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A linear stability study of stellar rotating spheres

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Abstract. Recent observations of globular clusters imposed major revisions to the previous paradigm, in which they were considered to be isotropic in velocity space and non-rotating. However, the theory of collisionless spheroids with some kinematic richness has seldom been studied. We present here a first step in this direction, owing to new results regarding the linear stability of rotating Plummer spheres, with varying anisotropy in velocity space and total amount of angular momentum. We extend the well-known radial orbit instability to rotating systems, and discover a new regime of instability in fast rotating, tangentially anisotropic systems.

Keywords. globular clusters: general, Galaxies: kinematics and dynamics, Instabilities, Methods: analytical

1. Introduction

Globular clusters have offered an essential empirical probe to develop and test theories of stellar dynamics. The long-term evolution of such self-gravitating systems generically comprises two stages. First, on a few dynamical times, as a result of strong potential fluctuations, the cluster can undergo a violent (collisionless) relaxation (Lynden-Bell 1967), allowing it to reach a steady state. Then, on longer timescales, the cluster will slowly explore subsequent thermodynamical equilibria, as a result of two-body relaxation sourced by finite-N fluctuations (Binney & Tremaine 2008). During the first stage of evolution, the globular cluster is insensitive to its finite number of constituents, and can therefore be treated as a collisionless system. It is also during this stage that the system can develop linear instabilities, that deeply rearrange the system's orbital structure, should the violent relaxation stage have left it in an unlikely (low entropy) configuration. This is where lies the interest of this study.

The analysis of the linear stability of spherically symmetric stellar systems has already been the subject of numerous investigations. For example, the Doremus-Feix-Baumann theorem (Binney & Tremaine 2008) states that radial modes of an ergodic spherical model are all stable if the gradient of the system's distribution function w.r.t. the energy is negative. On the contrary, systems that support too many radial orbits tend to undergo the so-called radial orbit instability (ROI, see, e.g., Polyachenko & Shukhman 1981; Saha 1991; Weinberg 1991). However, all these investigations were limited to configurations having a spherically symmetric velocity distribution (i.e., a distribution function which is invariant under rotation in configuration space). This study sets out to lift this restriction and investigate the linear stability of rotating, anisotropic equilibria, i.e. systems with a non-zero total angular momentum.

Indeed, recent astrometric measurements of globular clusters, including HST (Bellini *et al.* 2017) and Gaia DR2 (Bianchini *et al.* 2018; Sollima *et al.* 2019), revealed that a large fraction of them are rotating significantly. While refining our theories of formation and evolution of spherical stellar systems to include rotation becomes necessary, these observed globular clusters are often modelled by means of stationary dynamical models (e.g., with phase space distribution functions, Jeans, and Schwarzschild techniques), and it is important to explore the stability properties of the equilibria identified in these studies.

The methods, results and conclusions described here are extensively detailed in Rozier *et al.* (2019).

2. Method

System and phase space distribution function: We first designed a distribution function (DF) describing a rotating anisotropic Plummer sphere at equilibrium, with tuneable rotation and anisotropy. This was achieved through the application of "Lynden-Bell's demon" (Lynden-Bell 1960) to a parametric family of non-rotating distribution functions described in Dejonghe (1987):

$$F(\alpha, q, \mathbf{J}) = F_0(q, E(J_r, L), L) + \alpha F_0(q, E(J_r, L), L)\,\mathrm{Sign}(L_z), \tag{2.1}$$

where $\mathrm{Sign}(x)$ is the sign function, $|\alpha| \leqslant 1$ is a dimensionless parameter that controls the amount of rotation in the system, $q \leqslant 2$ is a dimensionless parameter that controls the amount of anisotropy in velocity space, $\mathbf{J} = (J_r, L, L_z)$ are the action variables, and the non-rotating DF is defined by:

$$F_0(q, E, L) = \frac{3\Gamma(6-q)}{2(2\pi)^{5/2}}(-E)^{\frac{7}{2}-q}\mathcal{F}\left(0, q/2, \frac{9}{2}-q, 1; -\frac{L^2}{2E}\right), \tag{2.2}$$

$$\mathcal{F}(a,b,c,d;x) = \begin{cases} x^a \dfrac{{}_2F_1(a+b, 1+a-c; a+d; x)}{\Gamma(c-a)\Gamma(a+d)}, & \text{if } x \leqslant 1; \\[2ex] \dfrac{1}{x^b}\dfrac{{}_2F_1(a+b, 1+b-d; b+c; \frac{1}{x})}{\Gamma(d-b)\Gamma(b+c)}, & \text{if } x \geqslant 1. \end{cases} \tag{2.3}$$

The matrix method: We applied to this series of equilibria a formalism based on the first order perturbation of the collisionless Boltzmann equation: the response matrix formalism, designed by Kalnajs (Kalnajs 1977). In this formalism, the linear response of a spherical equilibrium is described by its response matrix:

$$\widehat{M}_{pq}(\omega) = (2\pi)^3 \sum_{\mathbf{n}\in\mathbb{Z}^3} \int \mathrm{d}\mathbf{J}\, \frac{\mathbf{n}\cdot\partial F/\partial \mathbf{J}}{\omega - \mathbf{n}\cdot\mathbf{\Omega}(\mathbf{J})} \left[\psi_{\mathbf{n}}^{(p)}(\mathbf{J})\right]^* \psi_{\mathbf{n}}^{(q)}(\mathbf{J}), \tag{2.4}$$

where $\mathrm{d}\mathbf{J}$ is the volume element in action space, $\mathbf{\Omega} = (\Omega_r, \Omega_\phi, 0)$ are the intrinsic frequencies of an orbit in this volume element, $\psi_{\mathbf{n}}^{(p)}$ is a Fourier-transformed element of a given bi-orthogonal potential-density basis, and $\omega = \omega_0 + \mathrm{i}\eta$ is the (complex) temporal frequency at which the matrix is evaluated.

Searching for instabilities: A mode of the system is identified by finding a location in ω-space where the response matrix $\widehat{\mathbf{M}}$ has an eigenvalue equal to 1. The information on

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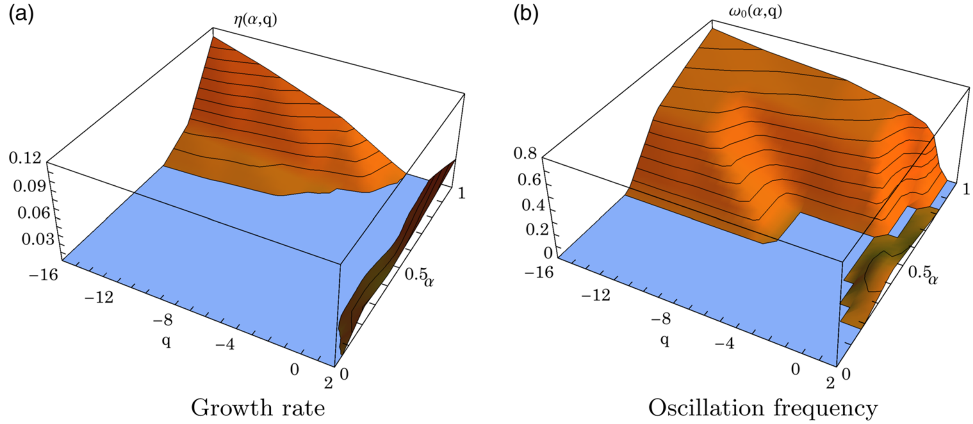

Figure 1. Illustration of the dependence of the growth rate η (left panel) and oscillation frequency ω_0 (right panel), as a function of the cluster's parameters (α, q). We searched for unstable modes on a (α, q)-grid composed of the locations $\alpha = 0, 0.2, 0.4, 0.6, 0.8, 1$ and $q = -16, -12, -6, -2, 0, 1, 2$. The blue plane represents our growth rate threshold, $\eta = 0.01$, for mode selection.

the shape of the mode is then contained in the corresponding eigenvector. When this mode is an instability, its growth rate is $\eta > 0$, and ω_0 is its oscillation frequency. For a mode with m-fold symmetry, the pattern speed Ω_{p} is linked to the oscillation frequency through $\Omega_{\mathrm{p}} = \omega_0/m$. In this study, unstable modes were identified through numerous computations of $\widehat{\mathbf{M}}$ on a fine grid in $\omega \in \mathbb{C}$, using the construction of Nyquist contours. The details of this method can be found in Rozier *et al.* (2019).

3. Results

The main results of our exploration are reported Fig. 1, where are represented the growth rates and oscillation frequencies of the m=2 instabilities which could be identified through the matrix method in a significant fraction of (α, q) space. Linear instabilities develop in surfaces coloured in orange, while the blue plane shows regions where no instability could be identified at the accuracy level of our method. Two main surfaces of instability stand out in this figure:

• The pyramid at the top of the $\eta(\alpha, q)$ panel ($\alpha \gtrsim 1/2, q < 0$) is specific to tangentially-biased and rotating systems.

• The rectangle in the bottom right part of the $\eta(\alpha, q)$ panel ($q > 0$). These instabilities are specific to radially-biased systems, and appear as an extension of the radial orbit instability to rotating systems.

Details on the exploration of the same equilibria with N-body simulations can be found in Rozier *et al.* (2019), and are in good agreement with the analytical results presented here.

When one focuses on the radial orbit instability ($q > 0$), it appears that two types of instabilities are actually competing. Those can be identified through either their pattern speeds, or more convincingly through the shape of the unstable mode in configuration space. At low rotation ($\alpha \lesssim 1/2$), bar-like instabilities develop with a relatively high oscillation frequency. We referred to this regime as the Fast ROI. At high rotation ($\alpha \gtrsim 1/2$), spiral-like instabilities develop with a relatively low oscillation frequency, which led us to refer to this regime as the Slow ROI. While both regimes seem to emerge from the same processes as the previously known radial orbit instability, i.e the alignment of near-radial orbits when the gravitational torque takes over the dispersion in their tumbling

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rates, it should be noticed that the Slow ROI can only dominate in fast rotators, hence a new distinction is required to explain the different behaviours. This distinction might be related to the different types of orbits involved in the instabilities.

4. Conclusions

We explored the linear stability of Plummer spheres in anisotropy-angular momentum space, using Kalnajs' matrix method. This led to the discovery of a new instability regime in tangentially anisotropic fast rotators, as well as the extension of the radial orbit instability to rotating systems. This first step in the theoretical study of rotating stellar spheroids opens some perspective in our dynamical understanding of globular and nuclear star clusters.

Our results clearly show that most systems with significant internal rotation are linearly unstable. The scenario implied by these findings is that a spherical system with high rotation will naturally evolve towards a triaxial or axisymmetric configuration, which can be transient or not on secular timescales, and convert part of its rotational support into random motions through the development of the linear instability.

On the theoretical side, it would be of interest to investigate the impact of angular momentum on the secular evolution of the stable equilibria. The effect of such an additional physical ingredient could be substantial, as rotation directly impacts the set of available frequencies allowing resonant relaxation to reshuffle the system's orbital structure. The investigation presented by Hamilton *et al.* (2018) could be naturally extended in that respect. Certainly, N-body studies show that the effect on the time to core collapse can exceed 20% (Breen *et al.* 2019).

On the observational side, our study can help constraining the measurement of anisotropy and rotation in globular clusters by providing a physically-based interpretation of such two-dimensional parameter space. This can be of some importance in the observation of unresolved stellar systems, where the mass-anisotropy degeneracy is a major issue. Similarly, our method can be used on Schwarzschild models for globular clusters and other spheroidal stellar systems to test their linear stability.

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