The book should be useful both as a convenient source of information about the present state of the problem and as a stimulus to further research. While it is true that abstract analysis (both harmonic and functional) has already had a significant influence on the development of the theory of the moment problem, it seems very likely that further fruitful interaction is possible here. For instance, the trigonometric moment problem seems to fit more easily into the general structure of abstract harmonic analysis than does the power problem; further work on abstract formulations of the power problem would evidently be of great interest. J. H. WILLIAMSON

YANO, K., *Differential Geometry on Complex and almost Complex Spaces* (Pergamon Press, 1965), xii + 323 pp., 90s.

The theory of complex and almost complex spaces belongs to one of the most attractive sections of modern Differential Geometry. Complex spaces first appeared in the early 1930's, in papers by Schouten and van Dantzig, and Kähler. Their studies had applications to algebraic varieties and these applications stimulated the interest of the differential geometers of the immediate post-war period. At that time, the emphasis in differential geometry was changing from local to global considerations and the development of the theory of complex spaces was influenced by this change. Nevertheless, tensors play an important part and a casual glance at the book under review is enough to show that it is necessary to have a working knowledge of the tensor calculus in order to understand the theory. The concept of an almost complex space arose in 1947 in the work of Weil and Ehresmann. The idea stems from the observation that a complex space admits a second order mixed tensor field whose square is minus unity, and that many properties of complex spaces depend only upon the existence of such a tensor. In an almost complex space, this property is taken as the starting point. A complex space is almost complex, but an almost complex space need not be complex, so that a wider class of spaces is being studied.

This book is indeed a masterly exposition of the subject, with an impressive wealth of detail. The patience needed to follow through the calculation in terms of tensor components should be rewarded by an appreciation of the beauty of the subject; there is much in complex and almost complex spaces that is mathematically attractive. Thus for the would-be differential geometer this book is a necessity; for others it could still be rewarding. The author has put in an immense amount of work. It might appear that he has left no avenue unexplored, but, as he himself states in the preface, the theory is a very fruitful one and he hopes that some of his readers will be encouraged to exploit this domain of differential geometry.

The first chapter is devoted to basic definitions concerning differentiable manifolds; it includes accounts of the concepts of Lie derivative and Killing vector. Green's theorem and some of its applications are discussed in Chapter II. These results are well worth a study in their own right, but are introduced here because of their wide applications in the present work. Complex manifolds are introduced in Chapter III. There are accounts of the basic properties of compex tensors and Hermitian and Kähler spaces. The study of Kähler spaces is continued in the next chapter, where the results of Chapter II together with Hodge's theorem on harmonic tensors are applied to prove a considerable body of theorems. The concept of an almost complex space appears in Chapter V. The question of the integrability of an almost complex structure (that is, whether the structure is induced by a complex structure) is studied early in Chapter V and several necessary and sufficient conditions are given. In particular, the important tensor introduced by Nijenhuis is discussed. Affine connexions in almost complex spaces are studied in the next chapter; an account of A. G. Walker's tensor differentiation is also given. Chapters VII and VIII are devoted to studies of almost Kähler spaces and almost Tachibana spaces respectively. Both types are special cases (with certain similar features) of almost Hermite spaces, which form the main topic of Chapter IX.

Many of the properties of complex and almost complex spaces can be studied in terms of two conjugate complementary distributions. In some ways, an almost complex space behaves like a kind of local product in which the components are not real. This suggests that concepts analogous to those studied in almost complex spaces might be of interest in real spaces which are locally products, or which behave in a similar way. This rather vague idea is clarified by studying what occurs in the almost complex case and making the obvious analogies. The situation is investigated in Chapters X and XI. The former deals with local product spaces and the latter with the concept of almost product space, which was initiated by Walker. Like an almost complex space, an almost product space is characterised by the existence of a mixed second order tensor field; however the square of this tensor is unity and not minus unity as in the almost complex case. One other different feature is that in the almost product case, the dimensions of the complementary distributions are not necessarily equal. In the final chapter, the author returns to almost complex spaces and studies holomorphically projective relations between connexions in such spaces.

The book is expensive, but no doubt the large numbers of formulae involving tensor components have resulted in high costs. Complex conjugates are denoted by the use of a bar; the weak-sighted may have a certain amount of difficulty with this. Apart from this, the book is very well produced and the printing is excellent. All in all it is an important addition to the literature on differential geometry and it is hoped that the author's wish that it should encourage further research is fulfilled.

E. M. PATTERSON

PONTRYAGIN, L. S., BOL'TANSKII, V. G., GAMKRELIDZE, R. S. AND MISCHENKO, E. F., *The Mathematical Theory of Optimal Processes*, translated by D. E. Brown (Pergamon Press), 338 pp., 80s.

A controlled process may be described by a system of ordinary differential equations

$$\frac{dx^i}{dt} = f^i(x^1, \ldots, x^n, u^1, \ldots, u^r), \quad i = 1, 2, \ldots, n,$$

where the x^i are the phase coordinates of the controlled entity and the u^j are the control parameters defining the course of the process. Given the initial and final values, $x^i(t_0)$ and $x^i(t_1)$, the problem of optimal control is to find the functions $u^j(t)$ which minimise an integral functional of the form

$$J = \int_{t_0}^{t_1} f^0(x^1, \ldots, x^n, u^1, \ldots, u^r)dt.$$

The classical calculus of variations is not adequate for solving problems of this type when, as is the case in modern applications, the u^j are subject to restrictive conditions such as

$$|u^1| \leqq 1 \text{ or } (u^1)^2 + (u^2)^2 \leqq 1,$$

and the optimal control turns out to be situated at the limits of such inequalities and to involve jumps between, say, $u^1 = 1$ and $u^1 = -1$ in the above example of $|u^1| \leqq 1$.

The book deals very thoroughly with the well-known and powerful Pontryagin Maximum Principle method for such problems. The presentation combines readability and rigour, and three of its seven chapters constitute an adequate account of the subject for engineers. Topics dealt with include linear time-optimal processes ($J = t_1 - t_0$), application to the approximation of functions, a pursuit problem,