and only if $\lambda(u) \sim v$;

B: if $(u, v, w) \in \mathcal{T}$ then $(v, w, u) \in \mathcal{T}$;

C: if $(u, v, w_1) \in \mathcal{T}$ and $(u, v, w_2) \in \mathcal{T}$, then $w_1 = w_2$;

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D: if $(u, v, w) \in \mathcal{T}$, then $\big(\lambda(w), \lambda(v), \lambda(u)\big) \in \mathcal{T}$;

E: if $\big(a, x, \lambda(b)\big), \big(b, y, \lambda(c)\big), \big(c, z, \lambda(a)\big) \in \mathcal{T}$ then $(x, y, z) \in \mathcal{T}$.

DEFINITION. Let Π be a projective space of dimension $n \geqslant 2$, let λ be an involution of Π, and let $\mathcal{T}$ be a set of triples compatible with λ. A *triangle presentation* is a group presentation with generating set

$$\{g_x | x \in \Pi\}$$

and relations

$$g_x g_{\lambda(x)} = 1 \text{ for all } x \in \Pi$$
$$g_x g_y g_z = 1 \text{ whenever } (x, y, z) \in \mathcal{T}.$$

THEOREM. *Let Π be a projective space of dimension $n \geqslant 2$, let λ be an involution of Π, and let $\mathcal{T}$ be a set of triples compatible with λ. Let Γ have triangle presentation*

$$\big\langle g_x | x \in \Pi : g_x g_{\lambda(x)} = 1 \textit{ for all } x \in \Pi \textit{ and } g_x g_y g_z = 1 \textit{ whenever } (x, y, z) \in \mathcal{T} \big\rangle.$$

Then the Cayley graph of Γ with respect to the generators g_x in this presentation is the one-skeleton of an $\widetilde{A}_n$ building, on which Γ acts simply transitively. The map $x \mapsto g_x$ is a type-preserving isomorphism from Π to the residue of the vertex 1 of Δ, and $g_x g_y g_z = 1$ if and only if $(x, y, z) \in \mathcal{T}$.

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Investment Analytics
Level 16
33 Alfred St
Sydney, NSW 2000
Australia