LETTERS TO THE EDITORS

Dear Sirs

In the *ASTIN Bulletin* Vol. 16, No. 2, p. 101–108, a paper by E. KREMER on "Recursive Calculation of the Net Premium for Largest Claims Reinsurance Covers" was published. I have a short and simple proof of the main result of this paper. Moreover, this proof shows that the result holds true for a more general assumption and that the size of the approximation error can be specifically stated. This proof might be of interest to the readers of the *ASTIN Bulletin*.

To be proven is KREMER's "approximate recursion" (3.3)

$$\mu_p \approx \mu_{p-1}(1+K_p) - \mu_{p-2}K_p$$

with $K_p = c_p/c_{p-1}$, thus

$$(*) \qquad \mu_p \approx \mu_{p-1} + (\mu_{p-1} - \mu_{p-2})c_p/c_{p-1}.$$

Here $c_1, c_2, \ldots, c_p$ are real constants depending on the nature of the cover and

$$\mu_p := \sum_{i=1}^{p} c_i E_i := \sum_{i=1}^{p} c_i E(X_{N:N-i+1})$$

is the net premium of the largest claims cover (see KREMER's equation (2.1)). Inserting this definition of μ_p in the right hand side of (*) yields

$$\begin{aligned} \text{RHS of } (*) &= \sum_{i=1}^{p-1} c_i E_i + (c_{p-1}E_{p-1})c_p/c_{p-1} \\ &= \mu_p - c_p E_p + c_p E_{p-1} \\ &= \mu_p + c_p(E_{p-1} - E_p) \\ &\approx \mu_p \end{aligned}$$

as $E_{p-1} - E_p$ is, in general, relatively small (see KREMER's examples).

While KREMER's proof uses the additional assumption (3.1)

$$p_n = p_{n-1}\ (a + b/n)$$

on the claim number distribution, the above proof shows that KREMER's result holds for every claim number distribution. In addition to this, the proof provides an expression for the size of the approximation error while KREMER in his Remark (2) could only derive the sign of this error in a special situation.

Yours sincerely

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Dear Sirs

Reading MACK (1987) made me look a bit closer at KREMER's (1986) approxi-

https://doi.org/10.2143/AST.17.2.2014975 Published online by Cambridge University Press

mate recursion (3.3). Using the notation of KREMER and MACK and letting μ_i^* denote the approximate value of μ_i, we write that recursion on the form

$$(1) \qquad \mu_p^* = \begin{cases} \mu_p & (p \leqslant 2) \\ \mu_{p-1}^* + (\mu_{p-1}^* - \mu_{p-2}^*)c_p/c_{p-1}. & (p > 2) \end{cases}$$

But then simple induction gives

$$\mu_p^* = E_1 c_1 + E_2 \sum_{i=2}^{p} c_i. \qquad (p \geqslant 2)$$

It seems simpler to apply this formula directly than to use the recursive form. Furthermore, we now easily see that KREMER's approximation actually consists in approximating E_i by E_2 for each $i > 1$ for which $c_i \neq 0$. Under reasonable regularity conditions one would typically have

$$\lim_{EN\uparrow\infty} (E_i - E_2) = 0 \qquad \lim_{EN\uparrow\infty} E_i/E_2 = 1, \qquad (i = 3, \ldots, p)$$

implying respectively

$$\lim_{EN\uparrow\infty} (\mu_p^* - \mu_p) = 0 \qquad \lim_{EN\uparrow\infty} \mu_p^*/\mu_p = 1,$$

and hence it is believed that the approximation is good if p is small compared to EN. It should however be noticed that KREMER's numerical examples seem to give a somewhat too flattering impression of the method. Instead of computing μ_p^* recursively by (1), he actually seems to apply

$$\mu_p^* = \mu_{p-1} + (\mu_{p-1} - \mu_{p-2})c_p/c_{p-1},$$

and thus he avoids cumulative errors.

Yours sincerely

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REFERENCES

KREMER, E. (1986). Recursive calculation of the net premium for largest claims reinsurance covers. *ASTIN Bulletin* **16**, 101–108.
MACK, T. (1987). Letter to the editors. *ASTIN Bulletin* **17**, 193.

Dear Sirs

In this issue of *ASTIN Bulletin* Dr. MACK and Dr. SUNDT give some comments to my paper "Recursive calculation of the net premium for largest claims reinsurance covers" (KREMER, 1986), which are published as letters to the editors. In the following I will make some remarks to these notes. In my opinion MACK's "proof" of my recursion is more a short heuristic argument than a strictly

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mathematical proof. I show in my paper how to apply some recursions of the theory of order statistics. Clearly SUNDT's reformulation of the recursive method to a nonrecursive one is of some interest. But I would prefer the recursive formula; it is in some sense an exponential smoothing technique:

$$\mu_p^* = \alpha_p \mu_{p-1}^* + (1 - \alpha_p)\mu_{p-2}^*$$

with often an atypical $\alpha_p > 1$.

Finally I take this opportunity of mentioning that the numerical results in the example of my paper are derived under the additional assumption of Poisson-distributed claims number.

Yours sincerely

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REFERENCES

KREMER, E. (1986) Recursive calculation of the net premium for largest claims reinsurance covers. *ASTIN Bulletin* **16**, 101–108.
MACK, T. (1987) Letter to the editors. *ASTIN Bulletin* **17**, 193.
SUNDT, B. (1987) Letter to the editors. *ASTIN Bulletin* **17**, 194.

https://doi.org/10.2143/AST.17.2.2014975 Published online by Cambridge University Press