ON ASYMMETRIC PERIODIC SOLUTIONS OF THE PLANE RESTRICTED PROBLEM OF THREE BODIES, AND BIFURCATIONS OF FAMILIES

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Previous work on the plane circular restricted problem of three bodies (Message 1958, 1959, 1970, and Fragakis 1973) has shown the existence, in association with each of the commensurabilities 2:1 and 3:1 of the orbital periods, of a pair of families of asymmetric periodic solutions, branching from the stable series of symmetric periodic solutions of Poincaré's second sort associated with that commensurability. (Each solution of either family is the mirror image, in the line of the two finite bodies, of a member of the other family of solutions associated with the commensurability.) The stability is transferred at the bifurcation to the two series of asymmetric orbits, each of which is therefore stable. Recent numerical integrations carried out by one of us (P.J.M.) have found such asymmetric periodic orbits associated also with the 4:1 commensurability, and quantities describing orbits of one of the two series are given in Table 1, showing the run of such orbits up to a second bifurcation with the same series of symmetric periodic orbits from which it sprang. Quantities describing some members of this series of symmetric orbits are given in Table 2. It is seen that stability is transferred back to the symmetric series at the second bifurcation. (The unit of distance is the distance between the two finite bodies, the unit of speed is the speed of their relative motion, and the initial conditions given (x_o, $\dot{x}_o$, $\dot{y}_o$) are for a crossing of the line of the two finite bodies, this line being taken as axis of "x" in a rotating Cartesian frame in the usual way. The mean values of the major semi-axis and eccentricity are denoted by $\bar{a}$ and $\bar{e}$, respectively, C is Jacobi's constant, and $\bar{y}_2$ is the mean value of the critical argument $y_2 = 4\lambda - \lambda' - 3\omega$. The mass ratio used is 0.000954927, T is the period of the solution in units of the period of the motion of the two finite bodies, and 2π c/T is the non-zero characteristic exponent.)

It was shown earlier (Message 1970) that a bifurcation of a series of asymmetric orbits from a series of symmetric orbits of the second sort implies a zero of $\dfrac{\partial^2 H^*}{\partial y_2^2}$, where H^* is the long-period part of the

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x_o	$\dot{x}_o$	$\dot{y}_o$	C	T	$\bar{a}$	$\bar{e}$	$y_2/(2\pi)$	$\cosh(2\pi c)$
3.02441	0.00186	-2.51016	-1.75377	4.00007	2.5225	0.2002	0.4541	0.99996
3.07723	0.02208	-2.57461	-1.74513	4.00002	2.5225	0.2252	0.3403	0.99783
3.13406	0.03133	-2.64379	-1.73496	3.99996	2.5226	0.2514	0.3104	0.99394
3.38456	0.05364	-2.94724	-1.67858	3.99956	2.5231	0.3617	0.2348	0.94319
3.63877	0.06320	-3.25382	-1.59915	3.99914	2.5236	0.4708	0.1947	0.80572
3.88892	0.07828	-3.55530	-1.49587	3.99911	2.5235	0.5763	0.1690	0.60221
4.09096	0.08154	-3.79879	-1.39372	3.99954	2.5229	0.6581	0.1564	0.41258
4.19809	0.08083	-3.92788	-1.33280	3.99989	2.5225	0.6995	0.1646	0.29455
4.33799	0.07645	-4.12082	-1.23200	4.00041	2.5219	0.7589	0.1821	0.09419
4.50418	0.06919	-4.29817	-1.12631	4.00076	2.5214	0.8113	0.2095	-0.06989
4.61277	0.06188	-4.43128	-1.03583	4.00087	2.5211	0.8495	0.2420	-0.11806
4.73631	0.05132	-4.58516	-0.91481	4.00079	2.5207	0.8925	0.2821	-0.01065
4.82849	0.04119	-4.70317	-0.80353	4.00057	2.5204	0.9245	0.3235	0.23279
4.92632	0.02567	-4.83421	-0.65220	4.00019	2.5201	0.9583	0.3842	0.66697
4.99326	0.00006	-4.93263	-0.50117	3.99993	2.5200	0.9817	0.4784	0.96223

Table 1

Initial conditions and other data for asymmetric periodic solutions associated with the 4:1 commensurability of periods

x_o	$\dot{y}_o$	C	T	$\bar{a}$	$\bar{e}$	$\cosh(2\pi c)$
2.56473	-1.94591	-1.78565	4.00015	2.5226	0.0179	1.00000
2.71867	-2.13659	-1.78098	4.00007	2.5227	0.0789	0.99985
2.91225	-2.37380	-1.76657	4.00004	2.5227	0.1556	0.99939
2.98435	-2.46153	-1.75873	4.00005	2.5226	0.1842	0.99960
3.06438	-2.55858	-1.74842	4.00008	2.5225	0.2160	1.00066
3.30311	-2.84634	-1.70724	4.00027	2.5222	0.3106	1.01487
3.78319	-3.42012	-1.57200	4.00064	2.5208	0.5009	1.11917
4.45657	-4.22863	-1.21423	4.00014	2.5200	0.7683	1.30153
4.56761	-4.36504	-1.12367	4.00000	2.5200	0.8125	1.30798
4.74135	-4.58334	-0.94761	3.99983	2.5199	0.8816	1.23146
4.97828	-4.90843	-0.54617	3.99988	2.5200	0.9757	1.01915
4.98689	-4.92219	-0.52111	3.99990	2.5200	0.9791	0.98854
5.00482	-4.95174	-0.46104	3.99997	2.5200	0.9862	0.90331
5.02001	-4.98065	-0.39602	4.00006	2.5201	0.9922	0.78708
5.03784	-5.02543	-0.26094	4.00032	2.5202	0.9992	0.41582

Table 2

Initial conditions and other data for symmetric periodic orbits associated with the 4:1 commensurability of periods

Table 3

Values of the mean eccentricity at zeros of $\frac{\partial^2 H^*}{\partial y_2^2}$ in the limit $m' \to 0$

Commensurability	mean eccentricities	
2:1	0.03652,	0.95927
3:1	0.12211,	0.97178
4:1	0.20112,	0.97792
5:1	0.26711,	0.98162
6:1	0.32184,	0.98413
7:1	0.36767,	0.98594
8:1	0.40657,	0.98732

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Hamiltonian function, and $y_2 = (p+q)\lambda - p\lambda' - q\omega$ is the critical argument. Recalculations of $\frac{\partial^2 H^*}{\partial y_2^2}$ by one of us (D.B.T.) have led to the finding of an error in the programme originally used in the calculation of this quantity, of such a type that the values previously given for the mean eccentricities at which such bifurcations can be expected need correction for those commensurabilities with $q \neq 1$. Corrected values of the mean eccentricity at which zeros occur, in the limit as the mass ratio tends to zero, for commensurabilities with $p = 1$ and $q = 1, 2, 3, \ldots, 7$, are given in Table 3. No zeros are now found for the cases with $p = 2$, $q = 1, 3, 5, 7$, with $p = 3$, $q = 1, 2, 4, 5$, with $p = 4$, $q = 1, 3, 5$, with $p = 5$, $q = 1, 2, 3, 4$, with $p = 6$, $q = 1$, with $p = 7$, $q = 1, 2$, and with $p = 8$, $q = 1$, so that no asymmetric periodic orbits are now indicated at these commensurabilities.

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