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How Can Data Science Contribute to Understanding the *Khipu* Code?

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Abstract
In "How Can Spin, Ply, and Knot Direction Contribute to Understanding the Quipu Code?" (2005), mathematician Marcia Ascher referenced new data on 59 Andean *khipus* to assess the significance of their variable twists and knots. However, this aggregative, comparative impulse arose late in Ascher's *khipu* research; the mathematical relations she had identified among 200+ previously cataloged *khipus* were specified only at the level of individual specimens. This article pursues a new scale of analysis, generalizing the "Ascher relations" to recognize meaningful patterns in a 650-*khipu* corpus, the largest yet subjected to computational study. We find that Ascher formulae characterize at least 74% of *khipus*, which exhibit meaningful arrangements of internal sums. Top cords are shown to register a minority of sum relationships and are newly identified as markers of low-level, "working" *khipus*. We reunite two fragments of a broken *khipu* using arithmetic properties discovered between the strings. Finally, this analysis suggests a new *khipu* convention—the use of white pendant cords as boundary markers for clusters of sum cords. In their synthesis, exhaustive search, confirmatory study, mathematical rejoining, and hypothesis generation emerge as distinct contributions to *khipu* description, typology, and decipherment.

Resumen
En 2005, la matemática Marcia Ascher utilizó nuevos datos sobre 59 quipus para evaluar la importancia de giros variables en sus cuerdas y nudos. Sin embargo, este impulso comparativo y agregativo surgió a finales de sus investigaciones; las relaciones matemáticas que había identificado entre más de 200 quipus previamente catalogados se presentaban sólo a nivel de ejemplares individuales. Este artículo propone una nueva escala de análisis, generalizando las "relaciones Ascher" para identificar patrones significativos en un corpus de 650 quipus —el mayor sometido hasta ahora a análisis informático—. Observamos que las fórmulas de Ascher caracterizan al menos el 74% del corpus, que presenta sumas internas regularmente arregladas. Además, las cuerdas superiores registran una minoría de relaciones aditivas; se identifican aquí por primera vez como indicadores de quipus "de trabajo" de bajo nivel. Reagrupamos dos fragmentos de un quipu roto utilizando propiedades aritméticas descubiertas entre las cuerdas. También se propone una nueva convención: el uso de cuerdas colgantes blancas como marcadores de los límites de grupos de cuerdas que registran sumas. En su síntesis, la búsqueda exhaustiva, estudios de confirmación, reagrupación matemática y la generación de hipótesis ofrecen contribuciones distintas a la descripción, la tipología y el desciframiento del quipu.

Keywords: *khipu* (quipu); data science; Andes; mathematics; decipherment

Palabras clave: *khipu* (quipu); ciencia de datos; Andes; matemáticas; desciframiento

In a 2005 study called "How Can Spin, Ply, and Knot Direction Contribute to Understanding the Quipu Code?" mathematician Marcia Ascher utilized newly available data from 59 samples to assess the significance of variable twists in *khipu* strings and knots. Based on straightforward frequency counts—for example, dozens of the *khipus* in question had uniformly S-twisted pendant cords—she deemed these elements largely "a characteristic of the quipu [that does] not represent choices being made by the quipu-maker on a cord-by-cord basis" (Ascher 2005:101).

Surprisingly, this aggregative, comparative impulse arose only toward the end of Ascher's research. Previously, she and Robert Ascher had produced descriptions of 235 *khipus*, 80 of which Marcia annotated with various arithmetic "relations" between the numbers knotted on the pendant strings (Ascher and Ascher 1972, 1978, 1988). Nonetheless, the mathematical statements were only defined for individual samples—they were never generalized to evaluate overarching patterns in *khipu* construction norms or recording techniques (Ascher 2005:100).[1]

Today, hundreds of additional *khipus* have been digitized, compelling us to take up a similar question to Marcia Ascher's. Prior to recent statistical analyses by Jon Clindaniel (2019), most published computational *khipu* research was carried out on some 300 digitized samples (e.g., Urton 2006). Our study expands the Open Khipu Repository (OKR Team 2022) to analyze 650 *khipus*. Compiled in Khosla's (2022) Khipu Field Guide, this corpus includes 24 previously unpublished specimens—21 cataloged by Medrano in seven museums in Spain, Switzerland, and Germany, as well as three recorded by Kylie Quave in Beloit, Wisconsin.[2]

In this article, we use the Aschers' mathematical relations as a springboard to demonstrate the multiple contributions of data science to understanding the *khipu* code; four are developed here. First is exhaustive search, in which every combinatorial possibility is generated and assessed: grouping the 80 Ascher relations into nine classes, we find that they generalize unexpectedly well. Though only identified by the Aschers for 34% of samples, one or more Ascher formulae characterize 76% (482/636) of *khipus* with knots (74% overall). Second, confirmatory studies: we revisit "top cords"—strings tied "upward" so as to lie vertically opposite to the pendants—a construction element often regarded as *khipus*' quintessential summing locus. We find such constructions represent only 28% (107/376) of all top cords, and less than 1% of all *khipu* sum relationships. Top cords are newly identified here as markers of low-level, "working" *khipus*, by which we refer to those employed in local and small-scale recordkeeping and administration. Third is the rejoining of broken *khipus*: we debut the analysis of internal sums as a means of reuniting separated specimens, as demonstrated by the discovery of mathematical relationships linking two *khipu* fragments in the Ethnologisches Museum, Berlin. Fourth is hypothesis generation and decipherment. Based on an exploratory analysis of pendant cord colors, we propose a decipherment of an arithmetic *khipu* convention: that white pendants mark the boundaries of clusters of sum cords, which themselves total the numerical values of summand cords that are tied elsewhere on the *khipu*. We discuss, within each section, the corresponding implications for our understanding of *khipu* semiosis.

Finally, on terminology: despite our use of the singular "code" in the article's title, we do not assume the existence of only one *khipu* logic (Brokaw 2010). We instead follow Robert Ascher (2002:106) in defining a *khipu*'s code as its "internal structure." Consequently, this study is distinct from the pathbreaking efforts of Carrie Brezine in searching for *khipu-khipu* (Urton 2005) or *khipu*-document (Urton 2006) "matches"—that is, sequences of identical numbers. By focusing on specimens' average makeup, we seek to make progress in a parallel lane—a ground-up "extraction" of meaning based on observed repetition and frequency (Houston 2004). Recognizing the initiatives of the OKR Advisory Board, we also employ the board's newly proposed *khipu* naming conventions (Brezine et al. 2024).

Khipu Studies at Scale

Our study comes at a time of increasing scholarly availability of machine-readable *khipu* data (Medrano 2021a:Chapter 6). As Matthew Jockers (2013:4) has argued elsewhere, "we have reached . . . an event horizon where enough text and literature have been encoded to both allow and, indeed, force us to ask an entirely new set of questions." In parallel, *khipu* research has begun to move from the identification of notable properties in individual specimens to the assessment of hypotheses using larger compilations of cords (e.g., Clindaniel 2019)—a commitment recently dubbed an "aggregative turn" (Medrano 2021b:312).

Nonetheless, we find it curious that recent *khipu* scholarship has de-emphasized the one element of the object that we can most consistently interpret: its numerical knots. Among the ambitious attempts to decipher the *khipu*'s purportedly "narrative" or nonnumerical elements, strong arguments exist concerning color patterning (Hyland 2016), knot direction (Hyland et al. 2014), and affixed needlework bundles (Hyland 2020), several of which find support in statistical testing on the

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OKR (Clindaniel 2019). Yet the *khipus*' numerical values have continued to play a largely ancillary role in such studies.

In contrast, the intensive analysis of numerical knots represents our attempt to upend their largely anodyne reputation. Despite recent appeals to cataloging additional *khipu* loci, here we return to numbers, which are recorded for all specimens with knots (98%). In this regard, the numerical values assigned by previous catalogers are used; we do not superimpose our own readings. Though this study does not include the Aschers' writings on potential calendrical specimens (e.g., Ascher and Ascher 1989) or their more minor treatments of multiplication and division, what remains is, nonetheless, ample fodder for committing oneself to "the actuality of the khipus" (Marcia Ascher, quoted in Cook 2003).

Exhaustive Search

The computer's potential for comprehensively searching the *khipu* corpus is not a recent realization. Some 60 years ago, Carlos Radicati di Primeglio (2006:243) affirmed that it would be precisely the "electronic computer"—with its ability to search "an infinity of combinatory tests of colors and knots"—that would aid in revealing the "key" to reading nonnumerical *khipus*. Something similar can be said of *khipus*' mathematical properties. Contemporaneous with the Aschers' (1969) first publication on the subject were calls to investigate internal sums by Radicati (who called them the *khipu*'s "parallelisms"; 2006:173–207) and anthropologist Carol Mackey (1970:53–54).

Nonetheless, it was with the publication of the Aschers' (1978, 1988) two *khipu* "Databooks" that specific properties were first identified for more than a handful of samples. Readers were alerted, for example, that cords one to four of *khipu* KH0109's seventh pendant cord group total the values of the corresponding indexed cords in the prior three cord groups; that is,

$$P_{7,i} = \sum_{j=4}^{6} P_{j,i} \text{ for } i = \{1, 2, 3, 4\},$$

where P, j, and i refer to pendant cord, cord group, and pendant cord index, respectively (Ascher and Ascher 1978:675). A systematic review has allowed us to generalize such statements to nine relations; KH0109's property, for example, qualifies as a so-called indexed pendant sum, in which a pendant cord (i.e., sum cord) registers the sum of similarly indexed pendant cords (i.e., summand cords), with the latter found in cord clusters that are contiguous to each other. The full list is reproduced in Table 1, which ranks the relations by the total number of *khipus* (out of 650) that exhibit each. The relations pertain to addition (numbers one, two, three, five, seven, eight, nine), subtraction (number four), and ordering by magnitude (number six). All accompanying data and code for the subsequent analyses are linked in the Data Availability Statement.

In the first instance, exhaustive search reveals that one or more Ascher formulae characterize 76% of *khipus* with knots (74% overall).[3] This raises previous estimates identifying 67% of *khipus* with numerical accounting functions (Urton 2017:49). The actual percentage could well be higher than 76% because some *khipus* are fragmentary.

Further, combining the distribution of Ascher relations with each *khipu*'s overall numerical magnitude contributes directly to an ongoing goal in *khipu* decipherment: the identification of samples from different levels of accounting hierarchies. Most famously associated with Inka decimal administration, the vertical summation and partitioning of *khipu* records facilitated accounting in the Andes both before and after the Spanish conquest (Julien 1988). Perhaps the most prominent example identified to date are seven archaeological *khipus* from the coastal site of Puruchuco that, based on summation relationships identified between them, have been described as pertaining to three levels (I, II, and III) of an Inka accounting hierarchy in the Rimac Valley (Urton and Brezine 2005).

Taking pendant-pendant sums (defined in Table 1, number one), the most numerous Ascher relation (8,088 individual occurrences, or 46% of the 17,414 total occurrences of relations numbers one to nine across the corpus), we find that high mean pendant value *khipus* exhibit very few sums (Figure 1). This seems to distinguish them as a class all their own; we hypothesize that they are consistent

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Table 1. Incidence of Generalized Ascher Relations in 650 Digitized, Inka-style *Khipus*.

Generalized Ascher relation	Definition of generalized Ascher relation (*see Data Availability Statement for full computational search parameters*)	Number of *khipus* (% of 650-*khipu* corpus) with at least one instance of relation
1. Pendant-Pendant Sums	Pendant cords that register the sum of a set of other pendant cords that are contiguous to each other, regardless of the colors of cords or the clusters in which the cords appear.	427 (66%)
2. Pendant Sums by Color	Pendant cords that register the sum of a set of other pendant cords of the same color, regardless of cord index (i.e., where in the sequence of cords in a cluster a cord appears). The summands do not need to be contiguous to one another, but they must be in cord clusters that are contiguous to each other.	261 (40%)
3. Pendant Sums by Index	Pendant cords that sum a set of similarly indexed pendant cords, with the latter found in cord clusters that are contiguous to each other.	228 (35%)
4. Pendant Subsidiary Difference	Contiguous pairs of pendant cords {A, B} in which pendant B registers the absolute value of the difference between pendant A and its subsidiary cord (or the sum of its subsidiary cords, if it has multiple).	145 (22%)
5. Subsidiary-Pendant Sums	Subsidiary cords that sum a set of pendant cords that are contiguous to each other, regardless of cord cluster.	117 (18%)
6. Decreasing Cluster Cord Values	Clusters whose individual cord values decrease roughly linearly from left to right, as measured by the slope of a least-squares line applied to the sequence of numbers.	90 (14%)
7. Cluster Sum Bands	Clusters in which the first half of cords (from left to right) sum to the same value as the remaining half of cords in the cluster. In other words, the cluster's left-half sum equals its right-half sum.	87 (13%)
8. Subsidiary Sums by Color Index	Subsidiary cords that sum a set of cords of the same color, with the latter found in clusters that are contiguous to each other. For sums ≥100, "off-by-one" matching is allowed (e.g., 1,341 and 1,241 are recorded as a match).	41 (6%)
9. Top Cord Sums and Double Sums	Top cords that register the sum, or twice the sum, of all cords in a cluster that is contiguous to the top cord, with off-by-one matching allowed.	29 (4%)

with higher-level "summary" accounts in *khipu* administration, as differentiated from lower-level "working" *khipus*, which have many internal checks and balances (i.e., Ascher relations). From Figure 1, we see the latter tend to comprise *khipus* with mean pendant cord values between 10 and 1,000; that is, up to the *waranka* level of Inka decimal accounting (assuming a one-to-one correspondence with the magnitude of this administrative unit).

This theory finds additional support in a corpus-wide search for untied "ghost knots" (Salomon 2004:169). In the clearest testimony on the matter, the conquistador Hernando Pizarro (1920 [1533]:175) wrote that as his men raided an Inka storehouse, the local *khipukamayuqs* "removed knots . . . and [re-]knotted them in another part [of the *khipu*(*s*)]" to reflect the changing inventories. Using references to the few untied knots in the corpus as a proxy for *khipus* subject to similar on-the-ground operations, we find matches to nine samples with mean pendant cord values between zero (for the fully untied KH0307) and 725 (KH0254)—falling within the numerical range identified above for "working" *khipus*.[4]

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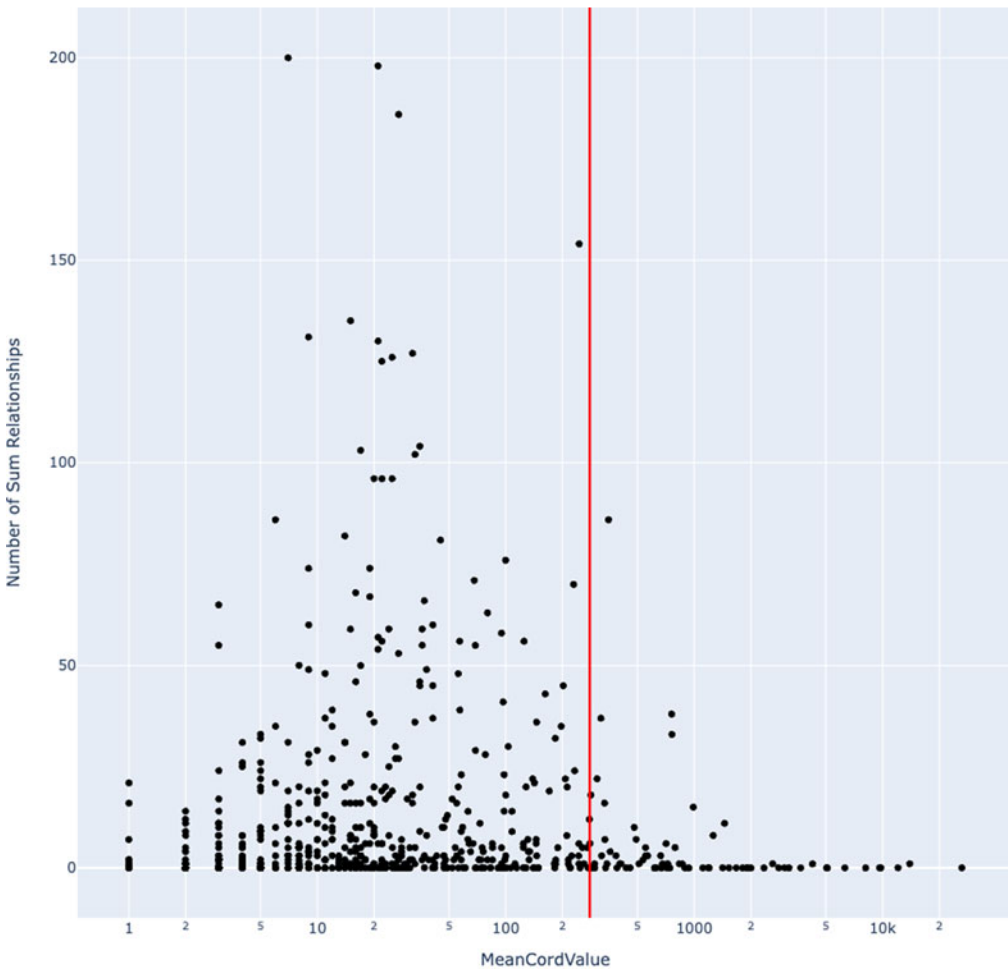

Figure 1. Number of pendant-pendant sum relations versus (log) mean pendant cord value. The corpus-wide average pendant value (279) is denoted with a vertical line.

Notably, Jon Clindaniel's (2019:111) computational study of *khipu* magnitude has identified the transition point between individual-level accounts (banded coloring) and aggregated ones (seriated coloring) with pendant cord values in the tens' place (i.e., in the range between 10 and 100). However, here we describe a transition point for "working" versus "summary" *khipus* somewhere in the thousands' place. How can the two be reconciled? It seems that, rather than contradicting Clindaniel's conclusion, our finding provides an identification mechanism for *khipus* at even higher accounting levels than those classifiable by their color patterning. That is, while the transition from *ayllu* to community-level records may have been marked by seriated *khipus* versus banded ones (Hyland 2016), "summary" level accounts at the *waranka* level and above were distinguishable by a density of values in the thousands' and ten thousands' places and an utter lack of internal sum relationships. The labeling of both banded and seriated *khipus* of certain magnitudes as "working" specimens may be a surprise; however, recall that the initial decipherment of color patterning that Clindaniel evaluated was based on modern *khipus* that aggregated labor contributions at the level of one Andean village (Hyland 2016:504). The Puruchuco case also demonstrates that even multiple seriated *khipus* could coexist in hierarchical relation to one another.

Together, these observations add much-needed nuance to the identification criteria for higher-order accounting *khipus*. When coupled with numerical magnitude and previously deciphered color

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schemas (Hyland 2016), internal sums better predict "summary" *khipus* than existing methods. Using the density of thousands' place values as a proxy for the latter, we can tentatively characterize the *khipus* of the Puruchuco hierarchy—which exhibit only a single thousands' place value on the highest level—as all essentially "local" in nature. This aligns with previous findings: even if the level III (highest level) *khipus* were "interface" specimens received from outside Inka authorities, their contents were ultimately delimited by Puruchuco alone (Urton and Brezine 2005). Just as color banded *khipus* may have underlain the hierarchy's lowest level (Hyland 2016:507n17), large-value dense, arithmetically poor *khipus* stored elsewhere could have summarized the Puruchuco hierarchy (and others like it) from above.

Validating (or Not) Existing Observationally Derived Hypotheses

In a recent analysis of early colonial *khipu* transcriptions, Medrano (2021b:330) argued that computational linguistic approaches enable the assessment of ethnohistoric hypotheses "resting dormant in [previous] studies." Data science allows us to say something similar about the surviving *khipus* as well. To do so, we revisit an often-repeated refrain: that top cords are *khipus*' quintessential summing structures, totaling the values on associated pendant strings (relation number nine, Table 1). Constituting less than 1% of sums in the corpus, top cords, per our searches, emerge more specifically as hallmarks of low-level "working" *khipus*.

The gradual conflation of top cords with sums is perhaps unsurprising because it was a *khipu* with several sum top cords (KH0405) that enabled L. Leland Locke's (1912) decipherment of numerical knots on Inka-style *khipus*. Marcia Ascher (1986:278) later characterized top cords as "almost exclusively carry[ing] the sums of the values of the groups with which they are associated." Slightly more measured descriptions of them "often" recording sums abound in the literature (e.g., Tun 2016:3685).

However, a search of the corpus reveals that of the 47 *khipus* with top cords (7.2% of the 650 eligible *khipus*), only 29 have one or more top cords recording sums or double sums, even using a capacious search criterion that allows for digits in each place value to be off by one (as they sometimes were for Locke). Though the 7.2% top-cord-bearing *khipus* aligns with previous estimates to this effect—for example, 10% (Ascher 2005:106) or 9.2% (Mackey 1970:45)—our analysis points to a basic overestimation of sum top cord frequency, confirming previous suspicions to this effect raised by Carol Mackey (1970:52–53). Database queries can thus add long-overdue nuance to widely held views of a "standard" Inka *khipu*. We are reminded in this regard of Galen Brokaw's (2005:588) admonition: in the search for conventionality in the *khipu* sign system, proposed decipherments of individual *khipu* elements must ultimately "produce results that complement numerical readings."

Most strikingly, aggregate analysis allows us to identify sum/double sum top cords—and even top cords more broadly—as a distinguishing characteristic of local, low-level accounting *khipus*. Returning to the *waranka* (1,000)-magnitude upper bound for "working" *khipus* proposed in the previous section, we find that 85 of 86 sum top cords in the corpus record values less than 1,000; the remaining top cord sum (on KH0405) has the value 1,417. All the 21 double sum top cords fall within the same interval, with a maximum value of 729. The 29 total *khipus* with these properties also record small quantities on the other strings: their mean pendant cord values range between one (KH0068) and 248 (KH0603). Equally suggestive is that top-cord-bearing samples are exceedingly likely to have banded coloring, a design feature previously shown to be a marker for ayllu-level *khipus* (Clindaniel 2019:Chapter 5; Hyland 2016). A manual review of the 46 computationally identified matches with available color data reveals that 44 *khipus* are all or majority banded, or have top cords associated with color bands. While banded *khipus* are estimated (Clindaniel 2019:109) to compose 16% of all *khipus* and 37.5% of *khipus* with discernible color patterning, they make up some 96% of *khipus* with top cords.

Among *khipus* with mixed color patterning, KH0607, in the Dallas Museum of Art, is particularly revealing. Composed of four primary cords tied together, the sample exhibits seriated coloring in the leftmost primary cord's pendants and banded coloring in the pendants of the remaining three primary cords (Figure 2). Not only does the *khipu*'s one sum top cord appear just after the transition to banded coloring, but the other (apparently non-sum) top cords also only appear on its banded portions.

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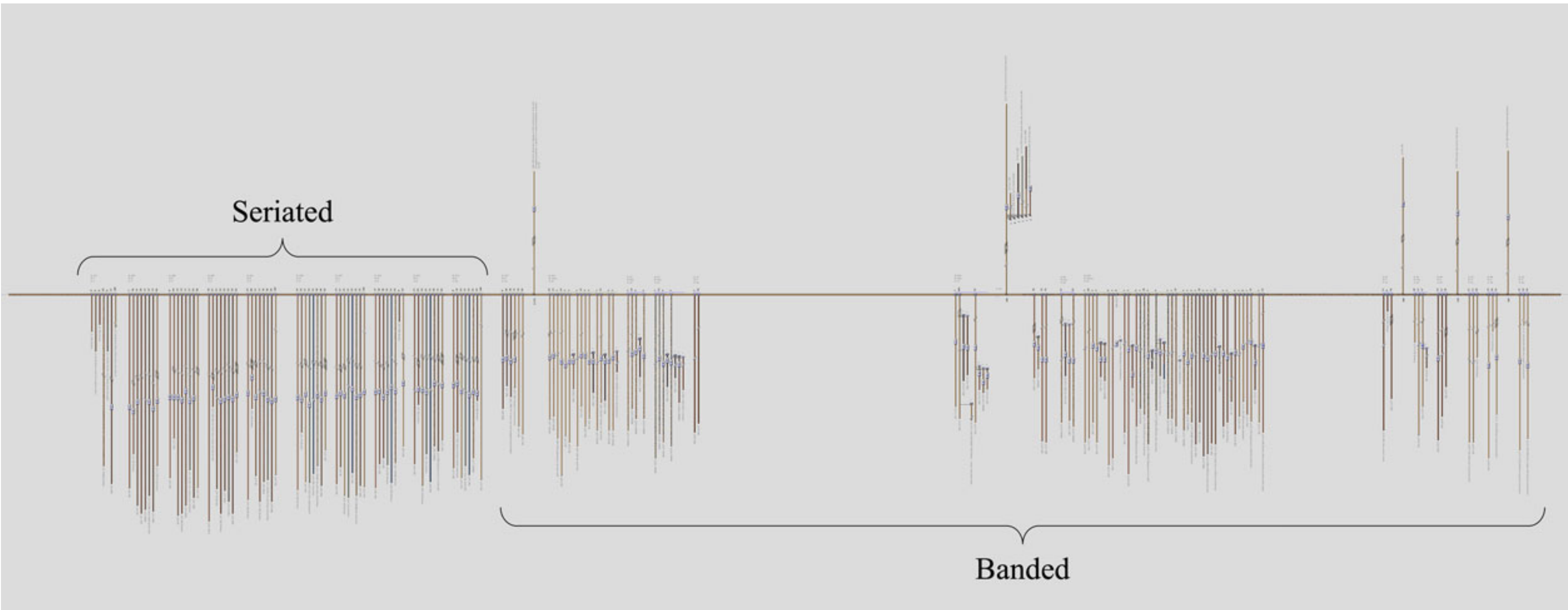

Figure 2. Dallas Museum of Art T41299.38 (*Khipu* KH0607). (Color online)

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A search for additional Inka-style *khipus* with top cords outside of our corpus yielded at least 19 results; with at most one exception (*khipu* VA22928, Ethnologisches Museum, Berlin [EMB]), all are color banded or have top cords associated with color bands (Locke 1927; Pancorvo 2020:332–333, 398–399; Tokovinine 2019; Tun 2016:3682; *khipus* B/8715 and 41.0/7304A, American Museum of Natural History [AMNH], New York; *khipus* VA42561a, VA44864b, VA47088, VA66842a, and VA66842b, EMB; *khipus* ML600131 and ML600007, Museo Larco, Lima; *khipu* 1940.469, Cleveland Museum of Art; *khipu* 80.1012, Stanford University Archaeology Collections; *khipu* [legacy number 002.49b], Manuscript Collection, Rare Books and Special Collections, McGill University Library; *khipu* O.4017, Nationalmuseet, Copenhagen, depicted in "Quipu nr 56: Köpenhamn," drawing, 1924–1925, F1-4, Världskulturmuseets Arkiv [VKMA], Gothenburg).

A brief turn to *khipus* with archaeological and ethnohistorical provenance provides further support for our hypothesis. Of the 11 *khipus* recovered in 2017 from Huacones-Vilcahuasi, a Late Horizon Guarco administrative center in the lower Cañete Valley, five have top cords, which register values in the ones' and tens' places. The site would have been an important stage for local, everyday "working" calculations, as suggested by the discovery of a *yupana* (an Inka abacus) in a large courtyard of the complex by Barraza Lescano and colleagues (2022:234–236). Sum top cords with values in the hundreds' place and below are found on three color-banded *khipus* (KH0072, KH0079, KH0080) among 16 samples from Quebrada de la Vaca, a settlement and Inka storage site near the southern Peruvian port city of Chala; two other *khipus* in the cache, excavated in 1954 by Francis A. Riddell and Dorothy Menzel, were also observed to be unknotted (Mackey 1970:Chapter 4). As for later *khipus* with ethnohistorical context, one sum top cord appears on KH0323, the largest of six color-banded *khipus* reportedly from the Santa Valley, in Ancash. These *khipus* have previously been correlated with ayllu-level registrations of tribute arising from a 1670 Spanish census visitation (Medrano and Urton 2018).[5]

Though our hypothesis does not rule out uses of top cords on higher magnitude *khipus* as possible aids to the strings' navigation, the inconsistency of top cord signification among smaller magnitude *khipus* is perhaps exactly what one would expect to see of idiosyncratic recording norms operating at lower levels of *khipu* hierarchies (Fossa 2019:125–127). It might thus be best to exercise caution around Leland Locke's (1923:31) claim of (sum) top cords being hallmarks of "the most highly developed form" of the *khipu*.

Leveraging Discovered Patterns to Reconstruct Broken *Khipus*

The next application returns to a question as old as *khipu* research itself: should we study broken and fragmentary samples; and if so, how? The Swedish anthropologist Erland Nordenskiöld (1925:10), for example, argued that "it is of the utmost importance in the interpretation of quipus to have complete specimens," an idea that was later rejected on face by Carlos Radicati di Primeglio (2006:161). While many *khipus* have a handful of broken pendant cords, those with broken primary cords are particularly vexing—these may have originally hosted tens if not hundreds of additional strings. Using the primary cord termination code of "broken" as a proxy for fragmentary *khipus*, a search of the dataset reveals 156 (24%) afflicted specimens. Although many of the matches to these fragments have been lost, others may be extant but merely dissociated. We can thus affirm that all available *khipus* should be studied (à la Radicati) while also seeing immense value in reuniting separated specimens (with a nod to Nordenskiöld).

Utilizing the generalized Ascher relations (Table 1) as computational constructs offers a novel path toward such reconstructions. To demonstrate, we present a base case in KH0468, comprising two mathematically associable *khipu* fragments (VA16135a and VA16135b) in the Ethnologisches Museum, Berlin. The attentive reader will notice that both *khipu* fragments (which we will refer to simply as A and B) are numbered similarly. They are in fact two of several *khipus* and *khipu* fragments reportedly from Pisco, on Peru's southern coast, acquired by the museum at the turn of the twentieth century from the collection of I. M. Bolivar (Ethnologisches Museum Berlin 2021). Fragment A is composed of 39 groups of pendant cords and fragment B is composed of 41. In total, the fragments comprise 954 strings arranged in 80 color-banded groups (Figure 3).

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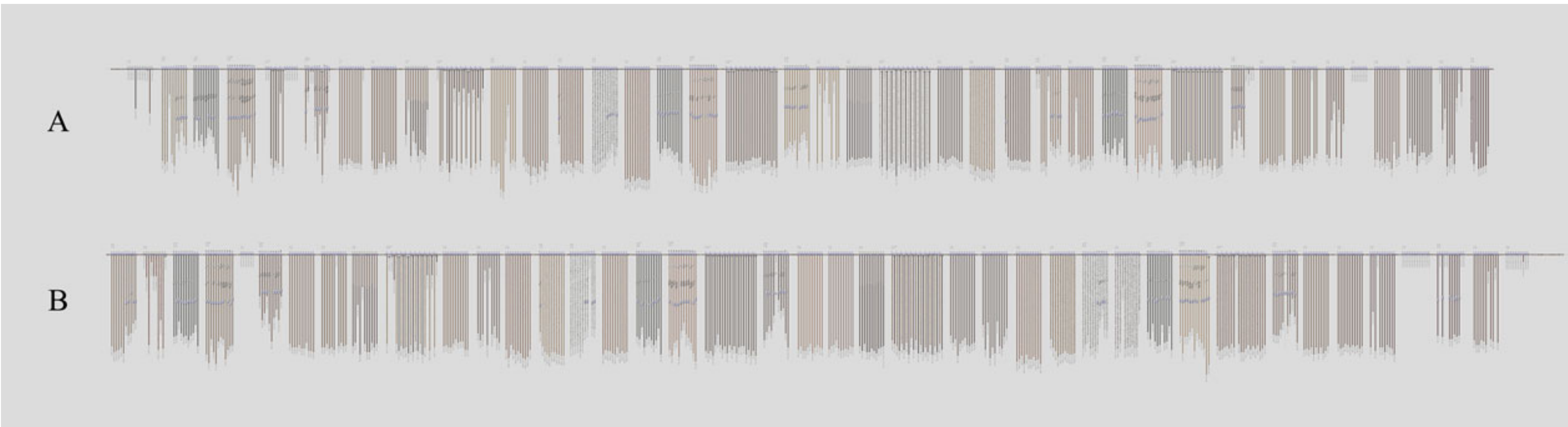

Figure 3. *Khipu* fragments VA16135a and VA16135b. (Color online)

It is unclear whether fragments A and B were physically connected when they entered the museum's collections; in any case, their rejoining may seem a foregone conclusion. Beyond having the same inventory number and provenance, the samples share the same primary cord structure, color patterning, and average pendant group size. They were entered into the OKR in 2013 as a single *khipu* (UR231, now KH0468). In our view, however, the presumed "obviousness" of the fragments' combination makes them an ideal proof of concept for our methodology. Because incorrect pairings of broken *khipus* can distort the scholarly record, it behooves us to develop additional means of corroboration beyond prevailing visual, ad-hoc methods (e.g., Ascher and Ascher 1978:551).

The computational combination of fragments A and B into a single *khipu* produces a wealth of cross-fragment, regularly arranged pendant-pendant sums, suggesting that the fragments were indeed previously connected—and that a third *khipu* fragment was not originally attached between them. We visualize this through a network graph (Figure 4), in which each column is one of the rejoined *khipu*'s cord clusters. The pendants in each cluster are represented in order (from top to bottom for each column) as rectangular nodes. The bolded nodes distinguish the cord clusters containing sum cords, which are not only visibly grouped but also appear with a consistent regularity and symmetry across the labeled break in the primary cord. Sums and summands are linked by connecting lines (see Figure 4's legend), which show the arithmetic interconnectedness of the rejoined *khipu*'s numerical values—of the 97 pendant-pendant sums we have identified, 30 of them involve summands and sum cords that are found on both fragments A and B (i.e., on either side of the break in the primary cord).[6] The proposed rejoining yields more pendant-pendant sums than does the combination of fragment A with any other comparable *khipu* in our corpus; visual inspection shows these sums to be more regularly arranged than any alternative (see accompanying analysis linked in data availability statement). At a high level, these findings imply that the visual and numerical inspection of pendant-pendant sums can be an additional line of evidence in arguing for the reunion of dissociated *khipu* fragments.

It should be noted that Figure 4, which might be likened to a sort of mathematical X-ray, only plots so-called right-handed sums, in which summands appear to the right of their associated sum. These can be represented by the formula $Y_n = x_m + x_{m+1} + \ldots + x_{m+c}$, where Y_n is a sum cord in position n, $x_{\{m,\ m+1,\ \ldots,\ m+c\}}$ are a sequence of summand cords contiguous to one another in positions $\{m, m + 1, \ldots, m + c\}$, c is a nonnegative integer, and $n < m$; this accords with the customary arrangement of equations in modern mathematics. However, 89 left-handed pendant-pendant sums also appear on KH0468; these are characterized by summands that appear to the left of their associated sum, as described by the formula $x_1 + x_2 + \ldots + x_m = Y_n$, where Y_n is again a sum cord in position n, $x_{\{1,\ 2,\ \ldots,\ m\}}$ remain a sequence of summand cords contiguous to one another in positions $\{1, 2, \ldots, m\}$, but $n > m$. Nineteen of the left-handed sums on the rejoined *khipu* span the break in the primary cord.

Decades ago, the Aschers identified a similar phenomenon. In their notes on a *khipu* (KH0217) with pendant-pendant sums in the AMNH, the duo stated that when the pendant positions of some cord groups were arranged in reverse order, a greater number of sums appeared, including some in which a given cord registered the sum of pendant cords in groups both preceding and following it (Ascher and Ascher 1978:1149). Here, exhaustive search provides the first large-scale confirmation of "sum handedness" in *khipus*, including a remarkably consistent asymmetry in right-handed versus left-handed sums across the corpus. In KH0468, for example, one finds that right-handed sums make up 52% of total pendant-pendant sums (97 out of 186). At the whole corpus level, we find an approximate 55/45 percentage split in right-handed versus left-handed sums for all four Ascher pendant sum relationships (Table 1): pendant-pendant sums (54/46), pendant sums by color (56/44), pendant sums by index (57/43), and subsidiary pendant sums (57/43). This is not to imply that some 45% of *khipus* have been recorded backward because in most cases (63% across the four named relationships), any given specimen presents both left-handed and right-handed sums.[7]

Reapplying our arithmetic diagnosis of KH0468 even allows some preliminary observations on genre and function. One of the fragments has previously been radiocarbon dated, yielding the conquest-spanning, 2σ- and IntCal09-calibrated age ranges of 1505–1589 cal AD ($p = 0.686$) and 1617–1648 cal AD ($p = 0.314$; Cherkinsky and Urton 2014:34). Fragment A's attached needlework

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Figure 4. Network graph of right-handed pendant-pendant sums in the "rejoined" *khipu* KH0468. (Color online)

bundle, however, provisionally identifies this rejoined *khipu* with an early accounting genre on the southern Peruvian coast. In a recent study of such bundles (termed *kaytes* in Huarochiri Quechua) as *khipu* subject markers, Hyland (2020:142, 150–151; see also Splitstoser 2014:62–64) proposed possible meanings for two variants: orange and yellow indicating recordings of ayllu-level "labor accounting," and bright crimson with silver thread and chevrons indicating matters of "the highest level of Inka royalty."[8] KH0468's red, yellowish-white, and black needlework bundle (Pancorvo 2020:426) adds to this inventory, designating a (Pisco-area) arithmetic register.

Further, other comparable *khipus* speak to regional accounting practices supported by active calculation. In his 1958 archaeological survey of the Pisco Valley, Dwight T. Wallace noted one site, PV 58-10, from which he recovered a *khipu*. The sample was associated with a possible *yupana*—a "peculiar stone" that Wallace (1971:86–87) described as "flat-convex with a series of shallow holes, which occupy all of its flat part." More recently, excavations at Inkawasi (in the nearby Cañete Valley) have yielded two needlework-bundle-bearing *khipus* (KH0502 and KH0505) among 14 specimens found covered with peanuts. Presumed to register storehouse deposits, both have been described as paradigmatic examples of "internal summing" on day-to-day working *khipus* (Urton 2017:161–166).[9]

These associations arise directly from the discovery of KH0468's internal arithmetic structure, expanding identification mechanisms for *khipu* genres beyond "physical correlations . . . analogies with post-Inka kaytes . . . associations with grave goods," or aspects of the primary cord (Hyland 2020:151). Given that only 16 samples (2%) in the corpus have *kaytes*, Ascher relations might fill some of the inherent gaps in such approaches to *khipu* typology. They can also contribute to understanding *kayte* variation: the high-magnitude, arithmetically poor nature of *khipu* KH0426 in the Musée du quai Branly, Paris, would provisionally identify its black needlework bundle with white bands (Pancorvo 2020:424) with the high-level "summary" specimens described previously.

In our view, arithmetically aided *khipu* reunion encourages as much an expanded search for other dissociated fragments as a reevaluation of specimens previously asserted to conjoin based on ad hoc, visual observation (e.g., Pancorvo 2020:450–451). With at least 156 potential *membra disiecta* sporting broken primary cords, we are optimistic that this initial attempt at a "*khipu* fragmentology" will make KH0468 only the first of other such matches to come (Davis 2016), particularly across different collections (see also Bjerregaard 2001).

Hypothesis Generation and Identifying *Khipu* Grammars

Particularly ambitious are numerical decipherments that claim to interpret *khipus* lacking significant archaeological provenience (e.g., Chirinos Rivera 2010:pt. 3). Quantitative aggregation and tailored data visualizations hold the potential to fast-track such initiatives, revealing *khipu* conventions more clearly discernible at high levels of abstraction. For example, our exhaustive search for Ascher relationships indicates that cords registering sums (i.e., sum cords, which total the values of summand cords) are often grouped together. In fact, 53% of all pendant-pendant sum cords in the corpus are contiguous to one another—KH0468, discussed in the preceding section, is one such example. Important to us, however, is identifying how such "sum clusters" are commonly denoted across large numbers of samples. Notably, in some 41% of cases, a white pendant cord begins a sum cluster, compared to just 32% for cord groups overall. The same holds for pendant sums by color (relation number two in Table 1), in which 40% of sum clusters begin with a white cord. In addition, we describe here, for the first time, the phenomenon of sum cluster clustering, in which sum clusters themselves co-occur, separated from surrounding groups of pendants by knotless cord groups. For any given sum cluster, 86% of the time, both of its neighboring groups are either additional sum clusters (75%) or knotless cord clusters (11%). Pendant sums by color exhibit the same property, with sum clusters also flanked by additional sum clusters (70%) or knotless groups (18%) in 88% of cases.

How do these findings advance *khipu* decipherment? The correlation of *khipu* structures with specific data organization practices allows us to sharpen previous allusions to "bookmarks" described as aiding readers in navigating the strings (Salomon 2013:32). White pendants and/or knotless neighboring groups would have enabled *khipukamayuqs* to quickly identify sum groups on certain numerical *khipus*. The regular arrangement of sums, as explored previously, suggests that these cluster identifications do not represent mere statistical noise. Sum clusters, and clusters of sum clusters, may well represent the *khipus*' "isolable units" (Urton 2005), physically and visually separated from the rest to diminish the risk of reading a summand instead of its associated total. Our characterization of white cords aligns with Andrés Chirinos Rivera's (2010:233–234) theory of color "elegance," according to which the *khipu* is made clearer to the user through visual redundancy. The use of knotless neighbors for visual emphasis has also been described by Carrie Brezine for the two "matching" *khipus* KH0128 and KH0137, in which the duplicate sections are set off from the other pendants by at least one zero-valued string on each side (Urton 2005:151).

We can again reconcile both ethnohistorical and archaeological evidence. Keeping the Pisco-area specimen KH0468 in mind, we venture a few dozen kilometers northward, where in 1544, a native cord-keeper showed a colonial administrator "certain cords in pairs, one white and one yellow, with some knots tied at intervals, saying that the yellow one was a count of gold and the white one a count of silver, and that each pair of cords recorded the account of what had been given [in tribute] in one year" on the royal encomienda of Chincha (qtd. in Hampe Martínez 1987:86). At least two plausible corresponding *khipu* morphologies arise: first, the summand cords for the Chincha specimen(s) were located elsewhere on the same *khipu*(*s*), supporting our sum cluster hypothesis; or second, the summands were recorded on other *khipus*, suggesting that the register at hand was an arithmetic-poor, high-level "summary" *khipu*. Indeed, with some 2,000 tributaries reported under the Chincha encomienda in the mid-1540s (Hampe Martínez 1987:87), this *khipu* would have easily surpassed the *waranka* (1,000)-level floor proposed for high-order summary *khipus*; with its white and yellow cords arranged "in pairs," the specimen also exhibited color seriation, which has been shown to designate aggregate accounts (Clindaniel 2019:Chapter 5; Hyland 2016). An explicit example of the second case comes from the Audiencia of La Plata (modern Sucre, Bolivia). In speaking to deliveries of goods and services under a corrupt encomendero between 1548 and 1551, two cord keepers were asked in 1578 exactly how many heads of livestock among the total were provisioned to the encomendero himself. They replied that "the *khipu* where they had it recorded summarized everything in the way they have said, and they cannot give an account of how many [heads] they gave him in particular to eat, as everything goes into the said sum" (Pärssinen and Kiviharju 2004:333).

Archaeologically, this finding allows us to amend and expand Jon Clindaniel's (2019:Chapter 4) hypothesis regarding white and light-colored cords involved in arithmetic sequences on storehouse *khipus* recovered from Inkawasi. Though Clindaniel (2019:95) argues that in two *khipus* from the site's major storehouse (sector A), KH0503 and KH0491, white and light-colored strings grammatically "signified the unmarked action of 'addition,'" we instead propose that white served as a functional marker for sums. On KH0503, for example, Clindaniel interprets the three-cord sequence 106 (white), 15 (amber brown), and 91 (mottled amber brown and dark brown) as $106 - 15 = 91$ or *light (addition) minus dark (subtraction) equals mottled light and dark (result)*. We reinterpret this sequence, and others like it in the Inkawasi corpus, as $106 = 15 + 91$, or more generally $a = b + c$ (i.e., a right-handed pendant-pendant sum) instead of $a - b = c$, with white denoting the sum.

At least two factors support our revision. First, exhaustive search reveals that three-cord "grammatical" color sequences of the type proposed by Clindaniel—*light cord minus dark cord equals mixed-color cord*—are exceedingly rare, appearing on only six other cords (i.e., two other times) in the 650-*khipu* corpus. Moreover, although white predominates in pendant-pendant sum relationships—which would initially support Clindaniel's (2019:83–97) description of light colors as an "unmarked" grammatical *khipu* element—the frequency distribution of pendant-pendant sum colors versus the overall frequency distribution for nonzero-valued pendant colors is within 4.5 degrees of similarity (by cosine similarity, a typical measure of the similarity between two vectors). Second, conceiving of the white cords in question as sum cords (rather than addition cords) explains more Inkawasi *khipus* with internal arithmetic. Particularly telling are samples in which more than two summands appear. For example, in KH0514, pendant group 8 comprises the seven-cord sequence 4,273 (white), 47 (yellowish brown), 260 (light brown), 393 (barber pole white/light brown), 472 (yellowish brown), 391 (barber pole white/dark brown), and 2,710 (mottled white/moderate brown). This can only be written as 4,273 = 47 + 260 + 393 + 472 + 391 + 2,710, with the solid white cord designating the sum. The same can be said of Inkawasi *khipus* KH0505, KH0506, KH0513, KH0502, and KH0504 (itself physically attached to the *khipu* analyzed by Clindaniel). Though the possibility of an additive formula has been acknowledged previously for these specimens (Urton 2017:161–166), our incorporation of color enables a broader statement: the Inkawasi *khipus* in question registered right-handed sums (rather than subtractive operations) in which white pendant cords recorded sum totals.

Sum relationships also appear on the fragmentary *khipus* recovered from *collcas* in Inkawasi's sector B, subsector one, an elite residential zone. Clindaniel (2019:54, Figure 3.3) wrote that these specimens (KH0608–KH0630) were "commonly found tightly bound and buried with large numbers recorded on them, suggesting that they may have been archival khipus that kept some sort of summary statistics for overseers at the site." Yet Ascher sum relations appear on 10 (43%) of the specimens—including some of the "tightly bound" examples—which suggests a more quotidian, "working" character. Tania Jiménez Mendoza (2022:162–166) agrees, interpreting at least four of these *khipus* (KH0609, KH0610, KH0613, KH0614) as unfinished accounts of the active "control and registration of products." As has been argued for *khipu*-wielding Guarco elites at nearby Huacones-Vilcahuasi (Barraza Lescano et al. 2022), Ascher sums would thus favor an interpretation closer to that of dynamic, "partially centralized" residential-administrative storage, in which local elites maintained registers of goods kept in private spaces separate from sector A, the primary storehouse in which Inkawasi's sitewide accounting was performed.[10]

Returning from Inkawasi to the corpus level, the identification of white cords as boundary markers for sum clusters (and by extension, as sum cords) confirms that the first pendant in a pendant group often played a distinct role in *khipu* semiosis. This is directionally supportive of conventions that have previously been asserted in *khipu* studies. Statements regarding the first pendant in a cord group recording "large" or presumably "important" values abound (Ascher 2005:103; Ascher and Ascher 1978:649n3, 816n3, 894n8, 896n10; Chirinos Rivera 2010:306–315; Mackey 1970:74), as do mentions of first pendants that total subsequent cords in various ways, at least at the individual *khipu* level (Ascher and Ascher 1978:850n5, 917–918n3, 1056n4, 1110n3).[11] Nonnumerical interpretations have also been raised, including one describing certain first pendants as "markers" separating different information zones on their respective *khipus* (Ascher and Ascher 1978:372n4, 1029n4). Another holds that via attachment knots, the first pendants of the colonial-era Santa Valley tributary *khipus* registered each individual's moiety affiliation (with the cord recording the associated tribute obligation; Medrano and Urton 2018).

Finally, a potential qualm should be addressed: is the observed incidence of white sum and sum cluster cords merely the product of pendant cords fading over time? Both before and after entering collections, archaeological *khipus* would have faded to varying extents from (in)direct exposure to light, heat, and (in the case of funerary contexts) human remains. Efforts to scientifically assess the differential degradation of both dyed and undyed Inka *khipu* cords due to these factors are only at their outset (Bjerregaard et al. 2010:28–29). Crucially, however, our finding arises from the comparison of pendant color distributions against themselves: the distribution of white sum cords, for instance, is measured against the baseline distribution of white cords in the same data set. This departs from previous computational *khipu* analyses, which have typically measured pendant color against an absolute lightness score (e.g., Clindaniel 2019:Chapter 4)—a methodological choice that has yielded results likely influenced, if not explained, by corpus-wide pendant fading. While our intervention cannot account for differential fading on a per-*khipu* basis, it better proxies overarching color conventions than arguments which depend on (or are at least reducible to) raw frequency counts.

Discussion

Our findings, while novel, are nonetheless rooted in long-established "truths" about the canonical, Inka-style *khipu*: it registered numbers using a decimal system and three types of knots;

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the numbers were aggregated across multiple specimens in administrative hierarchies; and a series of conventions aided *khipukamayuqs* in organizing data. Garcilaso's (1976 [1609]:1:112) colonial-era statement that the Inkas "added, subtracted, and multiplied by those knots" makes even the Aschers' mathematical observations lose a bit of their luster. Yet for us, it is precisely the anodyne, uncontroversial reputation of numerical *khipu* knots that both explains their dip into relative obscurity since the 1970s and belies their tremendous potential for advancing *khipu* decipherment—just nine Ascher "relations" have provided us a firm footing to computationally explore the boundaries of known *khipu* conventions while proposing new ones of our own.

Where, then, does this leave us? First are more subtle criteria for identifying *khipu* "genres" (Brokaw 2010) at multiple administrative levels. Of course, that a *khipu* records large values does not by itself guarantee that it is a high-order administrative specimen; a local *khipu* summing eggs or fish, for example, might plausibly record larger quantities than one registering the number of individuals involved in ritual labor obligations (see Salomon 2004:119). Our proposal achieves a more richly nuanced distinction: between low-level "working" *khipus*, which are generally color-banded, top-cord-bearing, low magnitude, and arithmetic-rich, versus higher-order, *waranka*-and-up "summary" *khipus*, which are generally seriated, top-cordless, high magnitude, and arithmetic-poor. The former reaffirms the importance of searching for mathematical properties within *khipus*, and not just between them (Urton and Brezine 2005); the latter identifies a group of *khipus* that seem to be in a class all their own, omitting the on-the-ground "noise" of numerical calculations.

Together, these findings force us to reconsider claims of an order of operations in *khipu* decipherment. Sabine Hyland (2020:152) has argued that "it is necessary to first understand Andean khipu typology . . . before the computational analysis of large datasets can make progress in khipu decipherment." However, some 116,246 knots and 55,720 strings instead suggest a two-way street—computational analysis can contribute to *khipu* classification by identifying large-scale patterns that crosscut the isolated genres identified through ad-hoc analysis. The contributions of such distinctions to feature engineering for machine learning-based approaches remain to be realized.

That said, even a cursory glance at Table 1 raises an obvious question: might the high incidence of Ascher relations simply be attributable to chance? Revealing here is a Monte Carlo simulation, a mathematical technique that uses repeated random sampling to compute numerical results for a complex system: an analysis of computationally rendered examples ("pseudo-*khipus*") generated from three sample distributions—random uniform, bin-sampling from the corpus, and bin-sampling from the corpus with jittering—confirms the distinctive nature of the reference distribution.[12] This is seen most clearly in comparing pendant-pendant sum handedness: while the actual corpus produces a 54%/46% asymmetry overall, the three sample distributions (also with 650 *khipus* each) consistently yield approximate 50%/50% splits. Simulating the jittered bin-sample *khipu* corpus 1,000 times, for example, the probability of the observed handedness asymmetry occurring by chance alone is $p < 0.001$ (Figure 5). We interpret this as a signal of intentional design decisions in the corpus.

More fine-grained differences also emerge. The sample distributions consistently yield sums with fewer summands, likely capturing spurious combinations of small handfuls of values (Figure 6). The simulated summands also appear impractically far from the pertinent sums along the primary cord—the reference corpus' summands are consistently closer together (reflecting functional usage), despite overall counts of pseudo-*khipu* pendant-pendant sums that are sometimes higher (bin-sampling and jittered bin-sampling) and sometimes lower (random uniform) than observed. Together, these observations render the fully random explanation untenable.

Finally, our results should not confer equal "legitimacy" on all Ascher sum relationships. For example, among pendant-subsidiary differences (relation number four, Table 1), which occur on 145 *khipus*, only 40/267 total occurrences (15%) involve an absolute value difference of greater than 10 between contiguous pendants. A number of spurious matches to small-valued neighbors thus seem likely. Crucially, however, only a "*khipu* studies at scale" allows for making such differential assessments.

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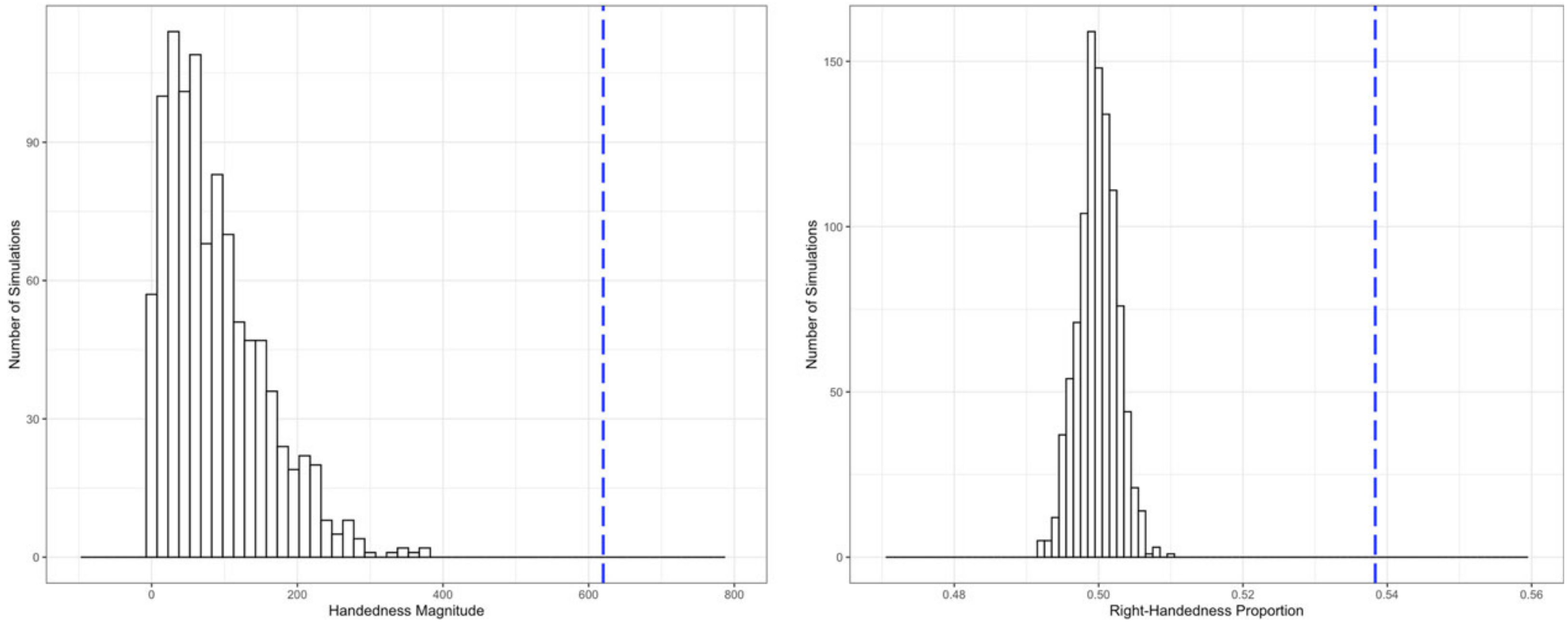

Figure 5. Monte Carlo simulation of pendant-pendant sum handedness, shown as both handedness magnitude and right-handed sum incidence (bin-sampling with jittering, 1,000 iterations; dashed line is observed).

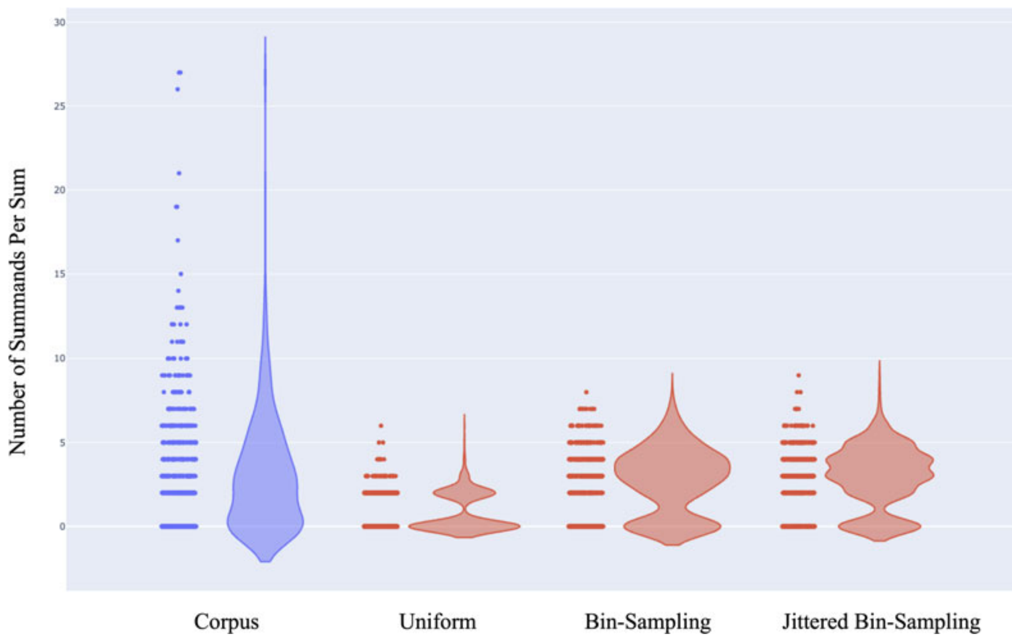

Figure 6. Violin plots comparing number of summands per pendant-pendant sum in reference distribution and three sample distributions.

Conclusion

In 1929, Erland Nordenskiöld replied to a *khipu*-related inquiry. "I should be very glad if my work were subjected to criticism from a mathematician," he explained, "with special regard to the theory of probabilities. [The physicist Paul] Langevin of Paris has declared, I understand, that the results at which I have arrived cannot be a mere matter of chance" (VKMA, E. Nordenskiöld to W. E. van Wijk, letter, May–June 1929, E1-21). Some 95 years later, these statements as much anticipate our intervention as reaffirm the importance of its careful application. Nordenskiöld was prescient in recognizing the value of statistically informed *khipu* research. Yet his far-fetched astronomical hypotheses were apparently endorsed by the director of a prestigious *grande école* before being summarily dismissed by the Aschers (1969:533) as nothing more than "far-fetched computation."

Our corrective lies in an aggregative assessment of *khipu* morphology that is equally attuned to the benefits and limitations of quantitative methodologies. Exhaustive search, confirmatory studies, *khipu* rejoining, and hypothesis generation emerge as distinct contributions to decipherment. Yet these analyses rest on a carefully compiled set of mathematical observations—the Ascher Databooks (1978, 1988)—generated from close studies of hundreds of individual specimens over four decades ago. Even earlier, the Aschers had recognized the "potential value of the digital computer as a tool in anthropological research" for its ability to facilitate "the discovery of meaningful relationships in sets of data" (Ascher and Ascher 1963:1045, 1050). That they never implemented this in their published *khipu* research gives us the hope that this study has successfully carried the torch, charting new paths for studying *khipus* in a digital age.

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Data Availability Statement. Underlying data for the 650 *khipus* described here, higher-resolution copies of the figures, as well as the code necessary to reproduce the analyses, are available at https://doi.org/10.5281/zenodo.10472429.

Competing Interests. The authors declare none.

Notes

1. The Aschers (1981:151–152) described the "body of arithmetic ideas used by the Incas" as including "addition, division into equal parts, division into simple unequal fractional parts, division into proportional parts, multiplication of integers by integers, and multiplication of integers by fractions." This is distinct, however, from examining the broader incidence and distribution of these operations.

2. We thank Kylie Quave for permission to publish her recordings of the Logan Museum of Anthropology's three *khipus* (one appears briefly in Quave and Heaney 2022:472–473). Those studied by Medrano are in the Museo de América, Madrid (one *khipu*), the Museum der Kulturen Basel (10 *khipus*), the MARKK Hamburg (three *khipus*), the Lippisches Landesmuseum Detmold (three *khipus*), the Linden-Museum Stuttgart (two *khipus*), the Niedersächsisches Landesmuseum Hannover (one *khipu*), and the Weltkulturen Museum Frankfurt (1 *khipu*).

3. A full description of the search criteria for each Ascher relation appears in the associated code; see Data Availability Statement.

4. A broader search for long knots with zero loops (implying that they are untied, though this can also reflect data entry errors) returns matches in 17 *khipus*, all of which have mean pendant cord values in the same, "working" *khipu* range. Ghost knots alone would not seem a sufficient criterion for "working" versus "summary" distinctions, however, as most *khipus* consistent with the former category lack them. Taphonomic factors may also be at play.

5. The Santa Valley *khipu* correspondence remains provisional, given inconclusive radiocarbon dates for several of the specimens (Medrano and Brokaw 2023). However, if upheld by future research, it is worth noting that these ayllu-level *khipus* are composed of groups of six pendant cords. The Aschers (1981:89) first observed that top cords and sum top cords appear disproportionately on *khipus* with six-cord pendant groups, which would further identify these specimens as local accounts (see also Chirinos Rivera 2010:293–301). Across the corpus, though six-cord clusters represent only 8% of all pendant groups, they make up 70% of all clusters associated with sum top cords.

6. Many of the sets of summand cords in KH0468 also either begin or end with the value one, designated by a so-called figure-eight knot.

7. *Khipus* are typically recorded beginning from the doubled, knotted, tufted, or tasseled "head" of the primary cord and proceeding to the other end, one pendant/subsidiary cord at a time (Urton 2017:223).

8. A third *kayte* type characterized by three triangle motifs is briefly hypothesized to indicate agricultural fields (Hyland 2020:144).

9. KH0505 is discussed directly in this regard, while KH0502 is indirectly identified through the discussion of another *khipu* with closely matching numerical values.

10. Of future interest are at least six unstudied *khipus* from Inkawasi's *kallanka*-lined sector C, subsector two, which has previously been interpreted as the site's Temple of the Sun (Ramos Vargas 2016:55n19).

11. Often referenced is Garcilaso de la Vega (1976 [1609]:2:24), who wrote that on tributary *khipus*, "items that did not have colors were placed according to their order, beginning with those of the highest quality and proceeding to those of the least, each according to their type."

12. Monte Carlo simulations performed by Clindaniel (2019:192–196) in his analysis of color matches between *khipus* and wrapped sticks from Inkawasi compared the observed frequencies of matches to random uniform-generated color pairings. We expand on this precedent by also juxtaposing cords randomly sampled from the corpus itself (bin-sampling) and with a proportional offset (bin-sampling with jittering), producing a distribution more closely resembling the actual one.

References Cited

Ascher, Marcia. 1986. Mathematical Ideas of the Incas. In *Native American Mathematics*, edited by Michael P. Closs, pp. 261–289. University of Texas Press, Austin.

Ascher, Marcia. 2005. How Can Spin, Ply, and Knot Direction Contribute to Understanding the Quipu Code? *Latin American Antiquity* 16:99–111.

Ascher, Marcia, and Robert Ascher. 1963. Chronological Ordering by Computer. *American Anthropologist* 65:1045–1052.

Ascher, Marcia, and Robert Ascher. 1969. Code of Ancient Peruvian Knotted Cords (Quipus). *Nature* 222:529–533.

Ascher, Marcia, and Robert Ascher. 1972. Numbers and Relations from Ancient Andean Quipus. *Archive for History of Exact Sciences* 8:288–320.

Ascher, Marcia, and Robert Ascher. 1978. *Code of the Quipu Databook*. University of Michigan Press, Ann Arbor.

Ascher, Marcia, and Robert Ascher. 1981. *Code of the Quipu: A Study in Media, Mathematics, and Culture*. University of Michigan Press, Ann Arbor.

Ascher, Marcia, and Robert Ascher. 1988. *Code of the Quipu Databook II*. University Microfilms, Ann Arbor.

Ascher, Marcia, and Robert Ascher. 1989. Are There Numbers in the Sky? In *Time and Calendars in the Inca Empire*, edited by Mariusz S. Ziółkowski and Robert M. Sadowski, pp. 35–48. BAR, Oxford.

Ascher, Robert. 2002. Inka Writing. In *Narrative Threads: Accounting and Recounting in Andean Khipu*, edited by Jeffrey Quilter and Gary Urton, pp. 103–115. University of Texas Press, Austin.

Barraza Lescano, Sergio, Rodrigo Areche Espinola, and Giancarlo Marcone Flores. 2022. By Stones and by Knots: The Counting and Recording of Chili Peppers Stored during the Inca Occupation of the Guarco Administrative Center of Huacones-Vilcahuasi, Lower Cañete Valley, Peru. *Andean Past* 13:221–264.

Bjerregaard, Lena. 2001. "Doubletten": Puzzles that Could Maybe Someday be Reconstructed. *Baessler-Archiv, Neue Folge* 49:187–92.

Bjerregaard, Lena, Ute Henniges, and Antje Potthast. 2010. Avoiding Nasty Surprises: Decision-Making Based on Analytical Data. In *North European Symposium for Archaeological Textiles X*, edited by Eva Andersson Strand, Margarita Gleba, Ulla Mannering, Cherine Munkholt, and Maj Ringgaard, pp. 26–30. Oxbow, Oxford.

Brezine, Carrie J., Jon Clindaniel, Ivan Ghezzi, Sabine Hyland, and Manuel Medrano. 2024. A New Naming Convention for Andean *Khipus*. *Latin American Antiquity*, in press. https://doi.org/10.1017/laq.2023.71.

Brokaw, Galen. 2005. Toward Deciphering the Khipu. *Journal of Interdisciplinary History* 35:571–589.

Brokaw, Galen. 2010. *A History of the Khipu*. Cambridge University Press, Cambridge.

Cherkinsky, Alexander, and Gary Urton. 2014. Radiocarbon Chronology of Andean Khipus. *Open Journal of Archaeometry* 2:32–36.

Chirinos Rivera, Andrés. 2010. *Quipus del Tahuantinsuyo: Curacas, incas y su saber matemático en el siglo XVI*. Editorial Commentarios, Lima.

Clindaniel, Jon. 2019. Toward a Grammar of the Inka Khipu: Investigating the Production of Non-numerical Signs. PhD dissertation, Department of Anthropology, Harvard University, Cambridge, Massachusetts.

Cook, Gareth. 2003. Scholar Sees Strands of Ancient Secrets. *Boston Globe*, July 4:A1.

Davis, Lisa Fagin. 2016. The Beauvais Missal: Otto Ege's Scattered Leaves and Digital Surrogacy. *Florilegium* 33:143–166.

Ethnologisches Museum Berlin. 2021. Khipu VA16135a. https://smb.museum-digital.de/object/28374, accessed July 7, 2023.

Fossa, Lydia. 2019. *Khipu*, *llautu*: Una familia de textos textiles. *Diálogo Andino* 59:119–130.

Garcilaso de la Vega, El Inca. 1976 [1609]. *Comentarios reales de los incas*. Edited by Aurelio Miró Quesada. 2 vols. Fundación Biblioteca Ayacucho, Caracas.

Hampe Martínez, Teodoro. 1987. Notas sobre la encomienda real de Chincha en el siglo XVI (administración y tributos): Fénix. *Revista de la Biblioteca Nacional* 32–33:80–95.

Houston, Stephen D. 2004. The Archaeology of Communication Technologies. *Annual Review of Anthropology* 33:223–250.

Hyland, Sabine. 2016. How Khipus Indicated Labour Contributions in an Andean Village: An Explanation of Colour Banding, Seriation and Ethnocategories. *Journal of Material Culture* 21:490–509.

Hyland, Sabine. 2020. Subject Indicators and the Decipherment of Genre on Andean Khipus. *Anthropological Linguistics* 62:137–158.

Hyland, Sabine, Gene A. Ware, and Madison Clark. 2014. Knot Direction in a Khipu/Alphabetic Text from the Central Andes. *Latin American Antiquity* 25:189–197.

Jiménez Mendoza, Tania. 2022. Identificando élites durante el Horizonte Tardío a través del estudio de un conjunto arquitectónico del Sector B, del sitio arqueológico Incahuasi, Cañete, costa sur central del Perú. Licentiate archaeology thesis, Professional School of Archaeology, Universidad Nacional Mayor de San Marcos, Lima.

Jockers, Matthew L. 2013. *Macroanalysis: Digital Methods and Literary History*. University of Illinois Press, Champaign.

Julien, Catherine J. 1988. How Inca Decimal Administration Worked. *Ethnohistory* 35:257–279.

Khosla, Ashok. 2022. The Khipu Field Guide. https://khipufieldguide.com/, accessed July 7, 2023.

Locke, L. Leland. 1912. The Ancient Quipu, a Peruvian Knot Record. *American Anthropologist* 14:325–332.

Locke, L. Leland. 1923. *The Ancient Quipu or Peruvian Knot Record*. American Museum of Natural History, New York.

Locke, L. Leland. 1927. *A Peruvian Quipu*. Contributions Vol. 7, No. 5. Museum of the American Indian Heye Foundation, New York.

Mackey, Carol J. 1970. Knot Records in Ancient and Modern Peru. PhD dissertation, Department of Anthropology, University of California, Berkeley.

Medrano, Manuel. 2021a. *Quipus: Mil años de historia anudada en los Andes y su futuro digital*. Planeta, Lima.

Medrano, Manuel. 2021b. Khipu Transcription Typologies: A Corpus-Based Study of the *Textos Andinos*. *Ethnohistory* 68:311–341.

Medrano, Manuel, and Galen Brokaw. 2023. Quipu. In *Oxford Bibliographies in Latin American Studies*, edited by Ben Vinson. Oxford University Press, New York. https://doi.org/10.1093/obo/9780199766581-0175.

Medrano, Manuel, and Gary Urton. 2018. Toward the Decipherment of a Set of Mid-Colonial Khipus from the Santa Valley, Coastal Peru. *Ethnohistory* 65:1–23.

Nordenskiöld, Erland. 1925. *The Secret of the Peruvian Quipus*. Comparative Ethnological Studies Vol. 6, Pt. 1. Elanders, Gothenburg, Sweden.

OKR Team. 2022. The Open Khipu Repository. *Zenodo*. https://doi.org/10.5281/zenodo.6908343, accessed October 28, 2023.

Pancorvo, Anel (editor). 2020. *Quipus y quipucamayoc: Codificación y administración en el antiguo Perú*. Apus Graph Ediciones, Lima.

Pärssinen, Martti, and Jukka Kiviharju. 2004. *Textos Andinos: Corpus de textos khipu incaicos y coloniales*. Instituto Iberoamericano de Finlandia and Universidad Complutense de Madrid, Madrid.

Pizarro, Hernando. 1920 [1533]. A los señores oydores de la audiencia real de su magestad. In *Informaciones sobre el antiguo Perú*, Vol. 3, edited by Horacio H. Urteaga, pp. 16–180. Sanmartí, Lima.

Quave, Kylie E., and Christopher Heaney. 2022. A Peruvian Central Coast Mortuary Assemblage in the Logan Museum of Anthropology, Beloit College. *Andean Past* 13:463–477.

Radicati di Primeglio, Carlos. 2006. La "seriación" como posible clave para descifrar los quipus extranumerales. In *Estudios sobre los quipus*, pp. 155–264. Universidad Nacional Mayor de San Marcos, Lima.

Ramos Vargas, Mario A. 2016. Quipus y *quipucamayos* en el registro arqueológico: Una evaluación desde Huaycán de Cieneguilla, valle de Lurín. *Cuadernos del Qhapaq Ñan* 4:36–65.

Salomon, Frank. 2004. *The Cord Keepers: Khipus and Cultural Life in a Peruvian Village*. Duke University Press, Durham, North Carolina.

Salomon, Frank. 2013. The Twisting Paths of Recall: Khipu (Andean Cord Notation) as Artifact. In *Writing as Material Practice: Substance, Surface and Medium*, edited by Kathryn E. Piquette and Ruth D. Whitehouse, pp. 15–43. Ubiquity Press, London.

Splitstoser, Jeffrey. 2014. Practice and Meaning in Spiral-Wrapped Batons and Cords from Cerrillos, a Late Paracas Site in the Ica Valley, Peru. In *Textiles, Technical Practice, and Power in the Andes*, edited by Denise Y. Arnold and Penelope Dransart, pp. 46–82. Archetype, London.

Tokovinine, Alexandre. 2019. Inka Khipu, Dumbarton Oaks (2016.71). 3D model. *Sketchfab*. https://skfb.ly/6ZpXv, accessed October 28, 2023.

Tun, Molly. 2016. Quipu. In *Encyclopaedia of the History of Science, Technology, and Medicine in Non-Western Cultures*, edited by Helaine Selin, pp. 3682–3687. Springer, Dordrecht, Netherlands.

Urton, Gary. 2005. Khipu Archives: Duplicate Accounts and Identity Labels in the Inka Knotted String Records. *Latin American Antiquity* 16:147–167.

Urton, Gary. 2006. Censos registrados en cordeles con "amarres": Padrones poblacionales pre-hispánicos y coloniales tempranos en los *khipu* Inka. *Revista Andina* 42:153–196.

Urton, Gary. 2017. *Inka History in Knots: Reading Khipus as Primary Sources*. University of Texas Press, Austin.

Urton, Gary, and Carrie J. Brezine. 2005. Khipu Accounting in Ancient Peru. *Science* 309:1065–1067.

Wallace, Dwight T. 1971. Sitios arqueológicos del Perú (segunda entrega): Valles de Chincha y de Pisco. *Arqueológicas* 13:1–131.

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