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MODELS OF FLOW AND DIFFUSION: SELECTIVE WITHDRAWAL FROM A STRATIFIED FLUID AND DISPERSAL OF HYDROGEN IN THE RETINA

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This dissertation is divided into two parts. In the first part, we investigate the withdrawal of fluid from a region of finite or infinite depth. The dispersal of hydrogen in the retina to model an experimental technique is considered in the second part.

Three related problems are studied in the first part and one in the second part. In the first problem, the flow induced by a line sink at an arbitrary location in a fluid of finite depth with a free surface, relevant to flows in reservoirs, lakes and cooling ponds, is examined [4]. A rigid-lid solution for small flow rates is obtained and a numerical method based on fundamental singularities techniques is applied to the full nonlinear problem. Both linear and numerical steady solutions are obtained for the shape of the free surface and show good agreement. The results suggest that steady nonlinear solutions are limited to flow rates below some critical value that depends on the sink location, the surface tension and the strength of the flow. A theorem is proved regarding the stagnation point location and some interesting surface shapes are obtained.

In the second problem, two-dimensional steady flow of an inviscid fluid induced by a line sink located at an arbitrary location in a region of infinite depth is computed [3]. The solution in the limit of small Froude number is obtained analytically, and numerically for the nonlinear problem. The asymptotic solution is found to have a property that if the horizontal location of the sink satisfies $x_S < 1$, there is only one stagnation point on the surface at the wall. However, if the horizontal location satisfies $x_S > 1$, a second stagnation point forms on the free surface. Numerical solution for the nonlinear problem confirms these properties. The effect of moving the sink horizontally has also been considered. The maximum Froude numbers at which steady

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solutions exist are computed and compared with previous work. The effects of surface tension are investigated.

The third problem considers the unsteady flow due to a line sink in a fluid of finite depth with surface tension, where the sink is situated at an arbitrary depth and location [5]. Here we focus on a critical value of the Froude number F that leads to a steady flow or drawdown, and also find the minimum F and the unknown free surface shape for a range of parameter values. A solution to the linear problem of an arbitrary location for the line sink in a duct is derived by using an integral equation method. The nonlinear equation is solved numerically by fundamental singularities techniques. The shape of the free surface is computed for a range of parameter values where the effects of surface tension are taken into account. It is shown that surface tension has an effect on the solution behaviour. An asymptotic solution is shown to be in a good agreement with the full nonlinear solution for small values of the Froude number.

In the second part, two simple mathematical models of advection and diffusion of hydrogen within the retina are discussed to assist in interpretation of the 'Hydrogen clearance technique' that is used to estimate blood flow [1, 2]. The first model assumes the retina consists of three, well-mixed layers with different thickness, and the second is a two-dimensional model consisting of three regions that represent the layers in the retina. Diffusion between the layers and leakage through the outer edges are considered. Solutions to the governing equations are obtained by employing Fourier series and finite difference methods for the two models, respectively. The effect of important parameters on the hydrogen concentration is examined and discussed. The results contribute to understanding the dispersal of hydrogen in the retina and, in particular, the effect of flow in the vascular retina. It is shown that the predominant features of the process are captured by the simpler model.

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