Part 3
$1/N$ Expansion

"Can I schedule parades and then call them off?"
"But just send out announcements *postponing* the parades. Don't even bother to schedule them."
J. HELLER, *Catch-22*

In many physical problems, especially when fluctuations of scales of different orders of magnitude are essential, there is no small parameter which could simplify a study. A typical example is QCD where the effective coupling, describing strong interaction at a given distance, becomes large at large distances so that the interaction really becomes strong.

't Hooft [Hoo74a] proposed in 1974 to use the dimensionality of the gauge group $SU(N)$ as such a parameter, considering the number of colors, N, as a large number and performing an expansion in $1/N$. The motivation was an expansion in the inverse number of field components N in statistical mechanics where it is known as the $1/N$-expansion, and is a standard method for nonperturbative investigations.

The expansion of QCD in the inverse number of colors rearranges diagrams of perturbation theory in a way which is consistent with a string picture of strong interaction, the phenomenological consequences of which agree with experiment. The accuracy of the leading-order term, which is often called multicolor QCD or large-N QCD, is expected to be of the order of the ratios of meson widths to their masses, i.e. about 10–15%.

While QCD is simplified in the large-N limit, it is still not yet solved. Generically, it is a problem of infinite matrices, rather than of infinite vectors as in the theory of second-order phase transitions in statistical mechanics.

We shall start this part by showing how the $1/N$-expansion works for the $O(N)$-vector models, and describing some applications to the four-Fermi interaction, the φ^4 theory and the nonlinear sigma model. Then we shall concentrate on multicolor QCD.