

account of the clarity of the exposition, the variety of its subject matter and the elegance in the presentation of what is sometimes rather cumbersome material. Most of them will deem it a privilege to have it on their shelves.

J. M. HYSLOP

SCHWARTZ, L., *Étude des sommes d'exponentielles* (Hermann, Paris, 1959), 152 pp., 1800 F.

This monograph, of considerable interest to the specialist, is a republication, somewhat revised, in three chapters of two works published separately in 1943.

Chapters I and III contain an investigation of the closed linear subspaces of $C(I)$ and $L^p(I)$ ($1 \leq p \leq \infty$) generated by sets of functions $\exp(-2\pi\lambda_\nu z)$ ($\lambda_\nu \in \Lambda$). Effectively the numbers λ_ν are real and I is an interval of the real axis (Chapter I) or imaginary axis (Chapter III): in the former case the subspace generated when the system is not total is completely characterised. Functiontheoretic applications are indicated including results on the location of singularities of functions represented in some sense by Dirichlet series.

The treatment involves a combination of transform theory and complex function-theory and is to some extent a development of methods introduced by R. E. A. C. Paley and N. Wiener, N. Levinson and V. Bernstein. Considerable use is made of an inequality for the coefficients, in representations by Dirichlet series, of the same general type as Mandelbrojt's "fundamental inequality". An important innovation is the introduction of some of the "classical" methods and results of functional analysis, for example the representation of linear functionals on $L^p(I)$.

In the short Chapter II the author considers

$$N_p(k, n, \Lambda) = \text{Max}_p \frac{|a_k|}{\|P\|_{L^p(0, \infty)}}$$

where

$$P(x) = a_0 + \dots + a_n e^{-2\pi\lambda_n x}.$$

The case $\lambda_n \equiv n$, $p = \infty$ was considered by S. Bernstein. It is shown that, as $n \rightarrow \infty$

$$\log N_p \sim 2\lambda_k \sum_1^n \lambda_p^{-1},$$

and conjectured (proved for $p = 2$) that

$$N_p \sim C_p(k, \Lambda) \exp \left\{ 2\lambda_k \sum_1^n \lambda_p^{-1} \right\}.$$

Since the original publication important related results have been obtained by the author and by A. F. Leontev, S. Mandelbrojt, J. P. Kahane and others. Extra references and a regrettably very brief account of recent work have been added in the new edition.

M. E. NOBLE

MICHAL, A. D., *Le Calcul Différentiel dans les Espaces de Banach*, Tome I (Gauthier-Villars, Paris, 1958), translated by E. Mourier, xiv+150 pp., 70 F.

This book is concerned with the extension of the differential calculus to embrace functions which map one Banach space into another, the basic notion of differentiation being that of the Fréchet differential and the associated Fréchet-Michal derivative. It is based primarily on a series of papers published by the author over the last twenty years, and is the first of two volumes of which the English manuscripts were almost

completed when the author died. It is now published in an excellent French translation (with some revision) by Mlle. E. Mourier and with a preface by M. Fréchet. The reader requires no special knowledge of functional analysis as all the relevant definitions are given, and the subject developed from first principles. Occasionally it has been necessary to quote general theorems or special results outside the scope of the book and adequate references are then given.

The book opens with an account of the classical theory of Volterra integral equations of the second kind and then introduces the basic ideas of polynomial (Fréchet) analytic function and (Fréchet) differentiation, in general Banach spaces. The techniques of the abstract differential calculus are then developed and in the later chapters applied to the theory of the classical Volterra and Fredholm integral equations and to the study of the solutions of systems of ordinary linear differential equations, regarded as functions of the coefficients occurring in the equations. The discussions are all extended to cover far-reaching generalisations of these notions. The final chapter is concerned with the exponential function in Banach algebras and general Banach spaces.

This first volume gives a clear and readable account of a theory of abstract differential calculus which is little known in this country. It is to be followed by a volume dealing with applications to geometry and theoretical physics, in which the slight deficiencies of the fairly exhaustive bibliography of the first volume will no doubt be remedied.

D. J. HARRIS

Library of Mathematics—(i) *Linear Equations* by P. M. COHN, (ii) *Sequences and Series* by J. A. GREEN, (iii) *Differential Calculus* by P. J. HILTON, (iv) *Elementary Differential Equations and Operators* by G. E. H. REUTER (Routledge and Kegan Paul, 1958), 5s. each.

This series of short text-books is edited by Dr Walter Ledermann of the University of Manchester and the above volumes are the first four to appear. They are paper-backed and consist of about seventy pages each. The publishers say that they are primarily intended for readers who study mathematics as a tool rather than for its own sake and the aim is to cover the topics which are usually included in course of mathematics for scientists, engineers and statisticians at universities and technical colleges. While this is true, a perusal of these first four volumes shows that they would be useful to students in the last year at school and in the first year at the university who propose to take a degree in mathematics itself.

The books are good value at their price and if the high standard of these four volumes is maintained, the series should be very successful. The outstanding features of all these books are the clarity of exposition and the care which has been taken to give a full and detailed discussion, so that the student is not held up by irritating gaps in the argument. A word or two will now be said about the scope of the individual volumes.

(i) In the book on *Linear Equations* the properties of vectors and matrices necessary for the solution of a set of linear equations are developed in some detail. Both the nature of the solution and a practical method of solving the equations are discussed with many simple illustrative examples. The book closes with a short chapter on determinants.

(ii) The volume on *Sequences and Series* is an excellent introduction to the ideas, which are often found so difficult by a beginner, associated with the convergence of sequences and series of real numbers. There are many simple examples to illustrate the theory. Students wishing to use infinite series as a tool will find useful the treatment of the estimation of approximations to the numerical value of a series.

(iii) *Differential Calculus* deals with the differentiation of functions of a single variable, including the mean value theorem, Taylor's theorem and Newton's method