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EDITORIAL: AT THE FRONTIER OF APPLIED DYNAMICAL SYSTEMS
SPECIAL ISSUE IN HONOUR OF PROFESSOR BERND KRAUSKOPF

(On the occasion of his 60th birthday)

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Dynamical systems are mathematical models of the time evolution of real systems, both natural and human, such as ecosystems, the Earth's weather and climate systems, neurons, lasers, electronic circuits and aircraft, to name but a few. These models are often process-based and can, therefore, be very powerful. For example, they can explain and predict new, often counter-intuitive, nonlinear phenomena of great importance for applications. Furthermore, such phenomena can be universal, bridging the gaps between different disciplines. Due to their nonlinearities and multiple scales, dynamical systems can be challenging to analyse. There are two distinct but complementary approaches to their analysis:

(i) rigorous qualitative theory, which provides insights into the behaviour of solutions without knowing them explicitly;
(ii) numerical methods developed to approximate the actual solutions.

Bernd Krauskopf's work pushes the boundaries of applied dynamical systems by combining these two approaches to develop new computational techniques for solving problems in applications that span many fields of science and engineering. In doing so, he has championed a new strand within the field of dynamical systems and made invaluable contributions that have had a significant impact worldwide.

Bernd started his career as a PhD student with Floris Takens at the University of Groningen in the Netherlands in 1991. The topic of his PhD research was suggested by none other than V. I. Arnold who posed the question: "What happens near a bifurcation where a closed orbit of a vector field loses stability in a 1:4 resonance?" Arnold and Takens had independently proposed a model in the form of a $\mathbb{Z}_4$-equivariant planar vector field, which Arnold claimed contained all versal unfoldings of such bifurcations. Takens probably had some ideas how to tackle this challenging conjecture, but neither he nor Arnold expected Bernd's novel approach, which involved not only numerical simulation, but the much more powerful tool of pseudo-arclength continuation using the software package AUTO[1] developed by Eusebius Doedel. Bernd was the first to use AUTO to compute bifurcation surfaces, rather than curves, which started his drive to push computational methods to their design limits, and beyond if necessary. By combining these computational techniques, especially for the continuation of global bifurcations, with the analysis of a codimension-three singularity at ∞, Bernd was able to produce all known unfoldings in Arnold's model; while not a complete proof, he provided compelling evidence for the validity of Arnold's conjecture.

After his PhD, from 1995 to 1998, Bernd worked with John Guckenheimer at Cornell University in Ithaca, USA, and then with Daan Lenstra at the Vrije Universiteit Amsterdam in the Netherlands. It was during his time in Amsterdam that Bernd became fascinated by the nonlinear dynamics of semiconductor laser systems. This fascination was a turning point in his early scientific career. It led to some of the first bifurcation diagrams for lasers with saturable absorbers, optical injection and external optical feedback. This new approach attracted much attention in the laser dynamics community and led to a number of new collaborations. Of particular importance were Bernd's collaborations, unusual for a mathematician, with laser physicists working on real laser experiments to compare their measurements with theoretical predictions. Ultimately, Bernd has made original and internationally recognized research contributions in the area of laser instabilities and numerical bifurcation analysis of delay differential equations that arise naturally as mathematical models of such laser systems. Nonlinear photonics remains one of Bernd's main research interests.

Bernd's arrival in Bristol in 1998 marked a period of growth in dynamical systems research; he played a key role in establishing the Bristol Centre for Applied Nonlinear Mathematics and building the research culture within the department. He was often found in the open-plan research offices engaged in animated discussions with PhD students and postdocs. His collaborative work on delay differential equations and algorithms for computing true and slow invariant manifolds for dynamical systems provided two of the core scientific themes within the group. The influence of this work extended world-wide, driven in part by his meticulous attention to high-quality visualizations of the intricate dynamic phenomena encountered, such as the geometry of slow manifolds near a folded node. To this day, colleagues in Bristol still have memories of Bernd bringing out a ruler to measure how well the different plots within a figure were aligned[2].

Over the years, Bernd's research interests expanded to include a wide range of engineering applications, with a particular focus on aircraft landing gear, wheel shimmy and ground dynamics in collaboration with Airbus. From unearthing

[1]"AUTO-07P: *Continuation and bifurcation software for ordinary differential equations*," by E. J. Doedel and B. E. Oldeman (2007), with major contributions from A. R. Champneys, F. Dercole, T. F. Fairgrieve, Yu. A. Kuznetsov, R. C. Paffenroth, B. Sandstede, X. J. Wang and C. H. Zhang; available at http://cmvl.cs.concordia.ca/auto.

[2]Some former PhD students might describe these as nightmares.

parameter combinations that created regions of tri-stability in the shimmy response of a landing gear to the discovery of a swallowtail catastrophe that governs the retraction operation of a novel landing gear concept, these works demonstrated the utility of computational methods that are at the core of Bernd's research. This demonstration led Airbus to adopt numerical continuation techniques to help them understand nonlinear phenomena that might arise in their future aircraft designs: phenomena such as the canard explosions in the ground handling behaviour of aircraft that Bernd's work discovered! Hybrid testing/real-time dynamic sub-structuring methods brought an impactful engineering-focused avenue for his work with delays and underpinned the "Making It Real" grant from the Engineering and Physical Sciences Research Council (EPSRC), which funded a significant number of postdocs. The development of control-based continuation techniques with Jan Sieber exemplifies his ability to bridge mathematics with experimental engineering, creating lasting impact in both fields.

When Bernd arrived at the University of Auckland in 2011, he immediately immersed himself in various novel projects, paving the way for new collaborations and cementing his commitment to training a new generation of researchers. His presence was felt from the onset, as he quickly became a driving force behind projects ranging from new techniques for computing slow manifolds and canard trajectories in slow–fast dynamical systems to the study of geometric structures associated with new types of chaotic behaviour. For the latter in particular, new techniques for the computation of heterodimensional cycles and blenders were developed, providing a bridge between theory and possible applications.

During this time, Bernd's research also continued to focus on nonlinear dynamics of optical systems and numerical bifurcation analysis of delay differential equations. His expertise in these areas, combined with his interdisciplinary approach, quickly positioned him as a member of the Dodd–Walls Centre for Photonics and Quantum Technologies, a New Zealand research centre of excellence. Here, he has worked on projects related to the modelling of pulse propagation in optical fibres, the analysis of semiclassical approximations of dissipative quantum systems and the study of the emergence of exotic dynamical behaviours in these systems. These often intricate and complex phenomena have been elucidated by the theoretical and numerical techniques that Bernd and his collaborators have developed and refined throughout his career. This work has significantly advanced the understanding of both fundamental and applied aspects of nonlinear photonics.

This special issue pays tribute to Bernd's broad and diverse research achievements in applied dynamical systems. Its initiative was born out of the desire to celebrate Bernd's 60th birthday in style: Bernd himself has been the (main) editor of several books and special issues in journals to mark such milestones for others, so it would only be fitting to honour him with the same. The collection of papers also reflects the flavour of the Workshop on "Frontiers in Applied Dynamical Systems" held at University College Cork in June 2024 in Bernd's honour. The issue features 11 papers, by authors with affiliations from nine different countries, on topics ranging from industrial dynamics, mathematical biology, electrical engineering and laser physics, to the core mathematical questions of global bifurcations, dynamics with delays, grazing phenomena and Poincaré embeddings. We are very grateful to all the authors for their contributions.

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