Bull. Aust. Math. Soc. **93** (2016), 223–230
doi:10.1017/S0004972715001379

REFINED MOTIVIC DIMENSION OF SOME FERMAT VARIETIES

SU-JEONG KANG

(Received 17 June 2015; accepted 27 September 2015; first published online 25 November 2015)

Abstract

Using the inductive structure of a Fermat variety by Shioda and Katsura ['On Fermat varieties', *Tohoku Math. J.* (2) **31**(1) (1979), 97–115], we estimate the refined motivic dimension of certain Fermat varieties. As an application of our computation, we present an elementary proof of the generalised Hodge conjecture for those varieties.

2010 *Mathematics subject classification*: primary 14C30; secondary 14C25.

Keywords and phrases: refined motivic dimension, generalised Hodge conjecture, Fermat varieties, inductive structures.

1. Introduction

The Fermat hypersurface in $\mathbb{P}^{n+1}$ of degree m, denoted by X_m^n, is the nonsingular hypersurface in $\mathbb{P}^{n+1}$ defined by the equation $x_0^m + x_1^m + \cdots + x_{n+1}^m = 0$. Thanks to the geometric and arithmetic properties that Fermat varieties possess, the classical Hodge conjecture for certain Fermat varieties has been known for years: the Hodge conjecture holds for X_m^n for m a prime or at most 20 [7, 8]. One approach to showing this was taken by Shioda. Using the inductive structure of Fermat varieties that Katsura and himself established [9], Shioda described the spaces of Hodge cycles and algebraic cycles in terms of eigenspaces of morphisms on $H^n_{\mathrm{prim}}(X_m^n, \mathbb{Q})$, induced by the action of the group of mth roots of unity on X_m^n. This eigenspace description gives rise to a system of linear Diophantine equations, and certain numerical conditions on the solutions of the system imply the Hodge conjecture for X_m^n. Shioda's numerical computation also implied the Hodge conjecture for X_m^n for $n \leq 10$ and $m = 21$ [8].

Given a complex smooth projective variety X, singular cohomology with rational coefficients carry two natural filtrations: the *coniveau* filtration $N^\bullet$ and the *level* filtration $\mathcal{F}^\bullet$. The pth degree of each filtration generalises the space of algebraic cycles and that of Hodge cycles, respectively. We say that the generalised Hodge conjecture (GHC) holds for X if the two filtrations coincide. In [5], we defined and explored the properties of the mth refined motivic dimension $\mu_m(X)$ of an algebraic variety X, which

© 2015 Australian Mathematical Publishing Association Inc. 0004-9727/2015 $16.00

https://doi.org/10.1017/S0004972715001379 Published online by Cambridge University Press

is the smallest integer n such that any $\alpha \in \mathcal{F}^m H^i(X,\mathbb{Q})$ vanishes on the complement of a Zariski closed set, all of whose components have codimension at least $(i-n)/2$. Our motivation for the study was to understand the refined motivic dimension as a tool to check the GHC for certain varieties. In this note, we apply our technique to provide an elementary proof of the GHC for X_m^n in codimension one for any m and n, and in codimension two if m and n satisfy a certain condition. As a corollary of the main result, we obtain the Hodge conjecture for a four-dimensional Fermat variety X_m^4 of *any* degree m.

We collect foundational material that we use throughout the note in Section 2, including a summary of Shioda's inductive structure of a Fermat variety. The GHC in codimension one and two for Fermat varieties are the contents of Sections 3 and 4, respectively. We finish the note with a general remark on the GHC.

All varieties will be defined over $\mathbb{C}$.

2. Foundational material

Given a nonsingular projective variety X, the cohomology $H^*(X,\mathbb{Q})$ of X carries two natural filtrations: the *level* filtration $\mathcal{F}^\bullet$ and the *coniveau* filtration $N^\bullet$. The pth level filtration $\mathcal{F}^p H^i(X,\mathbb{Q})$ is defined to be the largest sub-Hodge structure of $H^i(X,\mathbb{Q})$ contained in $F^p H^i(X,\mathbb{C}) \cap H^i(X,\mathbb{Q})$, where $F^\bullet$ is the Hodge filtration on $H^i(X,\mathbb{C})$. Alternatively, $\mathcal{F}^p H^i(X,\mathbb{Q})$ is exactly the largest rational sub-Hodge structure of $H^i(X,\mathbb{Q})$ of level at most $i-2p$. Here, the *level* of a pure Hodge structure $H = \oplus H^{pq}$ is defined by

$$\mathrm{level}(H) = \max\{|p-q| \mid \dim H^{pq} = h^{pq} \neq 0\} \overset{\text{set}}{=} \ell(H).$$

The pth coniveau filtration $N^p H^i(X,\mathbb{Q})$ is defined to be

$$\begin{aligned} N^p H^i(X,\mathbb{Q}) &= \sum_{\mathrm{codim}(S,X)\geq p} \ker[H^i(X,\mathbb{Q}) \to H^i(X-S,\mathbb{Q})] \\ &= \sum_{\mathrm{codim}(S,X)=q\geq p} \mathrm{im}[H^{i-2q}(\tilde{S},\mathbb{Q}) \to H^i(X,\mathbb{Q})], \end{aligned}$$

where the sum is taken over all subvarieties S of X of $\mathrm{codim}(S,X) \geq p$ and $\tilde{S} \to S$ is a desingularisation of S. The second description of $N^p H^i(X,\mathbb{Q})$, obtained using arguments of Deligne [2], easily implies $N^p H^i(X,\mathbb{Q}) \subseteq \mathcal{F}^p H^i(X,\mathbb{Q})$. We say that the GHC holds for i and p [4, 6] if the two filtrations coincide: that is,

$$\mathrm{GHC}(H^i(X,\mathbb{Q}),p) \quad \text{means } N^p H^i(X,\mathbb{Q}) = \mathcal{F}^p H^i(X,\mathbb{Q}).$$

We simply say that the GHC holds for X if $\mathrm{GHC}(H^i(X,\mathbb{Q}),p)$ holds for any i and p. In particular, $\mathrm{GHC}(H^{2p}(X,\mathbb{Q}),p)$ is the classical Hodge (p,p)-conjecture. The following lemma states that the GHC can be used to prove the Hodge conjecture.

LEMMA 2.1 [10]. $\mathrm{GHC}(H^{2p}(X,\mathbb{Q}),p-1)$ *implies* $\mathrm{GHC}(H^{2p}(X,\mathbb{Q}),p)$.

https://doi.org/10.1017/S0004972715001379 Published online by Cambridge University Press

In [5] we defined and explored a notion of the refined motivic dimension, having in mind its application to the GHC for certain varieties. For a *fixed* integer m, the mth *refined motivic dimension* $\mu_m(X)$ of X is the smallest nonnegative integer n such that any $\alpha \in \mathcal{F}^m H^i(X, \mathbb{Q})$ vanishes on the complement of a Zariski closed set all of whose components have codimension at least $(i-n)/2$. When $m = 0$, we recover the motivic dimension $\mu(X)$ of X [1]. The refined motivic dimension and the level of the cohomology have the following relation which we will use repeatedly throughout this note.

Lemma 2.2 [5, Lemma 2.1]. *Let X be a smooth projective variety of dimension n. For each $m \geq 0$,*

(a) $\mu_m(X) \geq \mu_{m+1}(X)$; *and*

(b) $\mu_m(X) \geq \ell_m \overset{\text{set}}{=} \operatorname{level}(\mathcal{F}^m H^*(X, \mathbb{Q})) \overset{\text{def}}{=} \max\{|p-q| \mid h^{pq} \neq 0, p \geq m\}$, *where the equality holds if* $\operatorname{GHC}(H^i(X, \mathbb{Q}), m)$ *holds for all* $i \geq 2m$.

Let X_m^n be the Fermat hypersurface in $\mathbb{P}^{n+1}$ of degree m: that is, X_m^n is the nonsingular hypersurface in $\mathbb{P}^{n+1}$ defined by the equation

$$x_0^m + x_1^m + \cdots + x_{n+1}^m = 0.$$

A Fermat variety X_m^n carries an inductive structure [9]: namely, for any positive integers r and s such that $r + s = n$, there exists a commutative diagram

$$\begin{array}{ccccccc} & & Z_m^{r,s} & \xrightarrow{\quad\pi\quad} & Z_m^{r,s}/G_m & & \\ & \swarrow^{\beta} & & \searrow^{\psi} & \swarrow^{\bar{\psi}} & & \\ Y \hookrightarrow & X_m^r \times X_m^s & & \overset{\phi}{\dashrightarrow} & X_m^n & \hookleftarrow & X_m^{r-1} \coprod X_m^{s-1} \end{array} \tag{2.1}$$

with the following properties.

(1) $\phi : X_m^r \times X_m^s \dashrightarrow X_m^n$ is a rational map of degree m defined by

$$\phi(x, y) = [y_{s+1}x_0 : \cdots : y_{s+1}x_r : x_{r+1}y_0 : \cdots : x_{r+1}y_s],$$

where $x = [x_0 : x_1 : \cdots : x_{r+1}] \in X_m^r$ and $y = [y_0 : y_1 : \cdots : y_{s+1}] \in X_m^s$ and the locus of indeterminacy of ϕ is given by

$$Y = \{(x, y) \in X_m^r \times X_m^s \mid x_{r+1} = y_{s+1} = 0\} \cong X_m^{r-1} \times X_m^{s-1}.$$

(2) $\beta : Z_m^{r,s} = \mathrm{Bl}_Y(X_m^r \times X_m^s) \to X_m^r \times X_m^s$ is the blow-up of $X_m^r \times X_m^s$ along the smooth centre Y (of codimension two).

(3) The composition $\psi = \phi \circ \beta : Z_m^{r,s} \to X_m^r \times X_m^s \dashrightarrow X_m^n$ is a morphism [9, Lemma 1.2].

(4) The group $G_m = \{\zeta \in \mathbb{C} \mid \zeta^m = 1\}$ of mth roots of unity acts on $X_m^r \times X_m^s$ via

$$(x, y) \mapsto ([x_0 : x_1 : \cdots : x_r : \zeta x_{r+1}], [y_0 : y_1 : \cdots : y_s : \zeta y_{s+1}]) \text{for } \zeta \in G_m.$$

This action extends naturally to the blow-up $Z_m^{r,s}$ and $\pi : Z_m^{r,s} \to Z_m^{r,s}/G_m$ is the quotient map,

https://doi.org/10.1017/S0004972715001379 Published online by Cambridge University Press

(5) $Z_m^{r,s}/G_m$ is a nonsingular variety of dimension n [9, Lemma 1.4].
(6) $\bar{\psi} : Z_m^{r,s}/G_m \to X_m^n$ is the blow-up of X_m^n along the smooth centre $X_m^{r-1} \coprod X_m^{s-1}$.
(7) $\phi \circ \beta = \psi = \bar{\psi} \circ \pi$.

In order to show the GHC of a Fermat variety X_m^n, we check the GHC for the blow-up $Z_m^{r,s}$ for suitable r and s by means of the surjective morphism ψ. The justification for this approach is the following lemma.

Lemma 2.3 [6, Lemma 13.6]. *Let $f : X \to Y$ be a surjective morphism of projective algebraic varieties of the same dimension. If* $\mathrm{GHC}(H^i(X, \mathbb{Q}), p)$ *holds, then* $\mathrm{GHC}(H^i(Y, \mathbb{Q}), p)$ *holds.*

As we mentioned earlier, our strategy to show the GHC of X_m^n is to estimate refined motivic dimensions of varieties appearing in the inductive structure. We will need the following two lemmas on refined motivic dimension.

Proposition 2.4 [5, Proposition 2.3]. *Let $\sigma : Y = \mathrm{Bl}_Z X \to X$ be the blow-up of a smooth projective variety X along a smooth centre Z. Then,*

$$\mu_m(Y) \le \max\{\mu_m(X), \mu_{m-c}(Z)\} \quad \textit{where } c = \mathrm{codim}(Z, X).$$

Lemma 2.5. *With the notation in diagram (2.1),* $\mathrm{GHC}(H^n(X_m^n, \mathbb{Q}), p)$ *holds if* $\mu_p(Z_m^{r,s}) \le n - 2p + 1$.

Proof. Although this lemma is basically [5, Lemma 3.1], we include the proof here. Suppose $\mu_p(Z_m^{r,s}) \le n - 2p + 1$. Then, by the definition of the pth motivic dimension, any $\alpha \in \mathcal{F}^p H^n(Z_m^{r,s}, \mathbb{Q})$ vanishes on the complement of a Zariski closed set, all of whose components have codimension $\ge (n - \mu_p(Z_m^{r,s}))/2 \ge (n - (n - 2p + 1))/2 = p - 1/2$. Hence $\alpha \in N^p H^n(Z_m^{r,s}, \mathbb{Q})$: that is, $\mathrm{GHC}(H^n(Z_m^{r,s}, \mathbb{Q}), p)$ holds, and Lemma 2.3 implies $\mathrm{GHC}(H^n(X_m^n, \mathbb{Q}), p)$. □

3. The generalised Hodge conjecture in codimension one

Throughout this section, we consider the following commutative diagram (derived from diagram (2.1) with $r = 1$ and $s = n - 1$).

$$\begin{array}{ccccc} & & Z_m^{1,n-1} & & \\ & \swarrow^{\beta} & & \searrow^{\psi} & \\ X_m^0 \times X_m^{n-2} \cong Y \hookrightarrow & X_m^1 \times X_m^{n-1} & \overset{\phi}{- - - - - - \to} & & X_m^n \end{array} \qquad (3.1)$$

We prove the GHC in codimension one, as mentioned in earlier.

Theorem 3.1. *The generalised Hodge conjecture* $\mathrm{GHC}(H^n(X_m^n, \mathbb{Q}), 1)$ *holds for any positive integer m. In particular, $\mu_1(X_m^n) = \ell_1(X_m^n) \le n - 2$.*

https://doi.org/10.1017/S0004972715001379 Published online by Cambridge University Press

PROOF. We use induction on the dimension n for $n \geq 2$, as the Lefschetz (1,1)-theorem implies $\mathrm{GHC}(H^2(X_m^2, \mathbb{Q}), 1)$. Assume $\mathrm{GHC}(H^d(X_m^d, \mathbb{Q}), 1)$ holds for all $d \leq n-1$. By applying Proposition 2.4 (or [5, Corollary 2.4]) to the blow-up $Z_m^{1,n-1}$,

$$\mu_1(Z_m^{1,n-1}) \leq \max\{\mu_1(X_m^1 \times X_m^{n-1}), \dim Y\} = \max\{\mu_1(X_m^1 \times X_m^{n-1}), n-2\}, \tag{3.2}$$

where $Y \cong X_m^0 \times X_m^{n-2}$ is the disjoint union of m Fermat varieties of degree m and dimension $n-2$. Furthermore, by [5, Proposition 2.2],

$$\begin{aligned}\mu_1(X_m^1 \times X_m^{n-1}) &\leq \max\{\mu_1(X_m^1) + \mu_0(X_m^{n-1}), \mu_0(X_m^1) + \mu_1(X_m^{n-1})\} \\ &\leq \max\{0 + \dim X_m^{n-1}, \dim X_m^1 + (n-3)\} = n-1,\end{aligned} \tag{3.3}$$

where the induction hypothesis induces the second inequality, as follows. The $\mathrm{GHC}(H^{n-1}(X_m^{n-1}, \mathbb{Q}), 1)$ implies (by Lemma 2.2)

$$\mu_1(X_m^{n-1}) = \mathrm{level}(\mathcal{F}^1 H^*(X_m^{n-1}, \mathbb{Q})) = \mathrm{level}(\mathcal{F}^1 H^{n-1}(X_m^{n-1}, \mathbb{Q})) \leq n-3, \tag{3.4}$$

since the cohomology of a hypersurface X_m^d in $\mathbb{P}^{d+1}$ is given by

$$H^i(X_m^d, \mathbb{Q}) = \begin{cases} 0 & \text{for odd } i \\ \mathbb{Q} & \text{for even } i \end{cases} \quad (\text{for } i \neq d = \dim X_m^d),$$

and $\mathcal{F}^1 H^{n-1}(X^{n-1}, \mathbb{Q})$ is the largest sub-Hodge structure of $H^{n-1}(X^{n-1}, \mathbb{Q})$ of level $\leq n-3$. Combining (3.2) and (3.3), we get

$$\mu_1(Z_m^{1,n-1}) \leq \max\{\mu_1(X_m^1 \times X_m^{n-1}), \dim Y\} = \max\{n-1, n-2\} = n-1$$

and the desired conclusion follows, by Lemma 2.5. □

The aforementioned Hodge conjecture is an immediate consequence of Lemma 2.1 and Theorem 3.1.

COROLLARY 3.2. *The generalised Hodge conjecture holds for a Fermat variety X_m^n of dimension three or four and any positive integer degree m.*

4. The generalised Hodge conjecture in codimension $p \geq 2$

We apply our method to show the GHC for $p = 2$ for Fermat varieties of small degree. Recall from [3] that the level of $H^*(T)$ for a complete intersection T of hypersurfaces of degree $d_1, d_2, \ldots, d_k$ in $\mathbb{P}^{n+k}$ can be computed by the formula

$$\ell(T) = \mathrm{level}(H^*(T)) = n - 2r \quad \text{where } r = \left[\frac{n - \sum_{i \neq s}(d_i - 1) + 1}{d_s = \max\{d_1, \ldots, d_k\}}\right].$$

In particular, for a Fermat hypersurface X_m^n in $\mathbb{P}^{n+1}$,

$$\ell(X_m^n) = n - 2r_{n,m} \quad \text{where } r_{n,m} = \left[\frac{n+1}{m}\right]. \tag{4.1}$$

https://doi.org/10.1017/S0004972715001379 Published online by Cambridge University Press

THEOREM 4.1. *For $m \le 4$, the generalised Hodge conjecture* $\mathrm{GHC}(H^n(X_m^n, \mathbb{Q}), 2)$ *holds if the GHC holds for* X_m^{n-2}.

PROOF. Note that the statement holds for $n \le 4$ for any m (Corollary 3.2). We fix an integer m where $m \le 4$, and we prove Theorem 4.1 by induction on the dimension n for $n \ge 5$. Referring to the diagram (3.1), we estimate $\mu_2(Z^{1,n-1})$ for $Z_m^{1,n-1}$ using the properties in [5, Proposition 2.1].

$$\begin{aligned}\mu_2(Z_m^{1,n-1}) &\le \max\{\mu_2(X_m^1 \times X_m^{n-1}), \mu_0(Y)\}\\ &\le \max\{\mu_1(X_m^1) + \mu_1(X_m^{n-1}), \mu_0(X_m^1) + \mu_2(X_m^{n-1}), \mu_0(X_m^{n-2})\}\\ &\le \max\{n-3, 1 + \mu_2(X_m^{n-1}), \mu_0(X_m^{n-2})\},\end{aligned} \tag{4.2}$$

where the last inequality holds by Theorem 3.1 and (3.4).

First, suppose $n = 5$. Since the GHC holds for X_m^n for $n \le 4$ for any m,

$$\mu_2(Z_m^{1,4}) \le \max\{2, 1 + \mu_2(X_m^4), \mu_0(X_m^3)\} \le \max\{2, 1 + 0, \ell(X_m^3)\} = 2 = 5 - 2(2) + 1,$$

where $\ell(X_m^3) \le 3 - 2r_{3,m} \le 3 - 2(1) = 1$ for $m \le 4$ by (4.1). Hence Lemma 2.5 yields $\mathrm{GHC}(H^5(X_m^5, \mathbb{Q}), 2)$. Furthermore, this together with Theorem 3.1, implies that the GHC holds for X_m^5 (for $m \le 4$) in any codimension, and hence

$$\mu_2(X_m^5) \le \mu_1(X_m^5) \le \mu_0(X_m^5) = \ell(X_m^5) = 5 - 2r_{5,m} \le 3.$$

Next, let $n > 5$ and suppose $\mathrm{GHC}(H^d(X_m^d, \mathbb{Q}), 2)$ holds for $d \le n-1$ and the GHC holds for X_m^{n-2}. This implies $\mu_2(X_m^{n-1}) = \ell_2(X_m^{n-1}) \le n-5$ and $\mu_0(X_m^{n-2}) = \ell(X_m^{n-2})$. Furthermore, we can estimate the level $\ell(X_m^{n-2})$, by (4.1), to be

$$\ell(X_m^{n-2}) \le (n-2) - 2r_{n-2,m} \le n-4 \quad \text{since } r_{n-2,m} = \left[\frac{n-1}{m}\right] \ge \left[\frac{5}{4}\right] = 1.$$

By substituting all these estimates into (4.2), we get

$$\begin{aligned}\mu_2(Z_m^{1,n-1}) &\le \max\{n-3, 1 + \mu_2(X_m^{n-1}), \mu_0(X_m^{n-2}) = \ell(X_m^{n-2})\}\\ &\le \max\{n-3, n-4, n-4\} = n-3 = n-2(2)+1.\end{aligned}$$

Once again, Lemma 2.5 finishes the proof of the Theorem. □

COROLLARY 4.2. $\mathrm{GHC}(H^n(X_m^n, \mathbb{Q}), 2)$ *holds for $n \le 8$ and $m \le 4$. In particular, the GHC holds for X_m^n for $n \le 6$ and $m \le 4$.*

PROOF. Corollary 3.2 implies $\mathrm{GHC}(H^n(X_m^n, \mathbb{Q}), 2)$ for $n \le 6$. Now Lemma 2.1 implies the GHC for X_m^n for $n \le 6$. Hence Theorem 4.1 yields $\mathrm{GHC}(H^n(X^n, \mathbb{Q}), 2)$ for $n-2 \le 6$, or, equivalently, for $n \le 8$. □

For the GHC in higher codimension, we present the following example, in which we use a different choice of r and s.

EXAMPLE 4.3. $\mathrm{GHC}(H^8(X_m^8, \mathbb{Q}), 3)$ holds for $m \le 4$.

https://doi.org/10.1017/S0004972715001379 Published online by Cambridge University Press

PROOF. We use $r = s = 4$ in the inductive structure of Fermat varieties. A similar computation to those above shows

$$\begin{aligned}\mu_3(Z_m^{4,4}) &\le \max\{\mu_2(X_m^4) + \mu_1(X_m^4), \mu_0(X_m^4), \mu_1(X_m^3) + \mu_0(X_m^3)\}\\ &\le \max\{\ell(X_m^4), \ell_1(X_m^3) + \ell(X_m^3)\} = 3 = 12 - 2(5) + 1.\end{aligned}$$

Lemmas 2.5 and 2.1 yields $\mathrm{GHC}(H^8(X_m^8, \mathbb{Q}), 3)$ for $m \le 4$. □

We finish the note by a few remarks on the GHC of Fermat varieties and that of a smooth hypersurface.

REMARK 4.4.

(1) The Hodge conjecture for a Fermat variety X_m^n has been known for m prime or $m \le 20$ [7, 8]. For $m = 21$, Shioda's argument also implies the Hodge conjecture for X_m^n of dimension $n \le 10$. Our approach proves the Hodge conjecture of X_m^4 without any restriction on m (Corollary 3.2).

(2) By considering hypersurfaces in $\mathbb{P}^{n+1}$ swept by projective spaces $\mathbb{P}^k$ of smaller dimension, Lewis obtained many hypersurface examples that satisfy the GHC [6, Ch. 13]. More precisely, $\mathrm{GHC}(H^n(X, \mathbb{Q}), k)$ holds for any smooth projective hypersurface $X \subset \mathbb{P}^{n+1}$ of degree m and dimension n if n, m and k satisfy the inequality

$$(k+1)(n+1-k) - \binom{m+k}{k} \ge n - 2k. \tag{4.3}$$

For the Fermat hypersurface X_m^n, this result implies $\mathrm{GHC}(H^n(X_m^n, \mathbb{Q}), 1)$ if $n + 1 \ge m$, while Theorem 3.1 implies the GHC for X_m^n in codimension one unconditionally. Furthermore, our method shows $\mathrm{GHC}(H^n(X_m^n, \mathbb{Q}), k)$ holds for $(n, m, k) = (5, m, 2), (6, m, 2)$ and $(8, m, 3)$ for $m \le 4$. These cases do not satisfy (4.3).

References

[1] D. Arapura, 'Varieties with very little transcendental cohomology', in: *Motives and Algebraic Cycles*, Fields Institute Communications, 56 (American Mathematical Society, Providence, RI, 2009), 1–14.

[2] P. Deligne, 'Théorie de Hodge. II, III', *Publ. Math. Inst. Hautes Études Sci.* **40** (1971), 5–57; **44** (1974), 6–77.

[3] P. Deligne and N. Katz (eds.), 'Groupes de monodromie en géométrie algébrique. II', in: *Séminaire de Géométrie Algébrique du Bois–Marie 1967–1969 (SGA 7 II)*, Lecture Notes in Mathematics, 340 (Springer, Berlin–New York, 1973), x+438.

[4] A. Grothendieck, 'Hodge's general conjecture is false for trivial reasons', *Topology* **8** (1969), 299–303.

[5] S.-J. Kang, 'Refined motivic dimension', *Canad. Math. Bull.* **58**(3) (2015), 519–529.

[6] J. Lewis, *A Survey of the Hodge Conjecture*, CRM Monograph Series, 10 (American Mathematical Society, Providence, RI, 1999).

[7] Z. Ran, 'Cycles on Fermat hypersurfaces', *Compositio Math.* **42**(1) (1980/81), 121–142.

https://doi.org/10.1017/S0004972715001379 Published online by Cambridge University Press

[8] T. Shioda, 'The Hodge conjecture for Fermat varieties', *Math. Ann.* **245**(2) (1979), 175–184.
[9] T. Shioda and T. Katsura, 'On Fermat varieties', *Tohoku Math. J.* (2) **31**(1) (1979), 97–115.
[10] J. H. M. Steenbrink, 'Some remarks about the Hodge conjecture', in: *Hodge Theory (Sant Cugat, 1985)*, Lecture Notes in Mathematics, 1246 (Springer, Berlin, 1987), 165–175.

SU-JEONG KANG, Department of Mathematics and Computer Science,
Providence College, Providence, RI 02918, USA
e-mail: skang2@providence.edu

https://doi.org/10.1017/S0004972715001379 Published online by Cambridge University Press