# THEORETICAL GRAVITY ANOMALIES OF GLACIERS HAVING PARABOLIC CROSS-SECTIONS

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Abstract. Equations and a graph are presented for calculating gravity anomalies on a two-dimensional glacier model having a horizontal upper boundary and a lower boundary which is a parabola with a vertical axis of symmetry.

Résumé. On donne des équations et un graphique pour le calcul des anomalies de gravité sur la base d'un modèle bi-dimentionnel de glacier ayant une surface horizontale et une base de forme parabolique à axe de symétrie vertical.

Zusammenfassung. Für die Berechnung der Schwereanomalien an einem zweidimensionalen Gletschermodell werden Gleichungen und eine graphische Darstellung angegeben. Das Modell ist oben durch eine Horizontale, unten durch eine Parabel mit vertikaler Symmetrieachse begrenzt.

Gravity measurements on valley glaciers have become more common during the last decade as investigators have attempted to determine subglacial bedrock configurations. The early work utilized very simple methods of analysis and only recently have more sophisticated procedures of two- and three-dimensional analysis been attempted. Kanasewich (1963) found that a two-dimensional parabolic shape for the bedrock was a good first approximation to use for comparison with the observed gravity values; however, he used an approximation method to determine the gravity variations across this model.

The gravity effects of a two-dimensional glacier model can be calculated exactly for the case where the upper boundary is horizontal and the lower one a parabola with a vertical axis of symmetry. Let $a$ be the half width and $b$ the maximum depth of the glacier (Fig. 1).

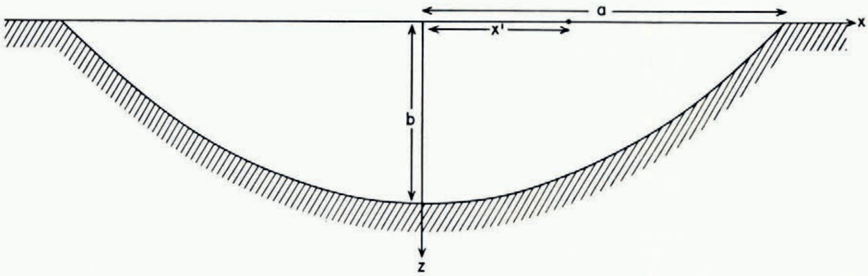


*Fig. 1. Notation used for the calculation of the gravity anomaly at point x'. The upper boundary of the glacier is horizontal and the lower boundary is a parabola with a vertical axis of symmetry*

The vertical component of gravity, $g_z$, at a point on the upper boundary of this body (Heiland 1940, p. 146) can be expressed as

$$2\gamma\rho a \int_{\xi=-1}^{\xi=1} \int_{\zeta=0}^{\zeta=\beta(1-\xi^2)} \frac{\zeta \, d\zeta d\xi}{(\xi-\chi)^2+\zeta^2},$$

where $\gamma$ is the gravitational constant, $\rho$ is the density difference between the glacier and bedrock, $\xi = x/a$, $\zeta = z/a$, $\beta = b/a$ and $\chi = x'/a$, $x'$ being the distance of the gravity station from the center line of the glacier. Integrating with respect to $\zeta$ yields

$$\frac{g_z}{\gamma\rho a} = \int_{-1}^{1} \ln[\xi-\chi)^2 + \beta^2(1-\xi^2)^2]d\xi - \int_{-1}^{1} \ln(\xi-\chi)^2 d\xi$$



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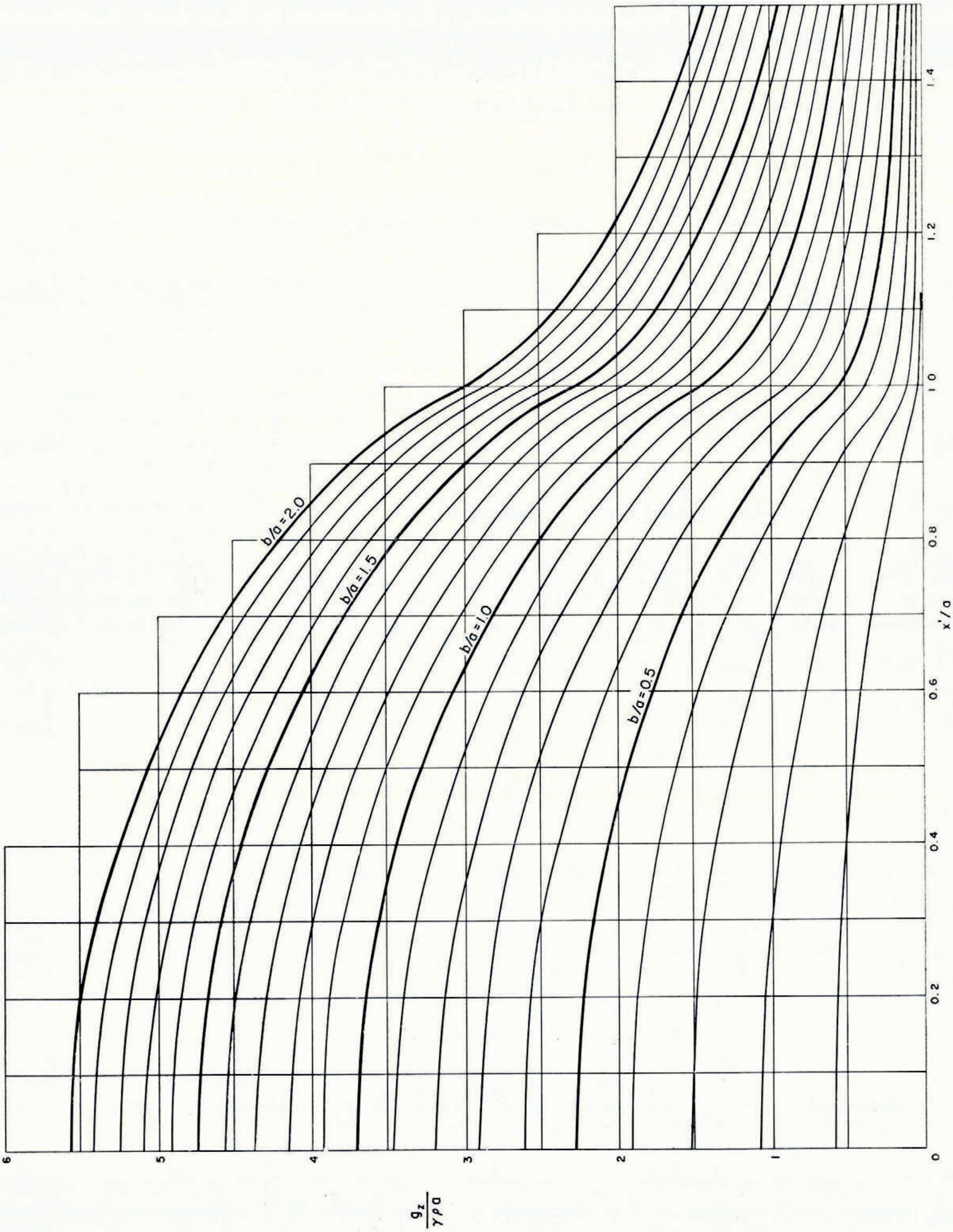


*Fig. 2. The function $g_z/\gamma\rho a$ plotted against $x'/a$ for values of $b/a$ from $0 \cdot 1$ to $2 \cdot 0$*

The final integration can easily be performed if the left-hand integrand is factored into

$$\ln\beta^2+\ln\left[\xi-\frac{i-(4\beta^2-1-4i\beta\chi)^{\frac{1}{2}}}{2\beta}\right]+\ln\left[\xi-\frac{i+(4\beta^2-1-4i\beta\chi)^{\frac{1}{2}}}{2\beta}\right]+$$

$$+\ln\left[\xi+\frac{i-(4\beta^2-1+4i\beta\chi)^{\frac{1}{2}}}{2\beta}\right]+\ln\left[\xi+\frac{i+(4\beta^2-1+4i\beta\chi)^{\frac{1}{2}}}{2\beta}\right],$$

where $i=(-1)^{\frac{1}{2}}$. By evaluating the integrals and re-arranging terms the following result is obtained when $\chi\neq 1$,

$$\frac{g_z}{\gamma\rho a}=\frac{\pi k}{\beta}-4+\chi\ln\left(\frac{1-\chi}{1+\chi}\right)^2+\frac{r}{2\beta}\ln\left[\frac{2\beta(t+\chi^2)+r(t+1)}{2\beta(t+\chi^2)-r(t+1)}\right]-$$

$$-\frac{s}{\beta}\left[\tan^{-1}\left(\frac{4\beta s-2r}{1-u}\right)+\tan^{-1}\left(\frac{4\beta s+2r}{1-u}\right)\right]$$

$$k=\begin{cases}0 & (\chi>1)\\ 1 & (\chi<1)\end{cases}\qquad r=\left(\frac{t-1}{2}\right)^{\frac{1}{2}}\qquad t=v+4\beta^2$$

$$s=\left(\frac{u+1}{2}\right)^{\frac{1}{2}}\qquad u=v-4\beta^2\qquad v=[(4\beta^2-1)^2+16\beta^2\chi^2]^{\frac{1}{2}}$$

and when $\chi=1$,

$$\frac{g_z}{\gamma\rho a}=\frac{\pi}{\beta}-4+\ln(1+4\beta^2)^2-\frac{2}{\beta}\tan^{-1}\left(\frac{1}{2\beta}\right).$$

The equations are shown graphically in Figure 2 for $0\cdot 1\leq\beta\leq 2\cdot 0$ and $0\leq\chi\leq 1\cdot 5$. (The calculation of the 320 points used in determining these curves was done on an IBM 7090 computer in less than one minute.) Comparison of the curves with observed gravity anomalies allows one to make a rapid first approximation for the depth of a valley glacier.


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