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FIXED POINTS THEOREMS FOR GENERALISED WEAKLY CONTRACTIVE MAPPINGS

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Abstract

In this paper, we establish a convergence theorem for fixed points of generalised weak contractions in complete metric spaces under some new control conditions on the functions. An illustrative example of a generalised weak contraction is discussed to show how the new conditions extend known results.

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1. Introduction

In 1997, Alber and Guerre-Delabriere [1] defined weakly contractive maps and established a fixed point theorem in Hilbert spaces. Later, Rhoades [8], by using the notion of weakly contractive maps, obtained a fixed point theorem in a complete metric space. Since then, the notions of weak contraction and generalised weak contraction have been widely studied. Further recent results related to fixed point theorems for generalised weak contractions can be found in [3–7, 9].

Let (X, d) be a metric space and $T : X \to X$ be a selfmap of X. T is said to be a φ-weak contraction if

$$d(Tx, Ty) \le d(x, y) - \varphi(d(x, y)), \quad x, y \in X,$$

where $\varphi : [0, +\infty) \to [0, +\infty)$ is a continuous and nondecreasing function with $\varphi(t) = 0$ if and only if $t = 0$. The following result was obtained by Rhoades [8].

Theorem 1.1 [8]. *If (X, d) is a complete metric space and T is a φ-weak contraction on X, then T has a unique fixed point.*

In 2008, Dutta and Choudhury [5] gave the first extension of Theorem 1.1 to more general φ-weak contractions.

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THEOREM 1.2 [5]. *Let (X, d) be a complete metric space and let $T : X \to X$ be a selfmap satisfying the inequality*

$$\psi(d(Tx, Ty)) \le \psi(d(x, y)) - \varphi(d(x, y)), \quad x, y \in X,$$

where $\psi, \varphi : [0, +\infty) \to [0, +\infty)$ are both continuous and monotone nondecreasing functions with $\psi(t) = \varphi(t) = 0$ if and only if $t = 0$. Then T has a unique fixed point.

To state the following theorems, define the quantity

$$M(x, y) = \max\{d(x, y), d(x, Tx), d(y, Ty), \tfrac{1}{2}[d(x, Ty) + d(y, Tx)]\}.$$

In 2009, Dorić [4] improved Theorem 1.2 as follows.

THEOREM 1.3 [4, Theorem 2.2]. *Let (X, d) be a complete metric space and let $T : X \to X$ be a selfmap satisfying the inequality*

$$\psi(d(Tx, Ty)) \le \psi(M(x, y)) - \varphi(M(x, y)), \quad x, y \in X,$$

where $M(x, y)$ is defined above and

(a) *$\psi : [0, +\infty) \to [0, +\infty)$ is a continuous monotone nondecreasing function with $\psi(t) = 0$ if and only if $t = 0$;*
(b) *$\varphi : [0, +\infty) \to [0, +\infty)$ is a lower semicontinuous function with $\varphi(t) = 0$ if and only if $t = 0$.*

Then T has unique fixed point.

In 2011, Choudhury *et al.* [3] gave the following fixed point theorem.

THEOREM 1.4 [3, Theorem 3.1]. *Let (X, d) be a complete metric space. Let $T : X \to X$ be a selfmap such that*

$$\psi(d(Tx, Ty)) \le \psi(M(x, y)) - \varphi(\max\{d(x, y), d(y, Ty)\}), \quad x, y \in X,$$

where $M(x, y)$ is defined above and

(a) *$\psi : [0, +\infty) \to [0, +\infty)$ is a monotone increasing and continuous function with $\psi(t) = 0$ if and only if $t = 0$;*
(b) *$\varphi : [0, +\infty) \to [0, +\infty)$ is a continuous function with $\varphi(t) = 0$ if and only if $t = 0$.*

Then T has unique fixed point.

In the case $M(x, y) = \max\{d(x, y), d(y, Ty)\}$, Theorem 1.4 becomes a special case of Theorem 1.3. In the same year, Popescu [7] extended the result of Dorić [4].

THEOREM 1.5 [7, Theorem 4]. *Let (X, d) be a nonempty complete metric space and $T : X \to X$ be a mapping satisfying*

$$\psi(d(Tx, Ty)) \le \psi(M(x, y)) - \varphi(M(x, y)), \quad x, y \in X,$$

where $M(x, y)$ is defined above and

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(a) *$\psi : [0, +\infty) \to [0, +\infty)$ is a monotone nondecreasing function with $\psi(t) = 0$ if and only if $t = 0$;*
(b) *$\varphi : [0, +\infty) \to [0, +\infty)$ is a function with $\varphi(t) = 0$ if and only if $t = 0$ and $\lim_{n\to\infty} \inf \varphi(a_n) > 0$ if $\lim_{n\to\infty} a_n = a > 0$;*
(c) *$\varphi(a) > \psi(a) - \psi(a-)$ for any $a > 0$, where $\psi(a-)$ is the left limit of ψ at a.*

Then T has unique fixed point.

Recently, Moradi and Farajzadeh [6] proved fixed point theorems for φ-weak and generalised φ-weak contraction mappings.

THEOREM 1.6 [6, Theorem 3.3]. *Let (X, d) be a complete metric space and let $T : X \to X$ be a selfmap satisfying*

$$\psi(d(Tx, Ty)) \le \psi(M(x, y)) - \varphi(M(x, y)), \quad x, y \in X,$$

where $M(x, y)$ is defined above and

(a) *$\varphi : [0, +\infty) \to [0, +\infty)$ is a mapping with $\varphi(0) = 0$ and $\varphi(t) > 0$ for all $t > 0$ and $\lim_{n\to\infty} \varphi(t_n) = 0$ for any bounded sequence $\{t_n\}$ with $\lim_{n\to\infty} t_n = 0$;*
(b) *$\psi : [0, +\infty) \to [0, +\infty)$ is a mapping with $\psi(0) = 0$ and $\psi(t) > 0$ for all $t > 0$;*
(c) *either ψ is continuous or ψ is monotone nondecreasing and, for all $t > 0$, $\varphi(a) > \psi(a) - \psi(a-)$, where $\psi(a-)$ is the left limit of ψ at a.*

Then T has a unique fixed point.

REMARK 1.7. Assumption (a) relating to φ is equivalent to the following conditions:

(a) $\lim_{n\to\infty} \inf \varphi(a_n) > 0$ if $\lim_{n\to\infty} a_n = a > 0$ (that is, (b) of Theorem 1.5);
(b) $\lim_{n\to\infty} a_n = 0$ or nonexistent if $\lim_{n\to\infty} \inf \varphi(a_n) = 0$.

From this observation, we see that condition (ii) is weaker than (a) of Theorem 1.6 (that is, $\lim_{n\to\infty} t_n = 0$ if $\{t_n\}$ is bounded and $\lim_{n\to\infty} \varphi(t_n) = 0$). Therefore, Theorem 1.6 is a special case of Theorem 1.5.

Inspired and motivated by these results, our purpose in this paper is to obtain fixed point theorems for generalised weak contractions that satisfy contractive conditions which are more general than those given in [3, 6, 7]. Our results also extend and generalise comparable results in [4, 5, 8] and part of [9].

2. The main results

THEOREM 2.1. *Let (X, d) be a complete metric space and $T : X \to X$ be a mapping such that for all $x, y \in X$,*

$$\psi(d(Tx, Ty)) \le \psi(M(x, y)) - \varphi(M(x, y)), \tag{2.1}$$

where $M(x, y) = \max\{d(x, y), d(x, Tx), d(y, Ty), \frac{1}{2}[d(x, Ty) + d(y, Tx)]\}$ and ψ, φ satisfy the conditions:

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(a) $\psi, \varphi : [0, +\infty) \to [0, +\infty)$ *are two functions with* $\psi(t) = \varphi(t) = 0$ *if and only if* $t = 0$;
(b) $\lim_{\tau\to t} \inf \psi(\tau) > \lim_{\tau\to t} \sup \psi(\tau) - \lim_{\tau\to t} \inf \varphi(\tau)$ *for all* $t > 0$.

Then T *has a unique fixed point.*

Proof. Let $x_0 \in X$ be arbitrary and let $\{x_n\}_{n=0}^{\infty}$ be the Picard iteration defined by $x_{n+1} = Tx_n \neq x_n$ for all $n \geq 0$. Taking $x := x_n$, $y := x_{n-1}$ in (2.1), we obtain

$$\begin{aligned}\psi(d(x_{n+1}, x_n)) &= \psi(d(Tx_n, Tx_{n-1}))\\ &\leq \psi(M(x_n, x_{n-1})) - \varphi(M(x_n, x_{n-1}))\\ &\leq \psi(M(x_n, x_{n-1})), \end{aligned} \tag{2.2}$$

where $M(x_n, x_{n-1}) = \max\{d(x_n, x_{n-1}), d(x_{n+1}, x_n)\}$. If $d(x_{n+1}, x_n) > d(x_n, x_{n-1})$ for some n, then it follows from (2.2) that

$$\psi(d(x_{n+1}, x_n)) \leq \psi(d(x_{n+1}, x_n)) - \varphi(d(x_{n+1}, x_n)),$$

which is a contradiction and so

$$d(x_{n+1}, x_n) \leq d(x_n, x_{n-1})$$

for all $n \geq 1$. Consequently, (2.2) implies that

$$\begin{aligned}\psi(d(x_{n+1}, x_n)) &\leq \psi(d(x_n, x_{n-1})) - \varphi(d(x_n, x_{n-1}))\\ &\leq \psi(d(x_n, x_{n-1})), \end{aligned} \tag{2.3}$$

whence $\{\psi(d(x_{n+1}, x_n))\}$ is monotone decreasing and bounded below. Thus there exist r, $R \geq 0$ such that

$$\lim_{n\to\infty} d(x_{n+1}, x_n) = r, \quad \lim_{n\to\infty} \psi(d(x_{n+1}, x_n)) = R.$$

If $r > 0$, then taking lower limits as $n \to \infty$ in (2.3) gives

$$\lim_{n\to\infty} \inf \varphi(d(x_n, x_{n-1})) \leq 0,$$

which contradicts the condition (b) in the theorem. Therefore $\lim_{n\to\infty} d(x_{n+1}, x_n) = 0$.

Next we prove that $\{x_n\}$ is a Cauchy sequence. Assume that this is not true. Then there exist $\epsilon > 0$ and subsequences $\{x_{m(k)}\}$ and $\{x_{n(k)}\}$ of $\{x_n\}$ such that $n(k)$ is the smallest index for which $n(k) > m(k) > k$ and $d(x_{m(k)}, x_{n(k)}) \geq \epsilon$. This implies that $d(x_{m(k)}, x_{n(k)-1}) < \epsilon$ for all $k \geq 1$. By the triangle inequality,

$$\begin{aligned}\epsilon \leq d(x_{m(k)}, x_{n(k)}) &\leq d(x_{m(k)}, x_{n(k)-1}) + d(x_{n(k)-1}, x_{n(k)})\\ &\leq \epsilon + d(x_{n(k)-1}, x_{n(k)}). \end{aligned} \tag{2.4}$$

Letting $k \to \infty$ in (2.4) gives

$$\lim_{k\to\infty} d(x_{m(k)}, x_{n(k)}) = \lim_{k\to\infty} d(x_{m(k)}, x_{n(k)-1}) = \epsilon.$$

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Similarly, it is easy to obtain

$$\lim_{k\to\infty} d(x_{m(k)-1}, x_{n(k)-1}) = \epsilon.$$

Again using (2.1),

$$\begin{aligned}\psi(d(x_{m(k)}, x_{n(k)})) &= \psi(d(Tx_{m(k)-1}, Tx_{n(k)-1}))\\ &\le \psi(M(x_{m(k)-1}, x_{n(k)-1})) - \varphi(M(x_{m(k)-1}, x_{n(k)-1}))\\ &\le \psi(M(x_{m(k)-1}, x_{n(k)-1})), \end{aligned} \tag{2.5}$$

where

$$\begin{aligned} M(x_{m(k)-1}, x_{n(k)-1}) &= \max\{d(x_{m(k)-1}, x_{n(k)-1}), d(x_{m(k)-1}, x_{m(k)}), d(x_{n(k)-1}, x_{n(k)}),\\ &\qquad \tfrac{1}{2}[d(x_{m(k)-1}, x_{n(k)}) + d(x_{m(k)}, x_{n(k)-1})]\}\\ &\le \max\{d(x_{m(k)-1}, x_{n(k)-1}), d(x_{m(k)-1}, x_{m(k)}), d(x_{n(k)-1}, x_{n(k)}),\\ &\qquad \tfrac{1}{2}[2d(x_{m(k)-1}, x_{n(k)-1}) + d(x_{n(k)-1}, x_{n(k)}) + d(x_{m(k)}, x_{m(k)-1})]\}\\ &\le d(x_{m(k)-1}, x_{n(k)-1}) + d(x_{m(k)-1}, x_{m(k)}) + d(x_{n(k)-1}, x_{n(k)}), \end{aligned}$$

which implies that

$$\begin{aligned} d(x_{m(k)-1}, x_{n(k)-1}) &\le M(x_{m(k)-1}, x_{n(k)-1})\\ &\le d(x_{m(k)-1}, x_{n(k)-1}) + d(x_{m(k)-1}, x_{m(k)}) + d(x_{n(k)-1}, x_{n(k)}). \end{aligned} \tag{2.6}$$

Letting $k \to \infty$ in (2.6),

$$\lim_{k\to\infty} M(x_{n(k)-1}, x_{m(k)-1}) = \epsilon.$$

By (2.5),

$$\inf_{i\ge k} \psi(d(x_{m(i)}, x_{n(i)})) + \inf_{i\ge k} \varphi(M(x_{m(i)-1}, x_{n(i)-1})) \le \sup_{j\ge k} \psi(M(x_{m(j)-1}, x_{n(j)-1})),$$

which yields

$$\liminf_{t\to\epsilon} \varphi(t) + \liminf_{t\to\epsilon} \psi(t) \le \limsup_{t\to\epsilon} \psi(t),$$

which contradicts the fact that $\epsilon > 0$. So $\{x_n\}$ is a Cauchy sequence and $\lim_{n\to\infty} x_n$ exists. Write $\lim_{n\to\infty} x_n = q$.

Finally, we show that q is the unique fixed point of T. If $q \ne Tq$, then $d(q, Tq) > 0$. Take $x := q, y := x_n$ in (2.1). This gives

$$\psi(d(Tq, x_{n+1})) = \psi(d(Tq, Tx_n)) \le \psi(M(q, x_n)) - \varphi(M(q, x_n)), \tag{2.7}$$

where

$$\begin{aligned} M(q, x_n) &= \max\{d(q, x_n), d(q, Tq), d(x_{n+1}, x_n), \tfrac{1}{2}[d(q, x_{n+1}) + d(x_n, Tq)]\}\\ &\le \max\{d(q, x_n), d(q, Tq), d(x_{n+1}, x_n), d(q, x_n) + \tfrac{1}{2}[d(x_n, x_{n+1}) + d(q, Tq)]\}\\ &\le d(q, x_n) + d(x_n, x_{n+1}) + d(q, Tq). \end{aligned}$$

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This implies that

$$d(q, Tq) \le M(q, x_n) \le d(q, x_n) + d(x_n, x_{n+1}) + d(q, Tq),$$

and so $M(q, x_n) \to d(q, Tq)$ as $n \to \infty$. From (2.7),

$$\inf_{i\ge n} \psi(d(Tq, x_{i+1})) + \inf_{i\ge n} \varphi(M(q, x_i)) \le \sup_{j\ge n} \psi(M(q, x_j))$$

for all $n \ge 1$. It follows that

$$\lim_{t\to d(q,Tq)} \inf \psi(t) + \lim_{t\to d(q,Tq)} \inf \varphi(t) \le \lim_{t\to d(q,Tq)} \sup \psi(t),$$

which is a contradiction and so $q = Tq$.

The uniqueness of the fixed point of T is clear. Indeed, if it is not the case, then there exists $p \in X$ such that $Tp = p \ne q = Tq$. Observe that

$$\begin{aligned} 0 &< \psi(d(q, p)) = \psi(d(Tq, Tp)) \\ &\le \psi(M(q, p)) - \varphi(M(q, p)) \\ &= \psi(\max\{d(q, p), \tfrac{1}{2}[d(q, Tp) + d(Tq, p)]\}) \\ &\qquad - \varphi(\max\{d(q, p), \tfrac{1}{2}[d(q, Tp) + d(Tq, p)]\}) \\ &= \psi(d(q, p)) - \varphi(d(q, p)) < \psi(d(q, p)) \end{aligned}$$

which is a contradiction and so $p = q$. The proof is complete. □

REMARK 2.2. Theorem 2.1 extends and improves [7, Theorem 4], [6, Theorem 3.3] and [4, Theorem 2.2] in the following sense.

(a) The assumption that ψ is monotone nondecreasing or continuous is unnecessary.
(b) The assumption that $\liminf_{\tau\to t} \varphi(\tau) > 0$ for all $t > 0$ is removed.
(c) The assumption that, for all $a > 0$, the function ψ has a left limit at a is not needed.

In particular, [4, Theorem 2.2] is a special case of Theorem 2.1.

EXAMPLE 2.3. Let $X = [0, 1) \cup \{1, 2, 3, 4, \ldots\}$ with the metric

$$d(x, y) = \begin{cases} |x - y| & \text{if } x, y \in [0, 1), \\ x + y & \text{if } x \in [0, 1), y \ge 1 \text{ or } x, y \ge 1 \text{ with } x \ne y, \\ 0 & \text{if } x = y. \end{cases}$$

Then $d(X, d)$ is a complete metric space [2]. Define a mapping $T : X \to X$ by

$$T(x) = \begin{cases} [x] & \text{if } x \in [0, 1), \\ x - 1 & \text{if } x = 1, 2, 3, \ldots. \end{cases}$$

Then the fixed point set of T is $F(T) = \{0\}$. Define $\psi, \varphi : [0, +\infty) \to [0, +\infty)$ by

$$\psi(t) = \begin{cases} t & t \in [0, 1), \\ (t - \frac{1}{4})^2 & t \ge 1, \end{cases} \qquad \varphi(t) = \begin{cases} \frac{1}{2}t^2 & t \in [0, 1], \\ t - \frac{17}{32} & t > 1. \end{cases}$$

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Then T, ψ, φ satisfy the conditions of Theorem 2.1 and the functions ψ and φ are neither continuous nor monotone nondecreasing. In order to prove that these facts hold, we consider the following possible cases. Without loss of generality, assume that $y < x$.

Case 1. Let $x, y \in [0, 1]$. Then

$$d(Tx, Ty) = 0, \quad \psi(d(Tx, Ty)) = 0.$$

Case 1.1. If $x, y \in [0, 1)$, then

$$\begin{aligned} M(x,y) &= \max\{d(x,y), d(x,Tx), d(y,Ty), \tfrac{1}{2}[d(x,Ty) + d(y,Tx)]\} \\ &= \max\{x - y, x, y, \tfrac{1}{2}(x+y)\} \\ &= x \end{aligned}$$

and

$$\psi(M(x,y)) = x, \quad \varphi(M(x,y)) = \tfrac{1}{2}x^2.$$

So

$$\psi(d(Tx, Ty)) \le \psi(M(x,y)) - \varphi(M(x,y)).$$

Case 1.2. If $x = 1$ and $y \in [0, 1)$. Then

$$\begin{aligned} M(x,y) &= \max\{d(x,y), d(x,Tx), d(y,Ty), \tfrac{1}{2}[d(x,Ty) + d(y,Tx)]\} \\ &= \max\{1 + y, 1, y, \tfrac{1}{2}(1+y)\} \\ &= 1 + y \end{aligned}$$

and

$$\psi(M(x,y)) = (1 + y - \tfrac{1}{4})^2 = (\tfrac{3}{4} + y)^2, \quad \varphi(M(x,y)) = 1 + y - \tfrac{17}{32} = \tfrac{15}{32} + y.$$

Thus

$$\psi(d(Tx, Ty)) \le \psi(M(x,y)) - \varphi(M(x,y)).$$

Case 2. Let $x \in \{2, 3, 4, \ldots\}$ and $y \in [0, 1]$. Then

$$d(Tx, Ty) = x - 1, \quad \psi(d(Tx, Ty)) = (x - \tfrac{5}{4})^2$$

and

$$\begin{aligned} M(x,y) &= \max\{d(x,y), d(x,Tx), d(y,Ty), \tfrac{1}{2}[d(x,Ty) + d(y,Tx)]\} \\ &= \max\{x + y, 2x - 1, y, \tfrac{1}{2}(x + y + x - 1)\} \\ &= 2x - 1, \end{aligned}$$

and

$$\psi(M(x,y)) = (2x - \tfrac{5}{4})^2, \quad \varphi(M(x,y)) = 2x - \tfrac{49}{32}.$$

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Now observe that, for $x \geq 2$, we have $96x^2 - 144x + 49 > 0$ which is equivalent to

$$(x - \tfrac{5}{4})^2 \leq (2x - \tfrac{5}{4})^2 - (2x - \tfrac{49}{32}),$$

that is,

$$\psi(d(Tx, Ty)) \leq \psi(M(x, y)) - \varphi(M(x, y)).$$

Case 3. Let $x, y \in \{2, 3, 4, \ldots\}$ with $x \neq y$. Then we have

$$d(Tx, Ty) = x + y - 2, \quad \psi(d(Tx, Ty)) = (x + y - \tfrac{9}{4})^2$$

and

$$\begin{aligned} M(x, y) &= \max\{d(x, y), d(x, Tx), d(y, Ty), \tfrac{1}{2}[d(x, Ty) + d(y, Tx)]\} \\ &= \max\{x + y, 2x - 1, 2y - 1, x + y - 1\} \\ &= 2x - 1. \end{aligned}$$

Consequently,

$$\psi(M(x, y)) = (2x - \tfrac{5}{4})^2, \quad \varphi(M(x, y)) = 2x - \tfrac{49}{32}.$$

Now we can verify that, for all x, y with $2 \leq y < x$,

$$(x + y - \tfrac{9}{4})^2 \leq (2x - \tfrac{9}{4})^2 \leq (2x - \tfrac{5}{4})^2 - (2x - \tfrac{49}{32}).$$

Hence

$$\psi(d(Tx, Ty)) \leq \psi(M(x, y)) - \varphi(M(x, y)).$$

Case 4. Finally, if $x = y$, then it is easy to verify that (2.1) of Theorem 2.1 holds. Further, it is apparent that the functions ψ, φ satisfy the other conditions of Theorem 2.1 except possibly when $t = 1$. In fact, this condition also holds because

$$\tfrac{9}{16} = \liminf_{t \to 1} \psi(t) > \limsup_{t \to 1} \psi(t) - \liminf_{t \to 1} \varphi(t) = 1 - \tfrac{15}{32} = \tfrac{17}{32}.$$

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