



ARTICLE

Logical Akrasia

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Abstract

The aim of this paper is threefold. Firstly, sections 1 and 2 introduce the novel concept *logical akrasia* by analogy to epistemic akrasia. If successful, the initial sections will draw attention to an interesting akratic phenomenon which has not received much attention in the literature on akrasia (although it has been discussed by logicians in different terms). Secondly, sections 3 and 4 present a dilemma related to logical akrasia. From a case involving the consistency of Peano Arithmetic and Gödel's Second Incompleteness Theorem, it's shown that either we must be agnostic about the consistency of Peano Arithmetic or akratic in our arithmetical theorizing. If successful, these sections will underscore the pertinence and persistence of akrasia in arithmetic (by appeal to Gödel's seminal work). Thirdly, section 5 concludes by suggesting a way of translating the Dilemma of Arithmetical Akrasia into a case of regular epistemic akrasia; and further how one might try to escape the dilemma when it's framed this way.

Keywords: Epistemic akrasia; logical akrasia; epistemic rationality; logical theories; Gödel's incompleteness theorem; the dilemma of arithmetical akrasia

1. Prologue

The Greek word “akrasia” translates literally as “lack of self-control,” but has come to be used as a general term for a *weakness of will*, i.e., a disposition to act contrary to one's own considered judgment. It will come as no surprise that such inability to act as one thinks right has interested ethicists since antiquity.

More surprising (perhaps) is the vast amount of attention that the analogous phenomenon *epistemic akrasia* has received lately.¹ A driving force behind this interest is the appealing thought that epistemic rationality requires coherence between: (A) an agent's doxastic attitudes in general, and (B) their specific beliefs about what doxastic attitudes are rational.² To illustrate, consider the medical resident Anandi who correctly

¹See Adler 2002; Brown 2018; Chislenko 2021; Christensen 2021, 2024; Daoust 2019; Greco 2014; Horowitz 2014, 2022; Jackson and Tan 2022; Kappel 2019; Kauss 2023; Kearl 2020; Lasonen-Aarnio 2014, 2020; Littlejohn 2018; Owens 2002; Ribeiro 2011; Roush 2017; Skipper 2019, 2021; Sliwa and Horowitz 2015; Smithies 2012; Titelbaum 2015, 2019; Williamson 2011, 2014; Worsnip 2018.

²Examples of *doxastic attitudes*: belief-tokens, credences, opinions, judgments, etc.

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figures that dosage $\langle p \rangle$ is appropriate for her patient; and thus believes that $\langle p \rangle$. Suppose she then learns she's been drugged herself, and further that the effects of the relevant drug very often lead to cognitive errors that are hard to detect from the inside. As a result, suppose she believes that $\langle \text{my belief that } p \text{ is irrational} \rangle$ but that she maintains her belief that $\langle p \rangle$ nonetheless. Other things being equal, Anandi's doxastic state should strike us as irrational because she believes against her own standards of rationality, or as recent epistemological parlance will have it; because she believes *akratically*.³

Examples like Anandi's drug case have led some epistemologists to argue for a general anti-akrasia constraint on epistemic rationality (Feldman 2005; Littlejohn 2018; Smithies 2012; Titelbaum 2015):

The Akratic Principle. No [epistemic] situation rationally permits any overall [doxastic] state containing both an attitude A and the belief that A is rationally forbidden in one's situation. (Titelbaum 2019: 227)^{4,5}

This principle is taken to imply that you should either have the attitudes you believe you ought to have, or stop believing that you ought to have those attitudes. Hence, in the name of rationality, the Akratic Principle forbids you to have certain combinations of attitudes such as *not* believing that $\langle p \rangle$ while believing that $\langle \text{believing } p \text{ is rationally required in one's situation} \rangle$; or having $\text{credence}(p) = 0.9$ while believing that $\langle \text{having credence}(p) = 0.9 \text{ is rationally forbidden in one's situation} \rangle$.

We'll return to the Akratic Principle in due course (cf. section 3), but for now let's consider a widely discussed case from the literature on epistemic akrasia to further guide our intuitions. The case concerns a sleep-deprived detective, Sam, who possesses misleading higher-order evidence (i.e., misleading evidence about what his first-order evidence supports):

Sleepy Detective. Sam is a police detective, working to identify a jewel thief. He knows he has good evidence – out of the many suspects, it will strongly support one of them. Late one night, after hours of cracking codes and scrutinizing photographs and letters, he finally comes to the conclusion that the thief was Lucy. Sam is quite confident that his evidence points to Lucy's guilt, and he is quite confident that Lucy committed the crime. In fact, he has accommodated his evidence correctly, and his beliefs are justified. He calls his partner, Alex. "I've gone through all the evidence," Sam says, "and it all points to one person! I've found the thief!" But Alex is unimpressed. She replies: "I can tell you've been up all night

³According to a popular *evidentialist* formulation, a subject is epistemically akratic when they are highly confident that proposition $\langle p \rangle$ is true while also believing that the higher-order proposition $\langle \text{my current evidence doesn't support } p \rangle$ is the case. So, if Anna believes that it's going to rain tomorrow while also believing that her evidence at the time doesn't support this, then Anna is in a state of epistemic akrasia. Prima facie – at least – Anna's overall doxastic state should strike us as irrational. Since believing against what one takes one's evidence to support just seems epistemically bad; if not outright paradoxical. To this end, the card-carrying evidentialist Richard Feldman wonders "...what circumstances could make [epistemic akrasia] reasonable..." (Feldman 2005: 109).

⁴Note that the Akratic Principle is sometimes referred to as the *Enkratic Principle* instead (see, e.g., Field 2019, 2021; Skipper 2019).

⁵For further details on Titelbaum's use of the term "rational" consult Skipper (2019) and Titelbaum (2015, 2019). See also Bradley (2021) and Carr (2021) for recent discussions of *ideal* versus *non-ideal* epistemic rationality.

working on this. Nine times out of the last ten, your late-night reasoning has been quite sloppy. You're always very confident that you've found the culprit, but you're almost always wrong about what the evidence supports. So your evidence probably doesn't support Lucy in this case." Though Sam hadn't attended to his track record before, he rationally trusts Alex and believes that she is right – that he is usually wrong about what the evidence supports on occasions similar to this one. (Horowitz 2014: 719)

Provided the information of sleepy detective – and the background assumption that respecting one's *total* evidence is an important standard of epistemic rationality – what is the rational response to Sam's predicament? In other words, what doxastic attitude should he hold with respect to the identity of the thief? And what should he believe about what his first-order evidence supports?⁶

The literature is divided into three main camps. According to *steadfast* views, Sam should simply stick to his guns. That is to say, he should keep both his high confidence that $\langle p \rangle$ (i.e., $\langle \textit{Lucy is the thief} \rangle$) and his belief that this is what his first-order evidence supports (Kelly 2005; Titelbaum 2015). A reason in favor of this response is that Sam actually got things right to begin with. So even though the later testimony from his partner Alex is higher-order evidence suggesting that his assessment of the first-order evidence is unreliable due to sleep deprivation, this is *in fact* misleading on the particular occasion.

In contrast, *conciliatory* views hold that Sam should reduce confidence both with respect to proposition $\langle p \rangle$ and the higher-order proposition stating that $\langle \textit{my first-order evidence supports } p \rangle$ (Christensen 2007; Feldman 2005).⁷ A reason in favor of this position is that from Sam's first-person perspective the higher-order evidence constituted by Alex's testimony seems undefeated. Since Sam rationally trusts Alex to be right about his unreliable track record in relevantly similar circumstances, he should reduce his confidence at both first- and higher-order level.⁸

Notice that although steadfast and conciliatory views disagree about the rational response to cases like sleepy detective, they agree that Sam's confidence in $\langle p \rangle$ shouldn't conflict with his belief about what the first-order evidence supports. That is, both camps accept that any such level-incoherence is epistemically irrational.

Level-splitting views dispute this. According to the level-splitter, it can sometimes be epistemically rational to have a high confidence that $\langle p \rangle$ while also believing the higher-order proposition $\langle \textit{my first-order evidence doesn't support } p \rangle$. Imagine, for instance, a long deductive proof written on a whiteboard, and suppose that Beth thinks through the proof and comes to rationally believe a series of claims from which she competently deduces their conjunction, $\langle p \rangle$.⁹ Assume (quite plausibly) that Beth comes to rationally

⁶Sam's *first-order* evidence includes (propositions about) the letters and photographs that he was looking through as he worked late at night.

⁷One should realize that while a higher-order proposition like $\langle \textit{my first-order evidence supports that } p \rangle$ has *positive* normative force with respect to proposition $\langle p \rangle$, i.e., it might make it rational for you to believe that $\langle p \rangle$; other higher-order propositions like $\langle \textit{any epistemic situation makes it rationally forbidden to believe that } p \rangle$ has *negative* normative force with respect to proposition $\langle p \rangle$.

⁸For canonical work on *defeaters* in epistemology, the reader should consult Pollock (1970, 1974, 1984, 1986, 1994). See Kelp (2023) for a comprehensive overview. Note also that Christensen might be said to lean toward a level-splitting rather than conciliationist view about akrasia in his more recent work on the topic.

⁹Denoting a conjunction using the symbol " p " might be thought to overload the notation, but we allow this here for the sake of simplicity.

believe $\langle p \rangle$ by these means. Yet Beth knows that people like her – in similar situations involving long deductions – often make inferential errors. So, it may well be highly probable on her higher-order evidence that she has made an inferential error in the current situation, which suggests that $\langle \text{her first-order evidence doesn't support } p \rangle$ after all (even though we can stipulate that her belief in the truth of $\langle p \rangle$ is *in fact* correct).¹⁰ To be sure, the intended interpretation here is that the knowledge Beth possesses about people's shortcomings in situations relevantly similar to hers should be taken as higher-order evidence against her first-order attitude toward $\langle p \rangle$, but according to level-splitting views, what goes on at higher-order level need not affect the rationality of Beth's first-order attitudes.¹¹ Thus – by level-splitting lights – this is a scenario where Beth can have a high confidence in $\langle p \rangle$ while also believing the higher-order proposition $\langle \text{my first-order evidence doesn't support } p \rangle$ and be rational nonetheless.

As with Beth's logic case, a level-splitting response to sleepy detective would have it that Sam should remain highly confident that $\langle \text{Lucy is the thief} \rangle$ and simultaneously believe that this isn't supported by his first-order evidence. Epistemologists such as Williamson (2011, 2014), Lasonen-Aarnio (2014, 2020), Wedgwood (2012), and Weatherson (2010), have all favored level-splitting views although their reasons for doing so diverge.

2. Logical akrasia

While epistemic akrasia is interesting in its own right, it will not be our main concern. Our primary focus will be on an analogous phenomenon in (formal) logic. The remaining sections aim to connect the discussion of epistemic akrasia from mainstream epistemology with another existing discussion in the philosophy of logic, which concerns the use of classical logic to prove metatheoretic results (such as soundness and completeness) about a weaker, non-classical logic. It will also be suggested that some level of akrasia is unavoidable in arithmetic because of Gödel's Second Incompleteness Theorem.

As a rough starting point, we'll take *logical akrasia* to consist of a mismatch between the deductive strength of the background logic one uses to prove metatheoretic results and the logical theory one believes (officially), i.e., a form of incoherence in logical theorizing akin to what we saw in the case of epistemic akrasia.¹² So, in other words, logical akrasia will occur when one explicitly appeals to (or at least implicitly commits to) a logical principle which is not endorsed by one's own theory.^{13,14} At this point, we

¹⁰Notice the structural analogy between Beth's logic case above and the well-known *Preface Paradox* (Makinson 1965; Sorensen 2020).

¹¹For further clarification of the distinction between first-order and higher-order evidence, see Christensen (2010), Horowitz (2022), and Skipper (2021).

¹²Logic L_i is *deductively stronger* than logic L_j whenever L_i can prove more, i.e., for every set of well-formed sentences, Γ , the deductive closure of Γ under L_j is a proper subset of the closure of Γ under L_i .

¹³A concrete example of an *implicit* commitment to a logical principle could be committing to the excluded middle via an explicit endorsement of Peirce's Law – i.e., $((\varphi \rightarrow \psi) \rightarrow \varphi) \rightarrow \varphi$, where lowercase letters from the Greek alphabet are metavariables. For more on the topic of implicit commitments in logical theorizing, the reader should consult Cieśliński (2017), Fischer *et al.* (2021), and Horsten and Leigh (2017).

¹⁴It's worth flagging that metatheoretical results (like soundness and completeness) use mathematical theories (about sets, functions, models, etc.) which are themselves the kinds of things we might explicitly endorse by our own lights, or not. Questions about the match of what we explicitly endorse and what we implicitly rely on arise at a number of levels in and around logic, both about theories that are properly *logic* (in the narrowest sense), and those formal theories we use in reasoning about logic. The example of

won't distinguish between *meta-logic* and *metatheory*, but we'll discuss the potential importance of drawing this distinction in section 2.1.

Now, to provide a concrete example of logical akrasia, consider the following passage from Beall and Restall:

The Intuitionist. What we have presented is a straightforward account of Tarski's model theory for classical predicate logic, and a simple account of truth conditions in a possible worlds semantics. We have claimed that such accounts deliver classical logic. Is this indeed the case? It is commonly thought that this is the case, but in present company we may have reason to question this thought. Upon an inspection of the usual soundness and completeness proofs, we shall see that the full power of classical logic is required to complete the proof. To show, for example, that in every model $AV\neg A$ is satisfied, we need to show, for each model \mathfrak{M} , that $\mathfrak{M}\models A$ or $\mathfrak{M}\models\neg A$. But this is an instance of the excluded middle! An intuitionist (for example) who rejects the law of the excluded middle will not endorse this reasoning. What can we say about this? (Beall and Restall 2006: 39)

One thing we could say – to answer Beall and Restall's query – is that the intuitionist is in a state of logical akrasia. The case of the intuitionist logician who, when doing the metatheory of intuitionistic logic, finds themselves using classical (nonconstructive) principles, is an illustrative example of logical akrasia as it involves a clear incoherence of the kind we are interested in. In sum: this logician happens to presuppose logical principles, when producing metatheoretic proofs, that are not endorsed by their own (intuitionistic) standards, and as was the case with the epistemic counterpart, logical akrasia seems self-undermining and irrational; or, to put this point more vividly: the intuitionist searching for an acceptable metatheory using a classical background logic seems akin to fixing a leaky roof by accustoming oneself to a wet floor.

2.1. Defining logical akrasia

So far, so good! Let's now define logical akrasia in a more regimented fashion:

Logical Akrasia. Subject S , believing logical theory T , is in a state of logical akrasia if and only if S commits to a logical principle that S 's logical theory T fails to endorse as valid.¹⁵

arithmetic we'll consider below is just the most simple example of a formal theory in this vicinity, and concerns around the epistemology of this theory and any akrasia involved therein, is therefore very much on topic, and not just an illustration of a similar phenomenon.

¹⁵Define a *logical theory* in the standard way. A logical theory is an ordered pair $T = \langle \mathcal{L}_i, \mathcal{L}_i^M \rangle$ such that \mathcal{L}_i is an object-logic and \mathcal{L}_i^M a metatheory.

On the one hand, we have *object-logic* $\mathcal{L}_i = \langle \Phi, \Sigma, P \rangle$, where Φ specifies a language of both logical and non-logical vocabulary while Σ gives a syntax for Φ (determining its well-formed formulas). P in turn provides a set of inference rules and/or axioms for syntactic manipulation of the symbolic strings that Φ gives rise to.

On the other hand, we find *metatheory* $\mathcal{L}_i^M = \langle \Phi_M, I, \models_\Phi, \vdash_\Phi \rangle$, the first element of which specifies a meta-language Φ_M . The second element I gives a semantics expressed in that meta-language, i.e., it lays down individual truth-conditions for the logical constants of Φ using Φ_M . The third element \models_Φ authorizes a semantic consequence relation between well-formed formulas of Φ . Finally, \vdash_Φ defines a syntactic consequence relation for well-formed formulas in Φ .

As this definition is too coarse-grained to capture all relevant cases, we further distinguish between:

Weak. *S*, believing logical theory *T*, is weakly akratic if *S* commits to a logical principle that *S*'s logical theory *T* fails to endorse as valid;

Strong. *S*, believing logical theory *T*, is strongly akratic if *S* commits to a logical principle that *S*'s logical theory *T* rejects as *invalid*.

Based on this weak/strong distinction, we get two non-equivalent versions of logical akrasia. To appreciate this, consider a case where an intuitionist commits to an instance of the excluded middle in a restricted situation, and suppose that their theory doesn't endorse this as valid. Then, the intuitionist can extend their theory at a later stage such that the instance of the excluded middle becomes endorsed as valid (just stipulate that the situation is decidable) – e.g., it happens to be a case concerning a quantifier-free sentence of arithmetic like $2 + 2 = 4$. Before the extension they were being weakly akratic, but not afterwards.¹⁶

Yet one should acknowledge that the tenability of the strong/weak distinction depends on the possible division: meta-logic/metatheory. There is, for instance, no difference between *failing to endorse as valid* and *rejecting as invalid* if one holds a formal and complete meta-logic rather than a non-formal and incomplete metatheory.¹⁷ If one's meta-logic is formal and complete over its domain, then the distinction between strong and weak logical akrasia collapses (since in that case everything which is not valid is simply invalid). When, say, an intuitionist holds a formal and complete meta-logic and the law of the excluded middle has counterexamples, then failing to endorse the excluded middle as valid and rejecting it as invalid amount to exactly the same.

2.2. Logical akrasia and incoherence

As we have seen above, states of logical akrasia seem to be incoherent. A point which is also frequently underscored in the literature:

If you take “logically valid” to obey a logic weaker than classical, you shouldn't ultimately be satisfied with developing your theory of that logic using inferences that are merely classically valid... (Field 2017: 14)

...what a strange approach to take, if one believes logic *X* is the correct logic. Why use an alien logic for one's metatheory – and if one does, why trust the result? (Read 2006: 208)

...it would be untoward in a logic to appeal in proof of its adequacy to principles in which the logic in question does not believe. (Meyer 1985: 13)

¹⁶Note that insofar as the distinction between weak/strong akrasia is relevant at all, it isn't just relevant to the case of intuitionism. In fact it seems relevant to many, perhaps even most, non-classical logicians like Field, Kripke, Ripley, Beall, etc. For they all take classical logic to be valid in non-problematic contexts. Some non-classical logicians are more “hardcore” and go non-classical all the way down (Priest, 2006; Weber *et al.* 2016), but this isn't the norm.

¹⁷For more on logic(s) and *formality*, see, for example, Beall and Restall (2006), Dutilh Novaes (2012), MacFarlane (2000), and Mortensen (2013).

If he rejects classical logic for the object language, how is he entitled to rely on it for the metalanguage? (Williamson 2020: 6)

These quotes notwithstanding logical akrasia are deeply entrenched in our contemporary logical theorizing. For it is no secret that classical logic serves as the golden standard in evaluations of non-classical logics (Schurz 2021), i.e., it's common practice to take classical (first-order) logic as the “neutral” backdrop against which we evaluate non-classical logics. Examples are: Łukasiewicz's three-valued logic, Kleene's (strong) three-valued logic, Brouwer's intuitionistic logic, Priest's paraconsistent logic, etc.¹⁸

Where does this leave us? Is the current modus operandi of non-classical theorizing severely misguided? Well, insofar as we want *reflective equilibrium* (Resnik 1985, 1996, 2004) between our logical theories and considered judgments about *logicality* (i.e., validity, consistency, implication, equivalence, etc.), there is a sense in which the answer is affirmative. According to Michael Resnik, one's preferred logical theory and considered judgments about logicality are in a state of reflective equilibrium when:

...the theory rejects no argument that one is determined to preserve and countenances no argument that one is determined to reject... (Resnik 1996: 493)

Thus, we have at least two interpretations of reflective equilibrium, depending on how we read the term “reject”: (1) one's theory judges valid every argument one is determined to preserve; and (2) one's theory doesn't judge invalid any argument one is determined to preserve. For weak logical akrasia to violate reflective equilibrium, we need interpretation (1); not interpretation (2) (cf. section 2.1); or, as one may otherwise put it, for weak logical akrasia to violate reflective equilibrium, we need the strong interpretation of reflective equilibrium, not the weak one. The ideal of strong reflective equilibrium is incompatible with states of weak logical akrasia. Hence, the golden standard of non-classical logicians – committing themselves to classical principles in their metatheoretical pursuits – appears to be a standard of fool's gold in at least one sense.¹⁹

2.3. The analogy with epistemic akrasia

If we return to the analogy between epistemic and logical akrasia for a minute, we can now appreciate how both the weak and strong version of logical akrasia resemble the standard definitions of epistemic akrasia in various ways.

Recall first the appealing thought from section 1 stating that epistemic rationality requires coherence between: (A) an agent's doxastic attitudes in general, and (B) their specific beliefs about what doxastic attitudes are rational. Epistemic akrasia occurs whenever *S* adopts doxastic attitudes that don't live up to *S*'s own standards of rationality. Logical akrasia, similarly, occurs when *S* commits to logical principles that don't live up to *S*'s own standards of logic. So, in both cases, the problem is one of not meeting one's own ideal (rather than pursuing a spurious ideal).

Adding further to the analogy, we have no trouble imagining what *steadfast* and *conciliatory* responses to logical akrasia would look like. If one asserts that logical akrasia

¹⁸Consult Priest (2008) for classical evaluations of each of the non-classical logics mentioned above. See Bacon (2013) for a discussion of *non-classical* metatheories for non-classical logics.

¹⁹See Priest (2006) for further discussion of this seemingly self-undermining practice of some non-classical logicians. Note also the potentially interesting distinction between sub-classical and contra-classical logics.

calls for a revision of one's logical commitments or theory, then it would count as a conciliatory view. If one, on the other hand, submits that no revision is required, it would be a steadfast view. One could also cook up a special *level-splitting* kind of steadfastness without noteworthy ingenuity. Assume, say, that one is a logical pluralist (of some sort), then one may argue that there are benign cases of logical akrasia. Given the pluralist's dictum that more than one logic can be correct, the (level-)incoherence of logical akrasia need not be problematic. Alternatively, one might be able to pull off a level-splitting response by appealing to logical instrumentalism (Haack 1974), i.e., the view that it doesn't make sense to think of logics as being correct or incorrect rather they are simply variously useful or not. It seems plausible to suggest that appealing to instrumentalism in some way could dissolve the tension in cases of logical akrasia to the level-splitter's satisfaction.^{20,21}

3. Enter Gödel: rationality and akrasia

As advertised earlier, the two sections 3 and 4 make use of Gödel's (in)famous Second Incompleteness Theorem (Gödel 1931) to pose a dilemma related to logical akrasia. It should be noted up front that while the puzzle of arithmetic raised in these sections is *related* to logical akrasia, we'll leave it open what exactly the relation consists of, e.g., if second-order arithmetic is "logic" as per the neo-logicians, then the puzzle is just another case of logical akrasia; if Quine was right about bounds of logic, however, then it's not.

Gödel's theorem establishes that: *assuming Peano Arithmetic (PA) is consistent, PA doesn't derive Con_{PA}*. In other words, if the theory PA is consistent, then PA cannot derive its own consistency.²²

²⁰The reader should consult Russell (2019) for an extensive overview of the many different kinds of logical pluralism on the market. It's also worth noting the close analogy between *weakness of will* in logic and recent discussions of the *normative significance of logic*. See, for example, "What is Logical Monism?" (forthcoming), where Clarke-Doane argues that logic doesn't settle what to infer, even bracketing Harman's complications of "Change in View" (1986), since it's a contingency of language that we happen to have the logical concepts that we do. Thus, even assuming that, say, the disjunction of the Riemann Hypothesis and its negation is valid (out of our mouths) it remains open whether to infer this disjunction, since it remains open whether to defer to the metalogical concepts that we actually find ourselves with. Thanks to an anonymous reviewer for pointing out this connection.

²¹Interestingly, Tim Williamson who is a card-carrying level-splitter with respect to epistemic akrasia, doesn't seem to be one when it comes to logical theorizing. See, for instance, his review of Kit Fine's compatibility semantics (Williamson 2020). To be fair, however, as Williamson discusses in his review, the issue with Fine's compatibility semantics is rather complicated and depends on the motivation for adopting the relevant non-classical logic in the first place.

²²Here we simply take the theory PA to be classical first-order logic extended with seven axioms. The language of first-order arithmetic can be specified as follows.

\mathcal{L}_{PA} : The language of first-order arithmetic contains the usual logical vocabulary (connectives, quantifiers, etc.) and auxiliary symbols such as brackets and punctuation marks. The set of primitive extralogical symbols is $\{+, \times, 0, S\}$ denoting addition, multiplication, zero, and the successor function, respectively.

Terms, formulas, and sentences, of \mathcal{L}_{PA} are also defined in the usual way. The formalized axioms of PA are: (Ax1) $\forall x(S(x) \neq 0)$; (Ax2) $\forall x\forall y(S(x) = S(y) \rightarrow x = y)$; (Ax3) $\forall x(x + 0 = x)$; (Ax4) $\forall x\forall y(x + S(y) = S(x + y))$; (Ax5) $\forall x(x \times 0 = 0)$; (Ax6) $\forall x\forall y(x \times S(y) = (x \times y) + x)$; (Ax7)

Consider now the following akratic puzzle:

Akratic Peano Arithmetic. In using Peano Arithmetic, PA, subject *S* is at least implicitly committing to (and relying on) PA's consistency. After all, if the theory were inconsistent it's no help in sorting out truths from falsehoods. But if *S*'s theory is PA, then the theory doesn't itself prove that PA is consistent (by Incompleteness). So, in that case *S*'s theory doesn't prove the claim that *S* is committed to, and consequently *S* is at least weakly akratic (cf. section 2.1). Of course, *S* can easily extend *S*'s theory (why not?), and then consider PA with Con_{PA} added as an axiom. This would provide *S* with a proof of Con_{PA} in a single line. But alas, now the issue of akrasia arises at the level of appeal to the consistency of that theory, i.e., $PA + Con_{PA}$. And so on ad infinitum...

In *S*'s use of PA it turns out that *S* is not just committed to the theory PA itself, but also the stronger theory $PA + Con_{PA}$. Ergo, *S* is committed to a "logical" principle that their theory cannot prove, and thus *S* is in a state of akrasia.²³ By construction, there is no way for *S* to avoid committing to Con_{PA} (or $Con_{PA+Con_{PA}}$, or...) and escaping their akratic state (on the pain of triviality). While this doesn't necessarily show that *S* is irrational when using PA, it does entail that *S* is committed to something which goes beyond *S*'s own theory in such situations.

Yet this is a *positive* kind of mismatch – i.e., *S*'s background logic can prove something which *S*'s believed theory cannot – rather than a violation of a *negative* rationality constraint such as the previously mentioned Akratic Principle. Recall the Akratic Principle stating that:

No [epistemic] situation rationally permits any overall [doxastic] state containing both an attitude *A* and the belief that *A* is rationally forbidden in one's situation. (cf. section 1)

$\{[\Phi(0) \wedge \forall x(\Phi(x) \rightarrow \Phi(S(x)))] \rightarrow \forall x(\Phi(x)) : \Phi(x) \in \mathcal{L}_{PA}\}$. Notice that axiom 7 is really the set of arithmetical sentences falling under the axiom schema of mathematical induction, i.e., it's an infinite set of axioms rather than just a single axiom. Obviously, \mathcal{L}_{PA} allows us to express claims about the natural numbers in the theory PA, e.g., claims concerning addition and multiplication, etc. But what is more important for our purposes below is that we'll tacitly assume some form of *coding*. As Kurt Gödel (1931) showed it is possible to define a procedure, starting with assigning natural numbers to primitive expressions of \mathcal{L}_{PA} , and then extending the assignment to more complex syntactical objects. Eventually, unique numbers become assigned to terms, formulas, and sequences of formulas; and in effect, we can then view some statements of first-order arithmetic as assertions about syntax. In other words, it becomes possible for us to use PA "introspectively." The most famous example of this is of course the *Gödel sentence* ("G"), which is at the heart of Gödel's Second Incompleteness Theorem. The sentence *G* states about itself (via such-and-such substitution operations) that it isn't a provable sentence in PA (Berto 2011: 92). While it isn't essential to us *how* the encoding from linguistic expressions to numbers is done – Gödel exploited the Unique Prime-Factorization Theorem to this end – it's important to note that it *can* be done. For below we'll appeal to the consistency claim – Con_{PA} – stating that the theory PA is consistent, which in a way is just a regular claim made in \mathcal{L}_{PA} , and yet, this is only the case *indirectly* via our tacit coding procedure.

²³Some might hesitate to admit that the case results in akrasia because they don't see Con_{PA} as a genuine *logical* commitment: *Why isn't Con_{PA} considered a further implicit, non-logical claim which *S* commits to?* The answer is straightforward. Con_{PA} is just a regular claim made in the language of PA (and definitely not a contingent empirical fact). There may of course be a sense in which PA doesn't count as strictly "logical" but rather as "mathematical." Even so the kind of commitment *S* holds with respect to Con_{PA} is not essentially different from the one *S* has toward the axioms of PA (though implicit). Hence – upon reflection – there is indeed a certain kind of akrasia (about the logic of the natural numbers if you like) arising in the puzzle.

As this is a negative principle in the sense that it involves an assertion about rationally forbidden states, the puzzle of akratic arithmetic is not in any obvious way a violation of it. Unless we are ready to grant that it's rationally forbidden to commit oneself to something which one cannot prove, of course, but this seems overly strong. Nobody in their right mind would suggest that provability is a plausible guide to rationality *simpliciter*.²⁴

Nonetheless, the puzzle of akratic arithmetic does illustrate a clash with our ideals concerning epistemic rationality insofar as reflective equilibrium is among them. As logically akratic states cannot be in reflective equilibrium – given interpretation (1) from section 2.2 – the case above does indeed suggest that S 's doxastic state is epistemically irrational in a certain sense. How bad this sort of irrationality looks to the epistemologist does of course depend on the kind of good they take reflective equilibrium to achieve. On one interpretation, reflective equilibrium merely indicates that a reasoner has done what is rationally required of them relative to their initial data (e.g., a set of intuitions about certain logical inferences), but it would take further argument to show that a reasoner's doxastic attitudes are also likely to be true. Under this interpretation, reflective equilibrium is a rational ideal regarding the *internal* coherence of doxastic states rather than truth-conducive rationality.²⁵

4. The Dilemma of Arithmetical Akrasia

The upshot of section 3 is what we might call the *Dilemma of Arithmetical Akrasia*:

Insofar we take our theories to be appropriately formal and complete, then either:²⁶

- (i) we must be agnostic about the consistency of PA (on the pain of triviality), which would be extremely odd at best;
- (ii) or we must accept being arithmetically akratic, i.e., accept that we are trapped in an inescapable, infinite hierarchy of akratic states.

Notice that while taking the second horn of the dilemma doesn't rule out the existence of a rational fixpoint somewhere on the theoretical ladder, it does eliminate the possibility of an akrasia-free state which is accessible to us (since Gödel's incompleteness results range over all axiomatizable theories).

²⁴Naturally it could be argued that provability is a plausible guide to rationality in a certain *narrow* sense. It's clear enough that we can rationally believe many contingent propositions that we cannot prove to be correct, but in the case above we are not concerned with any old contingent proposition. We are concerned with the proposition that $\langle PA \text{ is consistent} \rangle$, and it's not immediately clear how it could be rational to believe Con_{PA} given that it cannot be proved using one's theory. See, for example, Chow (2019) and Gentzen (1936) for further discussion of this non-trivial question. As an anonymous reviewer points out, another reason to think that provability cannot be a guide to rationality, even in mathematics, is that provability is always relative to a set of axioms, and thus an account of what makes belief in axioms rational will be needed.

²⁵For further discussion of positive epistemic evaluations and their connection to truth-conduciveness, see, e.g., Alston (1989), Berker (2013), Bonjour (1985), and Littlejohn (2012).

²⁶In this context, the term "*complete*" should be understood as follows: *Within a given domain, every question is answerable, i.e., for any φ in the domain, it holds that φ or not- φ .* This kind of completeness is also known as "syntactic completeness." Note that syntactic completeness doesn't entail that every particular answer has got the same epistemic status. The point is merely to suggest that we are committed to completeness in the sense that the question of whether PA is consistent has got an answer, but this is definitely not committing us to a theory which can decide every question. Different interpretations of the term "*formal*" can be found in Dutilh Novaes (2012) and MacFarlane (2000).

Further, the Dilemma of Arithmetical Akrasia is special in at least two ways. First, it involves an *unsolvable* case of akrasia while most cases of logical akrasia are clearly solvable, e.g., by converting to a fully classical theory. If the intuitionist we met in section 2 were willing to convert to a fully classical theory, then their akratic state would dissolve. But the case of PA is different as it looks more similar to an *epistemic blindspot*; where proposition $\langle p \rangle$ is an epistemic blindspot for subject S at time t if and only if $\langle p \rangle$ is consistent but unknowable by S at t (Sorensen 1988, 2020). Similarly, the consistency presumption is fundamental to our use of PA, but it just cannot be proved (and thus known) from within the bounds of the theory itself.

Second, unlike the akratic issues we focused on above (cf. section 2), the Dilemma of Arithmetical Akrasia cuts across the divide between classical and non-classical logicians. In the case of PA, it seems that we are all either agnostic (on the pain of triviality) or akratic!

So, in the end, taking the first horn doesn't sit well with our general intellectual outlook because we want to avoid being agnostic about the consistency of PA; on the other hand, going for the second horn is an unpleasant move as it reveals a boundary on arithmetical theorizing which seems to conflict with our rational ideals (e.g., reflective equilibrium).

5. Conclusion and perspectives: escaping the Dilemma of Arithmetical Akrasia

While the exact details remain to be worked out, this final section provides a quick and dirty proposal of how one can translate the Dilemma of Arithmetical Akrasia into a case of regular epistemic akrasia; and further how one might escape the dilemma when it's spelled out this way.

Let's first reformulate the dilemma such that it becomes a case of epistemic akrasia. Spelled out in terms of premises and conclusion(s), we get:

1. S believes PA [by Indispensability].
2. S believes Con_{PA} [by No-Miracles].
3. S believes Con_{PA} is a logical principle [in absence of reasons to the contrary].
4. S believes $\nVdash_{PA} Con_{PA}$ [by Incompleteness].
5. S believes in the strong interpretation of reflective equilibrium with respect to PA:
It's permissible for S to believe a logical principle only if PA proves it.
6. Therefore: S believes Con_{PA} and believes that \langle it's forbidden to believe Con_{PA} \rangle .
7. Ergo: S is epistemically akratic.

Now, it seems fair to suggest that we don't want to consider rejecting premises (1), (2), and (4); which leaves us with the possibility of rejecting one or both of (3) and (5) in order to escape the dilemma. That is to say:

1. ~~S believes PA [by Indispensability].~~
2. ~~S believes Con_{PA} [by No-Miracles].~~
3. S believes Con_{PA} is a logical principle [in absence of reasons to the contrary].
4. ~~S believes $\nVdash_{PA} Con_{PA}$ [by Incompleteness].~~
5. S believes in the strong interpretation of reflective equilibrium with respect to PA:
It's permissible for S to believe a logical principle only if PA proves it.
6. Therefore: S believes Con_{PA} and believes that \langle it's forbidden to believe Con_{PA} \rangle .
7. Ergo: S is epistemically akratic.

Consider first the possibility of rejecting (3), i.e., the logicity of Con_{PA} . The principle stated by Con_{PA} is certainly a well-formed sentence of PA; and in that specific sense it is logical. It

also expresses something essential to PA (or at least our use of PA). But perhaps Con_{PA} is still not logical in the right way for strong reflective equilibrium to apply to it. Is it really fair to expect strong reflective equilibrium to apply to paraconsistent logical theories, for example?

Consider next the possibility of rejecting (5) instead. While reflective equilibrium may be initially plausible when viewed as a philosophical *method* or as “the prima facie epistemology of logic” (Cohnitz and Estrada-González 2019: 137), it does seem like an overly demanding *output* to expect from applying the method in the context of logical theorizing and PA. Gödel’s theorem already suggests that epistemic principles of that kind are hopeless, because every theory with the same expressive power as PA (or more) has a Gödel sentence. Why insist on something impossible? (This is an “ought-implies-can” violation perhaps). Moreover, what reason do we have to think that strong reflective equilibrium is a norm of belief within arithmetical theorizing? It seems that you’ll need overly demanding bridge principles to establish the right connections between the formal sciences and epistemology in order to get this going (cf. MacFarlane 2004). Another way to put this point: the dilemma relies on it being an epistemic ideal that there is a reflective equilibrium between what one is committed to and what one’s believed theory can prove (call it “RE”). An alternative, and perhaps more plausible ideal is that there be a reflective equilibrium between what one’s committed to and what one’s epistemic practice can justify (call it “RE*”). What speaks in favor of RE over RE*?

Supposing that at least one of the rough strategies outlined above is successful in letting us escape the dilemma – when it’s framed in terms of epistemic akrasia – we are thus left to ask whether it’s still a problem if S is *arithmetically* akratic after rejecting either of these premises and avoiding epistemic akrasia. Some theorists of the post-Gödel era may simply shrug their shoulders and bite the bullet here. In a way what Gödel’s second incompleteness result tells us is that we can’t both have consistency and syntactic completeness when it comes to theories with a certain amount of expressive power. So, perhaps some level of akrasia is just something that working logicians, mathematicians, computer scientists, etc., have come to live with in the aftermath of Gödel. They may also want to suggest that there is an important difference between the cases of logical akrasia exemplified by the intuitionist rejecting the excluded middle (cf. section 2) and the specific case of PA. In the former, the intuitionist can’t combine their official theory and the background logic they are committing to into a jointwise consistent whole, whereas this is certainly possible in the latter – it’s just that the background logic must be stronger than the theory PA itself.

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