Small Galaxy Groups
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On the Formation of Small Groups of Galaxies at High Redshifts

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Abstract. Cosmological many-body clustering agrees with the spatial and velocity distribution functions of galaxies at low redshifts, and it can be extended to high redshifts $z \approx 3$ or more. The high redshift distribution functions are predicted to have a particular form. In the simplest case, there are no free parameters in this prediction, but the degree of clustering depends sensitively on Ω_0. Current observations of small groups at high redshifts suggest that $\Omega_0 = 0.3 \pm 0.2$ for Einstein-Friedmann cosmologies.

1. Introduction

The formation of groups of galaxies embodies many mysteries of our early Universe. It depends not only on the scale and amplitude of initial perturbations, but also on the global expansion of the Universe and on the nature and distribution of its dark matter. Many calculations and simulations have attempted to model these clusters. As befits the frontiers of our understanding, the results still remain controversial

Two general approaches to galaxy clustering continue to be widely discussed. In the "top-down" models, long wavelength perturbations with large amplitudes dominate. First they form massive clusters, possibly assisted by cosmic strings, and then these massive clusters fragment into smaller clusters that eventually fragment again into galaxies. In contrast, the "bottom-up" models start with large amplitude perturbations at short wavelengths. First they form galaxies, then the galaxies cluster gravitationally and eventually build up a a hierarchy of massive clusters and superclusters.

How do we distinguish between these approaches observationally? One method is through their influence on fluctuations of the cosmic microwave background. Another is by their residual effects on the morphology, structure, and stellar content of galaxies. A third is by their implied spatial and velocity distributions of galaxies.

Here I shall focus on some implications of the spatial distribution of galaxies. And, since this is primarily a meeting on dynamical interactions of a few galaxies,

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I'll concentrate on what a simple model of the formation of small groups may tell us about larger cosmological questions, especially the value of $\Omega_0 = \rho_0/\rho_c$ (where ρ_c is the critical density needed to just close the usual Einstein-Friedmann models of the universe and ρ_0 is the current density).

Recently a new impetus to examine the cosmological implications of small groups of galaxies has arisen from the discovery that these groups already exist at high redshifts $z \gtrsim 3.5$ (Steidel et al. 1999). This means that galaxies must have formed as dynamical entities and begun to cluster at even higher redshifts. In many "top-down" models, the consequent large scale clustering at high redshifts would have produced fluctuations in the cosmic microwave background which are not observed. This tilts the more plausible models, at least in the minds of many cosmologists, toward the hierarchical "bottom-up" approach.

Even within the hierarchical approach there are many variants. Nearly all of them rely on some unknown form of cold dark matter. In some models the CDM forms massive haloes within which several galaxies may condense as their baryonic component cools. This produces a ready-made small group. In other models each CDM halo surrounds a single galaxy, and then these galaxies cluster gravitationally. In a related model, the baryonic component first condenses into a galaxy which then attracts a CDM halo. These models are usually explored by detailed computer simulations which treat the dark matter as a fluid with growing density perturbations. They specify an initial perturbation spectrum, a value of Ω_0, a prescription for identifying regions which eventually represent galaxies, and a bias between the galaxies and the dark matter. This bias is usually determined by the average ratio of the variance of fluctuations of galaxies relative to those of dark matter, but it could of course also depend on higher moments, and it could be scale-dependent. These models may involve complex parameterized interactions among hot dark matter, cold dark matter, and the galaxies themselves. They can naturally produce small groups and clusters whose features depend on detailed, often ad hoc, properties of the models. So far the spatial distribution functions $f(N)$ which measure the statistical probabilities of voids, near neighbors, and counts-in-cells, are the most stringent tests of models. Unfortunately, when these models have been compared to the presently observed spatial distribution functions of galaxies, they have generally failed to reproduce the observations (e.g. Sheth e.al. 1994).

If we limit ourselves, however, to just the clustering of galaxies, rather than combining their formation with their clustering, it may be possible to bypass these detailed models in certain circumstances. The conditions under which this is likely are that the major processes of galaxy formation, whatever their details, left an initial galaxy distribution which had little coherence on large scales and whose motions were not dominated by residual inhomogeneous intergalactic matter. Most of the dark matter would be inside the galaxies and in individual dark haloes around each galaxy. Then the galaxies would cluster essentially as point masses, interacting like Newtonian particles in the background of the expanding universe. This is the cosmological many-body problem. We can solve it to obtain the probability of finding small groups of galaxies as a function of redshift, and compare the result with the new observations of small groups around $z \approx 3$. Here I shall summarize our results, which are discussed in more detail with many more references, and related to other models in Saslaw (1999) and Saslaw and Edgar (1999).

The Cosmological Gravitational Quasi-Equilibrium Distribution Function

If galaxies were distributed with a random Poisson spatial density - which astronomers have known they are not since at least the 1950's - then the probability of finding exactly N galaxies in a randomly placed volume V would be

$$f(N) = \frac{\bar{N}^N}{N!} e^{-\bar{N}} \tag{1}$$

where $\bar{N} = \bar{n}V$ and $\bar{n}$ is the average density. Thus the probability that a randomly placed volume is an empty void is just $f(0) = e^{-\bar{n}V}$. Although a Poisson distribution is not relevant today, something close to it may have been a good representation of the initial galaxy distribution. If this initial distribution had a roughly power law spectrum $P_k \propto k^n$ with $-1 \lesssim n \lesssim 1$ over a wide range of wavenumber, k, then its subsequent evolution in an Einstein-Friedmann universe would have occurred through a sequence of states in approximate thermodynamic equilibrium (e.g. Itoh 1990). The essential physical requirement for this quasi-equilibrium evolution is that there is no initial coherent structure on the scale of the volume V. Then the long range part of the galaxies' gravitational field (the mean field) is exactly cancelled by the global expansion of the universe and only the short range fluctuations affect galaxy orbits directly. Under these quasi-equilibrium conditions, statistical thermodynamics provides an excellent description of the state of the system (cf. Saslaw and Fang 1996).

The thermodynamic description provides equations of state which include the gravitational interactions of the point-mass galaxies. From these equations of state, and the first and second law of thermodynamics, one can calculate the distribution function for fluctuations (Saslaw and Hamilton 1984; Saslaw and Fang 1996; Saslaw 1999):

$$f(N) = \frac{\bar{N}(1-b)}{N!}[\bar{N}(1-b) + Nb]^{N-1} e^{-[\bar{N}(1-b)+Nb]} \quad . \tag{2}$$

Here

$$b = -\frac{W}{2K} = \frac{2\pi G m^2 \bar{n}}{3T} \int_0^R r\xi(r)dr \tag{3}$$

is the ratio of the gravitational correlation energy, W, to twice the kinetic energy, K, of peculiar velocities, v_i, for galaxies in a spherical volume of radius R. Volumes of other shapes can also be used by integrating the galaxy two-point correlation function, $\xi(r)$, over the desired volume. As usual, the effective temperature is given by

$$T = \frac{1}{2} \sum_i m_i v_i^2 \quad . \tag{4}$$

The limit $b = 0$ occurs for $W \to 0$, which is the case of a perfect non-interacting gas, or for $K \to \infty$, which implies that the velocities are unaffected

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by the interaction, or for $R \to 0$, which gives volumes so small that they usually contain either zero or one galaxy. For all these cases, the distribution function for equation (2) reduces to the Poisson distribution (1). As b increases, the distribution becomes more and more clustered until, when $b = 1$, the clustering is maximal. The form of equation (2) holds for general three-dimensional volumes, which can be cones projected onto the sky as two-dimensional area counts (cf. Sheth and Saslaw 1996).

This distribution has been tested extensively by many many-body simulations for different cosmologies, and for galaxies of different masses (see Saslaw 1999 for a detailed summary). Essentially, the distribution function of equation (2) acts like an attractor for a wide range of initial configurations and evolving cosmologies. Once attracted, the system evolves through a sequence of states described accurately by equation (2) with and evolving value of $b(t)$. The value of $\bar{N} = \bar{n}V$ is constant for comoving volumes. However the basis of attraction for this distribution has not been rigorously determined, and it remains a major unsolved problem.

It is a problem worth solving because equation (2) agrees very well with the observed distribution function of galaxies on the sky, as well as in three-dimensional volumes (again summarized in Saslaw 1999).This is a more stringent test than using just the two-point or three-point correlation functions since $f(N)$ depends on <u>all</u> the N-point galaxy correlation functions. Moreover, there are no free parameters in fitting equation (2) to the observations. The value of $\bar{N}$ is found directly from the average number density (or the average surface density for projected cells) of the observations. The value of b may be obtained directly from the variance of the observed counts in cells of a given size and shape since equation (2) implies

$$(\Delta N)^2 = \frac{\bar{N}}{(1-b)^2} \tag{5}$$

Therefore the self-consistent agreement of the observed values of $\bar{N}$ and b with the observed spatial distribution of equation (2) is a strong argument for the dominance of cosmological many-body clustering in the galaxy distribution. From analyses of several galaxy catalogs, it appears that at very low redshifts $b = 0.75 \pm 0.05$. We therefore try to predict its value at higher redshifts where small groups have evidently formed.

Evolution of Galaxy Clustering

The most common approach to the evolution of galaxy groups is to follow the development of clustering in computer simulations. This is the most useful technique for complicated multi-component models containing combinations of cold dark matter, hot dark matter, galaxies, different initial conditions, etc. It has the advantage that for a given realization of a model one can determine the clustering properties directly from the simulation, and convolve them with examples of selection effects if desired (eg. Steidel et al. 1999; Wechsler et al. 1998). However it has the drawback that to obtain accurate statistical distribution functions, many simulations are needed.

For the cosmological many-body problem as a model for galaxy clustering, we have the advantage of having the distribution function of equation (2) already

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in hand. Its modification by known selection effects can be calculated directly (Lahav and Saslaw 1992). Its value of b which describes the strength of the pattern of clustering may be found for quasi-equilibrium evolution from the growth of linear density fluctuations $\delta(t) =< [N(t) - \bar{N}] > /\sqrt{\bar{N}}$ (Zhan 1989; Inagaki 1991; Saslaw and Edgar 1999; Saslaw 1999) in comoving coordinates. If initially $b(t_i) = 0$ and there are no initial peculiar velocities, $\dot{\delta}(t_i) = 0$, then

$$b(t) = \frac{\delta(t) - 1}{\delta(t)} \tag{6}$$

implying $\delta(t_i) = 1$. This is consistent with an average initial Poisson distribution. Since the perturbation equation for the growth of $\delta(t)$ is linear, it applies to any normalization of $\delta(t)$. For the Einstein-de Sitter Universe ($\Omega_0 = 1$), the solution of the usual linear perturbation equation

$$\frac{d^2\delta(t)}{dt^2} + \frac{4}{3t}\frac{d\delta(t)}{dt} - \frac{2}{3t^2}\delta(t) = 0 \tag{7}$$

is therefore

$$\delta(t) = \frac{3}{5}\frac{a}{a_i} + \frac{2}{5}\left(\frac{a}{a_i}\right)^{-3/2} \tag{8}$$

with the metric scale $a \propto t^{2/3}$ related to the redshift by $1 + z = a_0/a$. This gives $b(z)$ as a function of the redshift at which clustering is observed, once we specify the present value of b_0. We shall use $b_0 = 0.75$ as an illustration here. Essentially the same procedure, though with different evolution equations instead of (7) for $\delta(t)$, applies to models with $\Omega_0 \neq 1$ or with $\Lambda \neq 0$.

Figure 1 (from Saslaw and Edgar 1999) illustrates the resultant evolution of $b(z)$ for the Einstein-Friedmann models ($\Lambda = 0$) having $0.05 \leq \Omega_0 \leq 1.0$ and $b(z = 0) = 0.75$. They have the important feature that $b(z)$ depends strongly on Ω_0, especially around $z \approx 3$ where observations are rapidly accumulating. As Ω_0 decreases, $b(z)$ increases, essentially because clustering in low Ω_0 models occurs more slowly compared with the Hubble expansion timescale at any given time. Therefore the lower Ω_0 models must have clustered for a longer period in order that they produce the presently observed $b(z = 0)$, and consequently their $b(z)$ must have been greater in the past for the same initial conditions.

Therefore, if cosmological many-body clustering was primarily responsible for the formation of galaxy groups, it would predict that the spatial distribution function at high redshift should have the same form as equation (2), but with a lower value of b. If so, then from $b(z \gtrsim 2)$ we could find Ω_0 from Figure 1. If not, then we may have to resort to more complicated evolutionary models and their computer simulations.

Note also, that smaller values of Ω_0 require that galaxies formed and started clustering at quite high redshifts for these models. Even for $\Omega_0 \lesssim 0.3$, galaxies must have started clustering at $z \gtrsim 10$. This conclusion is subject to the caveat that the initial conditions for galaxy clustering were close to a Poisson distribution independent of Ω_0. Early clustering on ~ 10 Mpc scales possibly left over from galaxy formation, could reduce the initial redshift when clustering started, but this has not yet been much explored.

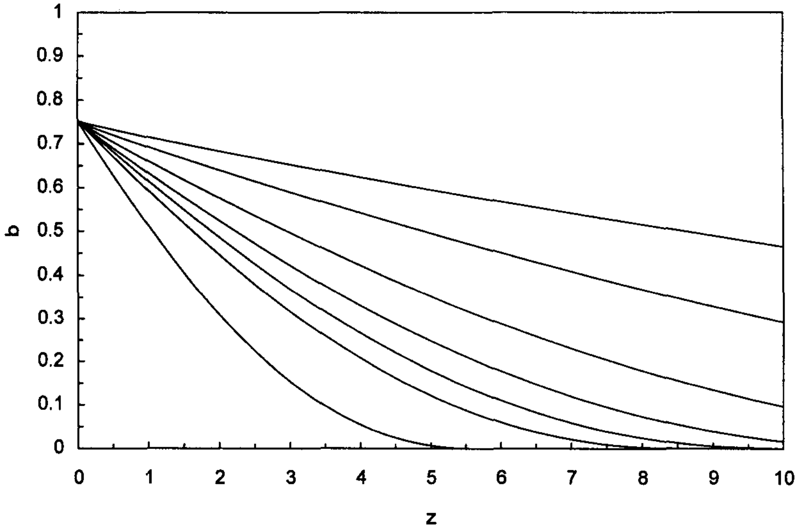

Figure 1. Evolution of $b(z)$ for $b_0 = 0.75$ and a range of Ω_0 (from Saslaw and Edgar 1999).

As an example of the distribution function of small groups, Figure 2 (from Saslaw and Edgar 1999) shows how $f(N)$ behaves for volumes expected to contain $\bar{N} = 3$ galaxies. A Poisson distribution with $b = 0$ would have a symmetric peak around $N = \bar{N} = 3$. As b increases, the peak shifts toward smaller values of N and the distribution becomes more skewed toward larger N. Although the amplitude is small for $N \gg \bar{N}$, it is relatively more sensitive to the value of b and therefore to Ω_0 since Ω_0 determines $b(z)$. Distribution functions for larger values of $\bar{N}$ have a qualitatively similar behaviour.

Observations and Implications for Ω_0

To measure the galaxy distribution function at high redshifts, we would ideally like to have a uniformly selected sample of about 10^3 galaxies located in contiguous areas of the sky in a relatively narrow redshift band with, say, $\Delta z \approx 0.2$. First steps toward such a sample have been made by Steidel et al. (1999) who developed an efficient technique that identifies likely candidates to have redshifts $2.5 \leq z \leq 3.5$. They use photometric color bands chosen to detect the redshifted Lyman limit break caused by intergalactic neutral hydrogen, by the Lyman continuum absorbtion within the distant galaxy, and by the intrinsic spectrum of early type stars. Then they measure the precise redshift of the candidate galaxy spectroscopically. So far, about 1,000 galaxies have measured redshifts in the range $2.5 \leq z \leq 3.5$ and more are accumulating.

Even before such a fine sample is formed, we can use these early results to estimate Ω_0, then see how its value changes with more data. There are two relatively simple ways to do this. One compares the existence of small groups of galaxies at high redshifts with the probability that they occur in particular models, and estimates whether this probability is consistent with the number

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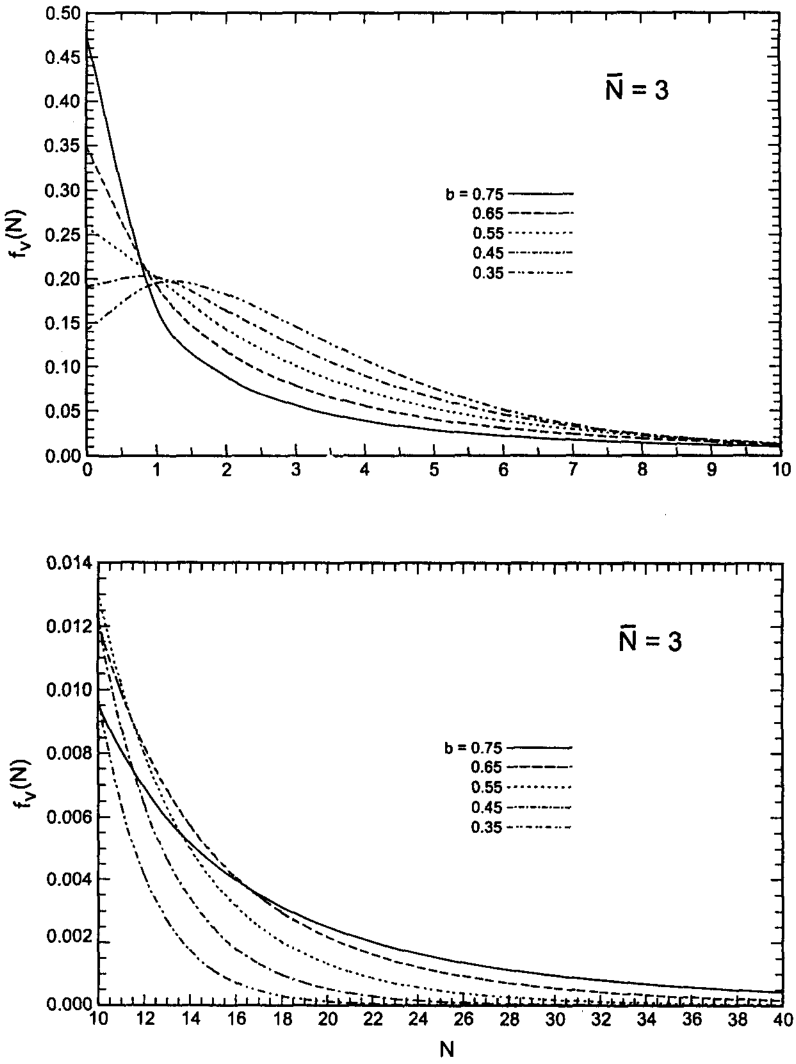

Figure 2. Spatial distribution function for small groups with $\bar{N} = 3$ and different values of b (from Saslaw and Edgar 1999).

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of groups observed. The second uses the variance of fluctuations in the number counts to estimate b from equation (5), assuming that the distribution (2) of many-body clustering applies at these early times. Fortunately, in neither case is it necessary to know whether these groups are bound or viralized. Next, I summarize the current tentative results from both these approaches.

Most earlier analyses using cold dark matter models concerned the initial discovery of 15 galaxies in a cube of roughly 10 Mpc at $z \approx 3.1$ where about 4.5 galaxies were expected, given the selection procedures of the sample (cf. Wechsler et al. 1998, and references therein). Most of these model simulations gave a reasonable probability for finding a high redshift group similar to the observed one, provided $\Omega_0 \lesssim 0.6$. These models all contain the bias ratio of the variance of the galaxy distribution to that of the dark matter distribution as an *ad hoc* free parameter. For some of the CDM models this bias was quite high, but for others it was small. Moreover, these models have generally not been tested against the currently observed galaxy distribution function, so it is not clear if their galaxy distributions agree with either present or past observations. To get an accurate estimate of these CDM distribution functions, especially in their low probability tails, would require many simulations of each model. Each simulation would have the same macroscopic average properties, but with different microscopic realizations.

For cosmological many-body clustering, however, we have the analytic distribution function of equation (2). Thus we can readily calculate probabilities for finding groups of a given number at a given redshift. These calculations also emphasize the importance of choosing an *a priori* range for determining such probabilities.

For example, suppose we wanted to ask for the *a priori* probability that a region of volume V at a redshift z contains exactly N galaxies. Then we would evolve b back to that redshift in a given cosmological model, as in Figure 1, and use equation (2) to find $f\left[N,\ \bar{N},\ b(z)\right]$. Next, suppose we relax the *a priori* redshift from a small range representing, say, 10 Mpc to a wider range, say $2.5 \leq z \leq 3.5$ where the Lyman break technique is most sensitive, in which such a $\sim$ 10 Mpc cube with N galaxies could occur anywhere. Then we would simply add the values of $f\left[N,\ \bar{N},\ b(z)\right]$ for all the z_i cells, assuming they were independent (and including the effects of selection on $\bar{N}$). This seems a reasonable approximation since the two-galaxy correlation function is likely to be small over distances $\gtrsim$ 10 Mpc. Suppose further that we relax the *a priori* question to also include the possibility that the overdense cell contains any number, N_i, of galaxies within some prescribed range. Then the previous sum over redshifts would itself be summed over the desired *a priori* range of numbers of galaxies. Clearly such sums will give larger *a priori* probabilities for finding an overdense (or underdense) cell within the range. By relaxing the conditions sufficiently, one can generally get results consistent with many models, as long as the observations are rather sparse. This is why one really needs the full distribution function before making definite claims.

Many astronomers find it irresistible to explore implications of the available data, and it is certainly interesting to see how these implications change as the data improves. So we will take that view here and illustrate some tentative implications of the observations for cosmological many-body clustering.

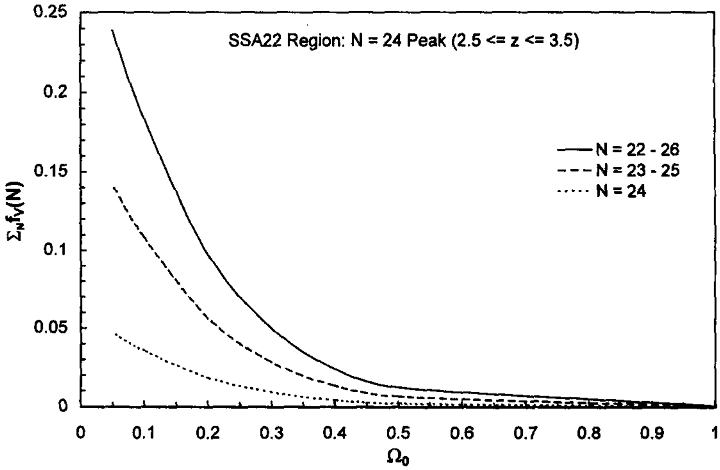

Figure 3. Probabilities for finding groups with $N = 24$, $23 \leq N \leq 25$ or $22 \leq N \leq 26$ Lyman break galaxies in ~ 10 Mpc cells in a redshift band $2.5 \leq z \leq 3.5$ for $b_0 = 0.75$ as a function of Ω_0 for $\bar{N} = 6$.

Figure 3 is an illustration for the SSA22 region where Steidel et al. (1999) have now found 24 galaxies in a cube approximately 10 Mpc on a side around $z = 3.1$. For the sample selection at this redshift, 6 galaxies would be expected in such a cell, so it is overdense by a factor of four. Figure 3 shows that the probability of finding a 10 Mpc cell with $N = 24$ in the range $2.5 \leq z \leq 3.5$ is quite small, only about 0.03 for $\Omega_0 = 0.1$, and it decreases considerably for larger Ω_0. Since a total of four regions comparable to SSA22 were observed, the discovery of an $N = 24$ cell in this redshift range would be unlikely. On the other hand, for a similar cell to contain from $22 - 26$ galaxies has a probability of ~ 0.18 for $\Omega_0 = 1$ and should occur in about 1/6 of the areas. Thus results depend strongly on the *a priori* range of N and z. Since the Steidel et al. (1999) work shows many large fluctuations of counts in cells over the whole $2.5 \leq z \leq 3.5$ redshift range in this and other areas of the sky, it is clear that the value of Ω_0 is not yet well-determined by this technique.

But Ω_0 will probably become much better determined in the near future. An illustration of how this may occur is given by Figure 4 which shows a similar diagram for the Westphal region of Steidel et al. (1999). This region contains 148 galaxies with measured $z > 2.4$. It has an overdense $\sim$ 10Mpc cell with 21 galaxies at $z = 2.9$ where 9 would be expected. The probability of this occurring is about 0.1 for $\Omega_0 = 0.1$. However it is nearly 0.4 for $20 \leq N \leq 22$, so such a spike should be observed, as it has been. If we widen the *a priori* range to include $19 \leq N \leq 23$, then almost every region should include such an overdense cell for low Ω_0. The observation that few regions include such overdensities would put a lower limit on Ω_0. Combined with the upper limit from the SSA22 region, this would restrict the possible values of Ω_0.

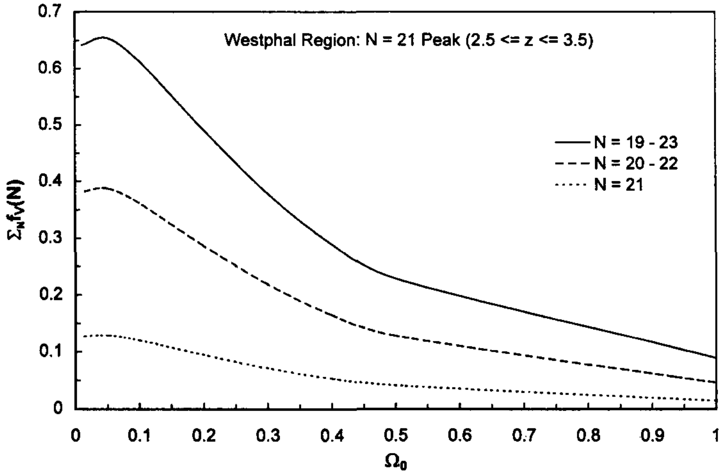

Figure 4. Probabilities for finding groups with $N = 21$, $20 \leq N \leq 22$ or $19 \leq N \leq 23$ Lyman break galaxies in ~ 10 Mpc cells in a redshift band $2.5 \leq z \leq 3.5$ for $b_0 = 0.75$ as a function of Ω_0 for $\bar{N} = 9$.

Of course this is only a stop-gap procedure until the complete distribution function is determined around $z \approx 3$. If it agrees with equation (2), we can determine $b(z)$ directly. If not, we will need to understand the astrophysical and dark matter modifications of cosmological many-body galaxy clustering which have occured. Meanwhile, another stop-gap approach is to use the variance of fluctuations in counts-in-cells at a particular redshift to obtain $b(z)$ from equation (4), and then Ω_0 from Figure 1 and its generalizations. Using the four regions examined by Steidel et al. (1999) gives an estimate of $\Omega_0 = 0.3 \pm 0.2$ (Saslaw and Edgar 1999). However, it should be emphasized that the observational sampling and completeness of these regions differ significantly, and the best way to weight them is not clear. Rather than playing with statistics, it seems much better to collect at least twice the number of present redshifts in these regions. At any rate, this type of analysis is relatively transparent and makes the clear prediction that increasing statistics will be consistent with equation (2) for $\Omega_0 \approx 0.3$. It is interesting that estimates of Ω_0 from many of the more complicated CDM computer simulations (cf. Wechsler et al. 1998, and references therein) also suggest $\Omega_0 \lesssim 0.5$ although this may be further complicated by the possibility that the cosmological constant $\Lambda \neq 0$.

There are many problems that remain concerning the formation of high redshift galaxy groups. Although simulations have modeled possible roles for dark matter, its actual role is unknown. Nor do we have much real understanding of the initial conditions of galaxy formation and how they interacted with the initial conditions for subsequent clustering. One method for exploring this would be to determine at what redshift small groups become gravitationally bound. So far, we just have indications of their overdensity. To estimate their binding energy would require very accurate peculiar velocities from secondary distance

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indicators, as well as independent criteria for group membership, all at present on a distant horizon.

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