BULL. AUSTRAL. MATH. SOC. 60G10
VOL. 15 (1976), 307-309. (60G15)

Abstracts of Australasian PhD theses

Excursions above fixed levels by random fields

Robert Joseph Adler

Let $X(\mathbf{t})$ be an n-dimensional random field, that is, a real valued random function whose parameter $\mathbf{t}$ takes values in the euclidean n-space R^n. This thesis is primarily concerned with excursions above fixed levels, or "level crossings", of such fields.

The generalisation of level crossings of a one-dimensional stochastic process to an n-dimensional random field clearly involves random point sets of the form $\{\mathbf{t} \in S \subset R^n : X(\mathbf{t}) = u\}$, which, for $n = 2$, form a family of contour lines in the plane. As is noted by Belyayev ([1], [2]) no technique has as yet been developed to satisfactorily study the distributional properties of these sets. We shall show that a full theory for level crossings in R^n can be developed by considering the above sets indirectly, via the "excursion sets" $A = A(S, u) = \{\mathbf{t} \in S \subset R^n : X(\mathbf{t}) \geq u\}$ which they bound. In R^1 it is true under mild conditions on $X(\mathbf{t})$ that the number of upcrossings of the level u by $X(\mathbf{t})$ in an interval $[a, b]$ is equal to the number of closed intervals in the set $A([a, b], u) \cap (a, b]$. This topological approach to upcrossings in R^1 leads to natural generalisations for R^n.

We begin in Chapter 1 by introducing our notation and the problems we shall consider. As previous work in this field is widely scattered

Received 5 January 1976. Thesis submitted to the University of New South Wales, December 1975. Degree approved, June 1976. Supervisor: Professor A.M. Hasofer.

throughout the literature, and a substantial amount is only available in Russian, we also include in this chapter a full literature survey.

In Chapter 2 we study some integral geometry, a field of mathematics concerned with the geometry of R^n . We show that using integral geometry it is possible to define a characteristic $\Gamma(A)$ of an excursion set $A(S, u)$ which generalises the notion of the number of upcrossings to R^n and has a significant topological meaning. When $n = 2$ and S is the unit square we show that $\Gamma(A)$ has a representation as a point process in R^2 which is readily amenable to probabilistic investigation.

Chapter 3 introduces a modified characteristic $\chi(A)$, motivated by seemingly more appropriate concepts from differential topology. The characteristic χ is closely related to the well known Euler characteristic of the set A . As for Γ we obtain a point process representation for χ , but for χ the representation is valid for all values of n and arbitrary compact $S \subset R^n$ which have boundaries of Lebesgue measure zero.

Chapter 4 is devoted to establishing a sequence of lemmata. These determine sufficient conditions for a random field to have realisations to which it is possible to apply the techniques of the previous two chapters.

Chapter 5 contains the most important results of the thesis. We obtain explicit formulae for the mean values of $\Gamma(A)$ and $\chi(A)$ when the underlying field is gaussian. We also study in this chapter the behaviour of $A(S, u)$ for arbitrarily large values of u , again only for the gaussian case. The only previous results for excursions in R^n are for arbitrarily high levels and we now compare our results to these.

In Chapter 6 we conclude the thesis with a central limit theorem for χ as $S \to R^n$ in the special sense. This is based on a general central theorem for φ-mixing processes on R^n .

References

[1] Ю.К. Беляев [Ju.K. Beljaev], Выбросы случайн.. Полей [*Excursions of random fields*] (Moscow Univ. Inter-Faculty Lab. Statist. Methods, Publication No. 29, 1972).

[2] Yu.K. Belyayev, "Point processes and first passage problems", *Proc. Sixth Berkeley Symposium*, III, 1-17 (University of California Press, Berkeley and Los Angeles, 1972).

https://doi.org/10.1017/S000497270002267X Published online by Cambridge University Press