elasticity, and bending of linearly elastic plates with transverse shear deformation. Several types of such techniques are analysed in depth for each model: the direct method (based on the Green–Somigliana representation formulae) and the so-called classical, alternative, modified and refined indirect methods, where the solution is assumed to have a convenient integral representation in terms of single layer and double layer potentials. Boundary value problems with Dirichlet, Newmann and Robin conditions for interior and exterior domains are reduced to integral equations on the boundary contour, which are then solved in spaces of Hölder continuous or Hölder continuously differentiable functions. The solution is based on an algebra of operators associated with the boundary values of the potentials and the spectral properties of these operators.

It is shown that two-dimensional problems are more difficult to handle than three-dimensional ones on two counts. First, there are boundary curves for which zero is an eigenvalue of the operator defined by the boundary values of the single layer potential. For the Laplace equation this happens when the logarithmic capacity of the curve is equal to one. The logarithmic capacity is generalized to a characteristic matrix of the boundary curve for the systems of plane strain and bending of plates. The 'pathology' occurring on curves for which this characteristic matrix is singular is eliminated through a modification of the fundamental solution (the modified indirect method) or the addition of a constant (rigid displacement) to the potential used in the integral representation of the solution (the refined indirect method). Second, the fundamental solutions do not decay to zero at infinity. To be able to derive Green–Betti and representation formulae, special asymptotic classes of functions are introduced for the far-field of the solution. These classes consist of functions of finite energy and they facilitate the construction of uniqueness theorems.

In the last chapter all direct and indirect techniques described in the book are compared to find out which one is best suited for numerical computations.

The text is written clearly and the proofs are given in detail. For fluency some of the necessary hard-analysis material is gathered in an appendix at the end. Overall, the book offers a comprehensive treatment of the subject matter and constitutes a very useful source of information for mathematicians and other scientists interested in boundary integral equation methods.

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SUNDER, V. S. *Functional analysis: spectral theory* (Birkhäuser Advanced Texts, Basel, 1998), ix+241 pp., 3 7643 5892 0 (hardback), DM 78.

The last 50 years have seen the appearance of several notable introductory texts on functional analysis, the emphasis of each reflecting its author's particular research interests. The author of this book works on von Neumann algebras, part of the more general field of operator algebras, the norm-closed algebras of operators on Hilbert spaces. It is no surprise that the book's mix of topics has particular relevance to this area. Despite the fact that operator algebras is one of the most active areas of functional analysis, this is, to my knowledge, only the second general text tailored particularly to the needs of those entering this field.

The treatment is mostly conventional, the exposition very clear and well explained. The main part of the book, amounting to three quarters of its length, is divided into five chapters. The first, 'normed spaces', is a concise but fairly complete treatment of the elements of Banach space theory. All the fundamental topics are here, including completeness, the dual of a normed space and spaces of bounded linear mappings. Several of the most important examples of Banach spaces are presented, and nice accounts of the Hahn–Banach, closed graph, open mapping and Banach–Steinhaus theorems are given. In the final part of the chapter, there is a short introduction to the theory of locally convex topological vector spaces, which sets the scene for the definition of the weak and weak*-topologies on a Banach space and its dual, and the statement and proof of Alaoglu's theorem.

The second chapter is a more or less standard introduction to the theory of Hilbert spaces and linear operators on Hilbert spaces. However, the chapter ends with a section on weak and strong convergence, a topic fundamental to von Neumann algebras, but not often encountered in a text at this level.

Chapter 3, the longest chapter, introduces Banach algebras and $C^*$-algebras. Again the treatment is very standard. There are accounts of the theory of abelian $C^*$-algebras, and of the fundamental aspects of the representation theory of general $C^*$-algebras up to the Gelfand–Naimark–Segal construction, the Double Commutant theorem and the Gelfand–Naimark theorems. A lengthy final section is devoted to the Hahn–Hellinger theorem on the classification of separable representations of separable abelian $C^*$-algebras, and to spectral measures.

Chapter 4 opens with an account of the spectral theorem for normal bounded linear operators on a Hilbert space, which is given an elegant proof using the machinery of the previous chapter. There follow sections on the polar decomposition of a bounded linear operator and on compact linear operators on Hilbert spaces. The chapter culminates in a useful exposition of the theory of Fredholm operators.

It became apparent in the late 1960s, with the appearance of the work of Tomita and Takesaki on modular Hilbert algebras, that closed, unbounded operators on Hilbert space had become an essential part of the operator algebraist's toolbox. The final chapter is devoted to their theory.

One of the author's aims is to make his treatment as self-contained as possible, in pursuit of which he devotes the final quarter of the book to a number of appendices covering the following topics: linear algebra; aspects of set theory such as Zorn's lemma and cardinality; elementary point set topology; compactness; the Lebesgue integral; the Stone–Weierstrass theorem; and the Riesz representation theorem. While the inclusion of much of this material, particularly of the final two topics, is very appropriate, it is a little difficult to understand the need to go back quite as far as elementary linear algebra. I would have preferred to see some of this space devoted to certain important topics which are not presented in the main part of the text, such as extreme points in the unit ball of a Banach space.

As a general introduction to functional analysis this book has much to recommend it. It is, however, fair to point out that, as well as such classics as the book of Rudin [**2**], which, though it does not embrace every topic covered in the present volume, has rather greater scope, there is another book with a similar target audience, *Analysis now* by Pedersen [**1**]. This covers an even more exhaustive range of material, including such topics as convexity and the Krein–Milman theorem. Nevertheless, Sunder's is an approach of exemplary clarity. His book is a good general introduction to functional analysis, and should find particular favour with students entering operator algebras.

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## References

1. Pedersen, G. K., *Analysis now* (Springer, 1989).
2. Rudin, W., *Functional analysis*, 2nd edn (McGraw-Hill, 1991).