# OBITUARY

## ERIC CHARLES MILNER

Eric Charles Milner was born on 17 May 1928 and brought up in London. His father was an engineer, but times were hard and work was often difficult to obtain. So his mother had to help out by working as a seamstress, and Eric was often looked after by his grandmother. At the age of 11, he won a scholarship to the Haberdashers' Aske's Boys' School, but never attended it in its permanent London buildings because the outbreak of the Second World War caused all London schools and their pupils to be evacuated to safer parts of the country. As a result, Eric, an only child and knowing none of his new schoolfellows, was billeted at a home near Reading where he was extremely unhappy. In despair, he ran away and returned to London, where, after unsuccessful attempts to find him another billet, he roamed the streets and missed school. After some time, he was eventually found another billet where he received kindness and was much happier. Despite these disruptions and the other inevitable shortcomings of a war-time education, Eric's intelligence more than sufficed to surmount such hurdles, and in later life he could speak and write better than most of us.

From 1946 to 1951, Eric attended King's College, London. He graduated with First Class Honours in 1949, when he was awarded the Drew Gold Medal as the most distinguished Mathematics student in that year, and a Research Studentship. He then studied for an MSc degree, taking 'Modern algebra' and 'Quantum mechanics (Wave mechanics)' as his selected subjects, his supervisors being Richard Rado (then a Reader at King's College) and Professor Charles Coulson. He received the MSc degree, with distinction, in 1950. This was followed by a year's research in quantum mechanics under the supervision of Professor Coulson.

Those who knew Eric only as a mature fellow-mathematician (and perhaps even worked closely with him) may not have realised what a many-sided person he was. A colleague has described him as 'a remarkable mixture of Cockney street smartness, wild adventurer and uncompromising mathematician'. He had considerable business acumen, and was also very athletic: he was a featherweight boxer for the University of London around 1947, and took immediately to skiing when he was nearly forty. In his early youth, he did not contemplate an academic career: nothing in his family background would have suggested such a possibility. At that time, National Service was compulsory in Britain, and Eric applied to join the Royal Navy, in which his academic record would normally have secured a commission and in which he might well have made his career. He was disappointed to be rejected for the Navy because he was found to be somewhat deaf, a fact which had not previously been noticed. Joining the Army did not appeal to him; but service in other Commonwealth countries was a permitted alternative. Possibly his deafness could have secured exemption from any form of National Service, but the foregoing circumstances and an adventurous spirit may explain a decision to go to Malaya in 1951 to work as a tin assayer for the Straits Trading Company, a tin mining and smelting company.

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At this time, Professor (later Sir) Alexander Oppenheim was Professor of Mathematics at the University of Malaya in Singapore. Having survived imprisonment during the Japanese occupation, Oppenheim now had the resources to build up a good Mathematics Department with a well-stocked library. However, available mathematicians were very hard to find. Richard Guy, himself a recent recruit to Oppenheim's department, met Eric socially in late 1951, and managed (with some effort) to persuade him to abandon his recently chosen way of life and join the department.

In 1954, Eric married Esther Stella Lawton (known as Estelle), whom he had known as a fellow-student in London and who now joined him in Singapore.

The Mathematics Department in Singapore was small and geographically somewhat isolated, but Oppenheim was able to arrange visits by a substantial number of notable mathematicians. Several visits by Paul Erdös, with his habit of disseminating interesting problems, probably stimulated Eric's interest in combinatorial set theory and had a decisive influence on his development as a mathematician.

In 1957, Oppenheim became Vice-Chancellor of the University of Malaya, leaving Richard Guy as Acting Head of the Mathematics Department during a lengthy interregnum. Rado had meanwhile become Professor of Mathematics at the University of Reading, and (at Erdös' suggestion) Eric spent a sabbatical leave there in 1958–59, working with Rado in combinatorial set theory. By the time Eric returned to Singapore, conditions there seemed to be deteriorating, possibly for economic reasons, and the Milners now had a child (Suzanne, born in June 1958). For these as well as mathematical reasons, a return to England seemed increasingly desirable. Fortunately, Rado was able to offer him a Lectureship at Reading (and was probably only too keen to do so, since British universities had difficulty in recruiting enough well-qualified Mathematics staff around that time). Eric took up this appointment in January 1961. He immediately began work on a PhD thesis 'Some combinatorial problems in set theory', which he submitted as an external student to the University of London towards the end of 1962. He apparently had no formally appointed PhD supervisor, but in practice Rado seems to have fully assumed this role. Eric was awarded the PhD degree in 1963, his examiners (Richard Rado and Roy Davies) being clearly in agreement that his thesis was 'of an exceptionally high standard'.

Eric's selection of his major research field was an extremely gradual process, and his research perhaps only gained momentum after his move to Reading in 1961 provided a second opportunity to work with Rado. However, his productivity thereafter more than compensated for a very slow start. His interest and activity in combinatorial set theory were reinforced by meeting the Hungarian mathematician András Hajnal, who proved to be a kindred spirit, in 1958. Later collaborations with many other mathematicians also proved very fruitful, as evidenced by the list of publications below. At the same time, he gave full attention to teaching, to which he clearly attached great importance even when the subject matter was remote from his research interests. I have been told that his undergraduate lectures were very clear and stimulating, beautifully presented and, indeed, inspiring.

Despite these merits, promotion was hard to come by in England. Meanwhile, Richard Guy and Peter Lancaster (another of Eric's former colleagues in Singapore) had moved to the University of Calgary, in Canada, where Guy became Head of Department, whilst Oppenheim retired in 1965 from the Vice-Chancellorship of what was now the University of Malaysia and became Visiting Professor at Reading for the next three years. Eric's former colleagues were keen to tempt him to Calgary, and in 1967 (with Oppenheim's encouragement) he accepted a Professorship there, approximately doubling his income, which must have significantly helped in supporting a family now containing his daughter Suzanne and three sons, Mark, Paul and Simon.

By now a distinguished mathematician with a growing international reputation, Eric spent the rest of his life in Calgary, apart from leaves of absence. He became a Canadian citizen in 1973. He bore his full share of administrative responsibilities at Calgary, including four years (1976–80) as Head of Department. He supervised six PhD students, Scott Niven (1968–70), S. P. Pethe (1970–71), Eva Nosal (1971–75), W. Lenihan (1972–74), Jean-Michel Brochet (1986–88) and Li Bo-Yu (1990–92), and several MSc students. He was in frequent demand as an invited lecturer at seminars and conferences all over the world. In particular, he was a Plenary Session speaker at the International Congress of Mathematicians in Vancouver in 1974, and was the Canadian Mathematical Society's Jeffery-Williams Lecturer in 1989. Further recognition included election as a Fellow of the Royal Society of Canada in 1976. Professional responsibilities outside his own University included being Problems Editor for the *Canadian Mathematical Bulletin*, Chairman of the National Research Council Mathematics Grants Committee in 1974–75, Convener of the Mathematics Section of the Royal Society of Canada for four years (1977–81), Chairman of its New Fellows Committee (Mathematics) for three years (1978–81), and Director on three occasions of Canadian Mathematical Congress Summer Research Institutes.

Eric held visiting appointments at the University of Cambridge (1971–72), the University of Tel Aviv (1972, 1979 and 1986), Merton College, Oxford (1978–79), the University of Singapore (1981 and 1984) and the Université Claude Bernard (Lyon I) (1984 and 1985–86). Notable evidence of his standing was the steady stream of distinguished visitors who came to work with him at Calgary, including Paul Erdös.

Sadly, Estelle (who had been working for a PhD in the English Department at Calgary) died of cancer in February 1975. In July 1979, Eric married Elizabeth Forsyth Borthwick, a school teacher who had formerly taught Innuit children within the Arctic Circle. Their son Robert was born in January 1985.

In addition to demanding professional duties and prolific research, both performed to the highest standards, Eric found time and energy for much else, and enjoyed life to the full. He made two very happy marriages, and his family occupied a great deal of his attention. He was also a devoted son to his mother, who outlived him. People tell with particular warmth of the welcome and hospitality extended to numerous guests in the Milner household (during both of his marriages). His recreations included rugger, tennis, squash, dancing, chess, Go and other board and card games, sailing, mountain walking and ski-trekking. He derived much enjoyment from mountains and, for a number of years, owned an A-frame wooden weekend cottage in Canmore, near Banff, Alberta.

Clearly, Eric did not let partial deafness (affecting the left ear more severely than the right) stand in the way of a very full life, and indeed I knew him for several years before becoming aware of this problem; but I am told that he sometimes failed to hear questions asked by students in class. He underwent surgery on one ear in 1968, and soon afterwards on the other, which may have helped to some extent.

Eric retired in 1996 with the title of Emeritus Professor. His many friends were devastated when he died on 20 July of the following year, at the relatively early age of 69, after a lengthy battle with cancer, faced calmly but realistically. He will be remembered for outstanding contributions to research and teaching, but particularly, among those closest to him, as a gentle, caring and exceptionally nice man. Readers may wish to know of the Eric Milner Scholarship Fund, which has been established by the University of Calgary to endow a scholarship in his memory: contributions can be sent to the Development Office, The University of Calgary, Calgary, Alberta, Canada T2N 1N4.

## *Mathematical work*

As already mentioned, the main theme of Eric Milner's research was combinatorial set theory, that is, the study of problems with a combinatorial flavour concerning (mainly infinite) sets. Within this broad area, his work on a wide variety of challenging problems shows something of the adventurous spirit which characterised his approach to life in general. Much of his work was done in collaboration with others, and probably discussion with other mathematicians often provided the impetus towards particular lines of investigation. The following is an attempt to convey something of the flavour of the work of Milner and his collaborators by means of examples.

An early product of Milner's collaboration with Rado was their joint paper [**5**] on 'The pigeon-hole principle for ordinal numbers'. Dirichlet's pigeon-hole principle states that if a finite set of cardinality $n$ is partitioned into fewer than $n$ subsets, then at least one of these subsets has more than one element. This may not seem a particularly profound truth. It is scarcely harder to notice that, more generally, partitioning a finite set of cardinality at least $n_1+n_2+\ldots+n_k-k+1$ into subsets $A_1,\ldots,A_k$ must give $|A_i| \geqslant n_i$ for some $i$. However, Milner and Rado noticed that replacing positive integers by ordinal numbers in this innocent remark leads to a non-trivial and interesting problem. In other words, if $k$ is an ordinal and $\alpha_\kappa$ is an ordinal for each $\kappa < k$, what is the least ordinal $\alpha$ such that partitioning a well-ordered set of order type $\alpha$ into sets $A_\kappa$ $(\kappa < k)$ must result in $\operatorname{tp} A_\kappa \geqslant \alpha_\kappa$ being true for at least one $\kappa$ (where 'tp' means 'the order type of')? In [**5**], Milner and Rado obtain an explicit answer when $k$ is finite, an explicit answer when all the $\alpha_\kappa$ are equal, and an algorithm in the general case which yields the least $\alpha$ after a finite number of well-defined steps.

Erdös and Rado ⟨**10**⟩ pointed out that, more generally, the subject of 'partition calculus' might be regarded as studying extensions of Dirichlet's pigeon-hole principle which are certainly far from trivial. F. P. Ramsey's famous theorem states that if $r$, $s$ are positive integers, $A$ is a countably infinite set, $[A]^r$ is the set of all $r$-subsets of $A$ and $[A]^r$ is partitioned into $s$ subsets, then one of these subsets contains $[B]^r$ for some infinite $B \subseteq A$. More picturesquely: 'if the $r$-subsets of a countably infinite set $A$ are coloured with $s$ colours, then there will necessarily be a infinite subset $B$ of $A$ whose $r$-subsets are coloured monochromatically'. In the 'partition calculus' notation introduced by Erdös and Rado ⟨**9**⟩, this could be expressed more concisely as $\aleph_0 \to (\aleph_0, \aleph_0, \ldots, \aleph_0)^r$, where $\aleph_0$ appears $s$ times between the brackets, or still more concisely as $\aleph_0 \to (\aleph_0)^r_s$. In general, if $\alpha$, $\alpha_1$, $\alpha_2$, $\ldots$, $\alpha_s$, $n$ are cardinal numbers, then $\alpha \to (\alpha_1, \alpha_2, \ldots, \alpha_s)^n$ means 'if $A$ is an $\alpha$-set and the set $[A]^n$ of all $n$-subsets of $A$ is partitioned into subsets $\mathscr{C}_1$, $\mathscr{C}_2$, $\ldots, \mathscr{C}_s$, then for some $i \in \{1, \ldots, s\}$, there will exist an $\alpha_i$-subset $B$ of $A$ such that $[B]^n \subseteq \mathscr{C}_i$'. This notation is abbreviated to $\alpha \to (\beta)^n_s$ if $\alpha_1 = \alpha_2 = \ldots = \alpha_s = \beta$. The notations $\alpha \to (\alpha_1, \alpha_2, \ldots, \alpha_s)^n$ and $\alpha \to (\beta)^n_s$ can be similarly defined when $\alpha$, $\alpha_1$, $\alpha_2$, $\ldots$, $\alpha_s$, $\beta$ are ordinals or, more generally, order types: in this case, we take $A$ to be an ordered set of order type $\alpha$. Thus, in [**5**], Milner and Rado were investigating the least ordinal $\alpha$ satisfying a condition which, in partition calculus notation, might be written as $\alpha \rightarrow (\alpha_1, \alpha_2, \ldots)^1$, where $\alpha_1, \alpha_2, \ldots$ is deemed to be a transfinite sequence of length $k$, or as $\alpha \rightarrow (\alpha_\kappa; \kappa < k)^1$.

Given the influence of Erdös, Hajnal and Rado, it is not surprising that partition calculus features substantially in Milner's work. His paper [**19**] brings considerable technical ingenuity to bear in proving partition calculus results concerning countable ordinals, mostly of the form $\omega^\alpha h + m \rightarrow (n, \omega^\beta + p)^2$ or $\omega^\alpha h + m \nrightarrow (n, \omega^\beta + p)^2$, where $h$, $m$, $n$, $p$ denote non-negative integers and $\alpha$, $\beta$ denote countable ordinals, and of course $X \nrightarrow Y$ denotes the negation of a statement $X \rightarrow Y$. For example, he proved that $\omega^4 \rightarrow (3, \omega^3)^2$. A further result of this sort was proved by Erdös and Milner in [**30**], namely, that $\omega^{1+\nu h} \rightarrow (2^h, \omega^{1+\nu})^2$ if $h < \omega$ and $\nu < \omega_1$. In ⟨**10**⟩, Erdös and Rado proved that $\psi \rightarrow (\omega + k, 4)^3$ if $k$ is a non-negative integer and $\psi$ is a 'real' order type, that is, the type of an uncountable ordered set containing no subset of type $\omega_1$ or $\omega_1$ reversed. In [**61**] and [**69**], respectively, Milner and Prikry proved that $\omega_1 \rightarrow (\omega + k, 4)^3$ for any non-negative integer $k$, and that $\omega_1 \rightarrow (\omega 2 + 1, 4)^3$, observing also that the truth or falsity of many similar statements remains unsettled. Their proofs used a somewhat surprising idea from axiomatic set theory attributed to Baumgartner and Hajnal ⟨**3**⟩. This involves observing that statements of a certain kind (namely statements which are 'absolute with respect to countable chain condition forcing extensions') are automatically true in 'ordinary' set theory (Zermelo–Fraenkel set theory with the Axiom of Choice) if they are true in a suitable model of set theory which may incorporate additional assumptions. Thus, in [**61**], Milner and Prikry first proved that $\omega_1 \rightarrow (\omega + k, 4)^3$ for every non-negative integer $k$ if Martin's Axiom $\mathrm{MA}_{\omega_1}$ is true, and then used the idea from ⟨**3**⟩ to show that the additional assumption $\mathrm{MA}_{\omega_1}$ can be discarded. In [**69**], they proved that $\omega_1 \rightarrow (\omega 2 + 1, 4)^3$ if $\mathrm{MA}_{\omega_1}$ and $\omega_1 \rightarrow (\omega_1, \omega 2 + 1)^2$ are both true, and then used the same device to discard these two additional assumptions.

In addition to producing numerous papers himself, Milner was unusually diligent in reading those of others. Proving that $\omega^\omega \rightarrow (\omega^\omega, 3)^2$ became recognised as a particularly challenging problem in the partition calculus, and the proof eventually obtained by Chang ⟨**6**⟩ was an impressive *tour de force*. Milner had the stamina to read this long and difficult proof, and saw how it could be generalised to prove that $\omega^\omega \rightarrow (\omega^\omega, n)^2$ for every positive integer $n$. (He was much impressed when J. A. Larson ⟨**12**⟩ subsequently found a much shorter proof of this.)

Erdös and Rado ⟨**10**⟩ introduced an extension of the partition calculus in which we replace the above set (or ordered set) $A$ by a Cartesian product $A \times B$ of two sets (or ordered sets). This leads to so-called 'polarised partition relations' such as

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \rightarrow \begin{pmatrix} \alpha_0 & \alpha_1 \\ \beta_0 & \beta_1 \end{pmatrix},$$

which (when $\alpha$, $\beta$, etc. are order types) means that if $A$, $B$ are ordered sets of types $\alpha$, $\beta$, respectively, and $A \times B$ is partitioned into sets $\mathscr{C}_0$, $\mathscr{C}_1$, then $A$, $B$ will necessarily have subsets $A_0$, $B_0$, respectively, such that $\operatorname{tp} A_0 = \alpha_0$, $\operatorname{tp} B_0 = \beta_0$, $A_0 \times B_0 \subseteq \mathscr{C}_0$, or subsets $A_1$, $B_1$, respectively, such that $\operatorname{tp} A_1 = \alpha_1$, $\operatorname{tp} B_1 = \beta_1$, $A_1 \times B_1 \subseteq \mathscr{C}_1$. Polarised partition relations were investigated by Erdös, Hajnal and Milner in [**20**], [**21**] and [**45**].

A somewhat surprising result in [**5**] states that if $\alpha$, $\lambda$ are ordinals and $\lambda < \omega_{\alpha+1}$, then $\lambda \nrightarrow (\omega_\alpha, \omega_\alpha^2, \omega_\alpha^3, \ldots)^1$, where the sequence $\omega_\alpha, \omega_\alpha^2, \omega_\alpha^3, \ldots$ has length $\omega$ (that is, it is an ordinary infinite sequence). This amounts to saying that any well-ordered set $S$ with $\operatorname{tp} S < \omega_{\alpha+1}$ can be partitioned into $\aleph_0$ sets $S_n$ ($1 \leqslant n < \omega$) which are 'small' in the sense that $\operatorname{tp} S_n < \omega_\alpha^n$ for every positive integer $n$. We might describe this as a 'paradoxical covering property' of the set $S$. In [20], the authors explain why this result is equivalent to saying that $\binom{\omega}{\lambda} \nrightarrow \begin{pmatrix} 1 & \omega \\ (\omega_\alpha)^\omega & 1 \end{pmatrix}$ if $\lambda < \omega_{\alpha+1}$; and this appears to motivate much of the work in [20] and [45]. The 'paradoxical covering property' discovered in [5] raises a host of questions as to which other statements of a similar kind are true, and such questions are studied in [45]. Some of them have different answers in different models of set theory.

Transversal theory is another set-theoretic theme which caught Milner's interest: it was the subject of his plenary session lecture [38] at the International Congress of Mathematicians in 1974. A *transversal* of an indexed family $(S_i: i \in I)$ of sets is a family $(s_i: i \in I)$ such that $s_i \in S_i$ $(i \in I)$ and $s_i \neq s_j$ when $i \neq j$. As Milner points out in [38], the starting point of transversal theory is the 'marriage theorem', which states that a *finite* family $(S_i: i \in I)$ has a transversal if and only if $|\bigcup_{i \in K} S_i| \geqslant |K|$ for every subset $K$ of $I$. It is considerably harder to characterise those infinite families of sets which have transversals. For countably infinite families, the first such characterisation was established by Damerell and Milner [37], by proving a conjecture of mine. (At a later date, a somewhat different alternative characterisation for countably infinite families was proved in ⟨**17**⟩, and uncountable families were treated in ⟨**1**⟩ and ⟨**2**⟩.)

Since the known necessary and sufficient conditions for infinite families of sets to have transversals are rather complicated, one might ask whether there are simpler *sufficient* conditions which at least cover many cases likely to be of interest. In this spirit, Milner and Shelah [35] proved that a family $(S_i: i \in I)$ has a transversal if $|S_i| \geqslant d(x)$ whenever $i \in I$ and $x \in S_i$, where $d(x) = |\{j \in I: x \in S_j\}|$.

Two sets $A$, $B$ are said to be *almost disjoint* if $|A \cap B| < \min(|A|, |B|)$. A well-known theorem of Sierpinski states that an infinite set of cardinality $\alpha$ has more than $\alpha$ subsets which are pairwise almost disjoint. (Perhaps a helpful illustration is given by a countably infinite tree $T$ in which each vertex is incident with at least 3 edges: uncountably many infinite paths in $T$ start at a particular vertex, and the sets of vertices of these paths are pairwise almost disjoint subsets of the set of vertices of $T$.) Sierpinski's theorem is easily seen to be equivalent to saying that $\alpha$ disjoint infinite sets of cardinality $\alpha$ have more than $\alpha$ transversals which are pairwise almost disjoint. The papers [14] and [15] investigate the truth or falsity of various other statements of a similar type concerning pairwise almost disjoint transversals, and related matters.

Experience shows that results in transversal theory can often be generalised to results concerning matroids. A *matroid* or *independence space* is a pair $(S, \mathscr{I})$ such that $S$ is a set and $\mathscr{I}$ is a non-empty set of subsets of $S$, called *independent* sets, such that:

(i) every subset of a member of $\mathscr{I}$ is a member of $\mathscr{I}$;
(ii) if $A, B \in \mathscr{I}$ and $|B| = |A| + 1 < \aleph_0$, then $A \cup \{b\} \in \mathscr{I}$ for some $b \in B \setminus A$;
(iii) $\mathscr{I}$ has finite character, that is, a set is independent if all its finite subsets are independent.

A theorem of Podewski and Steffens ⟨**18**⟩ states that a countably infinite family $(S_i: i \in I)$ has a transversal if and only if there is no pair $K \subseteq I$, $i \in I \setminus K$ such that $S_i \subseteq \bigcup_{k \in K} S_k$ and every transversal of $(S_k: k \in K)$ uses all the elements of $\bigcup_{k \in K} S_k$. In [41], Milner generalised this theorem to one concerning the existence of a transversal $(s_i: i \in I)$ of $(S_i: i \in I)$ such that the set $\{s_i: i \in I\}$ is independent in a specified matroid.

In a similar sense, Györi and Milner [**43**] established a matroid generalisation of a theorem of Woodall ⟨**24**⟩ (closely related to a similar theorem of Brualdi and Scrimger ⟨**5**⟩) which gave necessary and sufficient conditions for $(S_i: i \in I)$ to have a transversal in the case in which only finitely many of the sets $S_i$ are infinite.

Partially ordered sets feature in many of Milner's papers, starting with [**16**]. More precisely, [**16**] refers to quasi-ordered sets, but the difference between 'partially ordered' and 'quasi-ordered' is an unimportant technicality. A quasi-ordered set $(Q, \leqslant)$ is said to be *well-quasi-ordered* (wqo) if for every infinite sequence $q_1, q_2, \ldots$ of elements of $Q$, there exist positive integers $i, j$ such that $i < j$ and $q_i \leqslant q_j$. In [**16**], Milner considered any quasi-ordered set $(Q, \leqslant)$ such that the elements of $Q$ are transfinite sequences of ordinal numbers, and $q \leqslant q'$ means that some subsequence of $q'$ dominates the sequence $q$. He proved that $(Q, \leqslant)$ is wqo if every element of $Q$ is a transfinite sequence of length less than $\omega^3$, and conjectured that this length restriction could be dropped. I was grateful to him for posing this interesting problem, because I was able (in ⟨**16**⟩) to prove this conjecture by using ideas that I had just developed while working on other problems about well-quasi-ordering.

A partially ordered set $(P, \leqslant)$ is said to be *well-founded* if it contains no infinite descending chain $x_1 > x_2 > \ldots$. Then the *height* $h(x)$ of an element $x$ of $P$ is the ordinal defined recursively by $h(x) = \sup\{h(y)+1 : y < x\}$, and the *height* of $(P, \leqslant)$ is $\sup\{h(x)+1 : x \in P\}$. There are well-founded partially ordered sets of arbitrary height containing no infinite chain (totally ordered subset), but these must contain, in some sense, a large antichain (set of mutually incomparable elements) if the height is large. In [**46**], Milner and Sauer define $\alpha \to [\beta, \gamma]$ to mean that every partially ordered set of height $\alpha$ contains either a chain of type $\beta$ or an antichain $A$ such that the order type of $\{h(x) : x \in A\}$ is $\gamma$. They prove a number of results about when $\alpha \to [\beta, \gamma]$ is true and when it is false. They point out that $\alpha \to (\beta, \gamma)^2$ implies $\alpha \to [\beta, \gamma]$: so we might think of the latter as a weakened version of a partition calculus relation.

The *depth* of a partially ordered set $(P, \leqslant)$ is the least ordinal $\gamma$ such that no chain in $P$ has order type $\gamma^*$ (the reverse of $\gamma$): for example, a well-founded partially ordered set has depth at most $\omega$. The *width* of $(P, \leqslant)$ is the smallest cardinal $\mu$ such that $P$ contains no antichain of cardinality $\mu+1$. In [**58**], Milner and Prikry study the question: given an ordinal $\gamma$ and a cardinal $\lambda$, can every partially ordered set of depth $\gamma$ be decomposed into $\lambda$ parts of depth less than $\gamma$? They also prove that for any cardinal $\lambda$ and any infinite cardinal $\nu$, there is a partially ordered set of width $\nu^+$ (the least cardinal greater than $\nu$) which cannot be decomposed into $\lambda$ parts of smaller width. In [**52**], they prove some other results about partially ordered sets which might be harder to summarise briefly, but once again partition calculus conditions of the form $\alpha \to (\beta, \gamma)^2$ play a role. In particular, when $\kappa$ denotes a cardinal number, a theorem of [**52**] states that $\kappa \to (\kappa, \kappa)^2$ is equivalent to the statement: 'Whenever a partially ordered set $(P, \leqslant)$ contains no antichain of cardinality $\kappa$ and $|P| = \kappa$, there exists a cofinal subset of $P$ which is the union of fewer than $\kappa$ chains'. (A subset $A$ of $P$ is *cofinal* if for every $x \in P$ there exists $y \in A$ such that $x \leqslant y$.) We remark that a cardinal $\kappa > \aleph_0$ such that $\kappa \to (\kappa, \kappa)^2$ must (see ⟨**7**, Chapter 7, Theorem 3.1⟩) be strongly inaccessible and hence 'very large': the non-existence of any such cardinals is consistent with the Zermelo–Fraenkel Axioms for set theory and the Axiom of Choice.

It seems natural to try to extend some of the theory of partially ordered sets to closure systems. A *closure system* is a pair $(E, \phi)$ such that $E$ is a set, $\phi$ is a function from the set of subsets of $E$ into itself, $X \subseteq \phi(X) = \phi(\phi(X))$ for every $X \subseteq E$, and

$\phi(X) \subseteq \phi(Y)$ whenever $X \subseteq Y \subseteq E$. A partially ordered set $(P, \leqslant)$ can be identified with the closure system $(P, \phi)$, where $\phi(X) = \{y : y \leqslant x \text{ for some } x \in X\}$ when $X \subseteq P$.

Closure systems appear in the papers **[60]** and **[83]** of Milner and Pouzet. In **[60]**, they extended to closure systems a result of Pouzet which stated that a partially ordered set must contain an infinite antichain if its cofinality (that is, the minimum of the cardinalities of its cofinal subsets) is a singular cardinal. A partially ordered set $(P, \leqslant)$ is *up-directed* if for all $x, y \in P$ there exists $z \in P$ such that $x \leqslant z$ and $y \leqslant z$. Erdös and Tarski ⟨**11**⟩ proved that a partially ordered set which contains no infinite antichain must be the union of finitely many up-directed partially ordered sets, and in **[83]** Milner and Pouzet established a neat extension of this result to closure systems.

It is well known that every totally ordered set $(E, \leqslant)$ has a cofinal subset $A \subseteq E$ such that for every subset $B$ of $A$, $B$ is cofinal in $(E, \leqslant)$ if and only if $|B| = |A|$. This motivated Galvin, Milner and Pouzet **[75]** to make the following definition for a closure system $(E, \phi)$: a family $\mathscr{A}$ of subsets of $E$ is called a *cardinal representation* of $(E, \phi)$ if, for every set $X \subseteq \bigcup \mathscr{A}$, we have

$$\phi(X) = E \quad \Leftrightarrow \quad (|X \cap A| = |A| \text{ for every } A \in \mathscr{A}).$$

In **[75]**, the authors proved a number of results about cardinal representations, and illustrated their possible usefulness by using them to give a new proof of a theorem of Duffus and Pouzet concerning the gaps in a lattice of finite breadth.

Given a partially ordered set $(P, \leqslant)$, we call a function $f: P \to P$ *order-preserving* if $f(x) \leqslant f(y)$ whenever $x \leqslant y$. We say that $(P, \leqslant)$ has the *fixed point property* if every order-preserving function $f: P \to P$ has a 'fixed point', that is, an element $z \in P$ such that $f(z) = z$. Tarski ⟨**23**⟩ proved that every complete lattice has the fixed point property, and Rival ⟨**20**⟩ and Duffus and Rival ⟨**8**⟩ explored what can be said on similar lines about finite partially ordered sets in general. They introduced a notion of 'dismantling' a finite partially ordered set $(P, \leqslant)$ by removing, one by one, elements which are in some sense not essential to the structure of $P$ (or of what remains of $P$ at the relevant stage in the process). The part of $P$ left at the end of this process is called its *core*, and any two cores of $P$ are isomorphic as partially ordered sets. Moreover, $P$ has the fixed point property if and only if its core has the fixed point property. In **[81]**, Li and Milner extended this idea to infinite partially ordered sets which are *chain complete*, that is, in which every chain has an infimum and supremum: they defined a transfinite sequence of steps which progressively pick out smaller and smaller subsets of $P$ until we are left with a 'core'. If $P$ is chain complete and has no infinite antichain, then the process behaves nicely: for example, any two cores of $P$ are once again isomorphic, and $P$ once again has the fixed point property if and only if its core has the fixed point property. However, this approach is not particularly useful in deciding whether $P$ has the fixed point property in cases in which (as can easily happen) the core of $P$ turns out to be $P$ itself. This motivated Li ⟨**13**, **14**⟩ to introduce a somewhat different procedure for progressively picking out smaller and smaller subsets of $P$, the set left at the end of this process being called the *ANTI-core* of $P$. This raised questions as to whether the end result of this process would once again be unique up to isomorphism, and in what way it might be helpful in deciding whether $P$ has the fixed point property. Under suitable conditions on $P$, such questions are answered fairly satisfactorily by Li and Milner in **[90]**, **[91]** and **[92]**.

A striking result about *totally* ordered sets (which does not seem to extend to partially ordered sets in any obvious way) was obtained by Aharoni, Hajnal and Milner in **[85]**. Let a family of intervals in a totally ordered set $S$ be called a $\kappa$-*cover* of $S$ if each element of $S$ belongs to at least $\kappa$ of these intervals: then any $\kappa$-cover of $S$ contains $\kappa$ disjoint 1-covers of $S$. The method of proof depends on whether $\kappa$ is finite or infinite, but is highly non-trivial in both cases.

Milner's ability to learn and use new techniques is seen in his expository paper [**86**], where he describes the concept of 'elementary substructures' (compare ⟨**4**, Chapter 4⟩) and suggests that they provide 'a powerful technique which can and should be part of the working mathematician's toolbox'. A *relational structure* $(A, \mathscr{C})$ consists of a set $A$ and a collection $\mathscr{C}$ of finitary relations on $A$. By a *substructure* of $(A, \mathscr{C})$, we mean a relational structure $(B, \mathscr{C})$ where $B \subseteq A$ and the relations in $\mathscr{C}$ are now (by an abuse of notation on my part) regarded as relations on $B$. This substructure is said to be *elementary* if, roughly speaking, all true statements about $(B, \mathscr{C})$ which can be formulated in predicate calculus are also true statements about $(A, \mathscr{C})$. For example, $(\mathbb{Q}, <)$ is an elementary substructure of $(\mathbb{R}, <)$ if $\mathbb{Q}$, $\mathbb{R}$ denote the sets of rational and real numbers, respectively. Under suitable conditions, one can prove that a given relational structure $(A, \mathscr{C})$ must have elementary substructures of certain infinite cardinalities (and, if necessary, with certain additional properties) by starting with any subset of $A$ and repeatedly enlarging it to include elements which would be needed in a substructure of the required kind. In [**86**], Milner gives three examples of the use of such ideas in infinite combinatorics. Two of them use elementary substructures of relational structures of the form $(A, \in)$, where $A$ is a suitable set of sets and $\in$ is the usual relation of membership, to prove a result in partition calculus and answer a question of Pouzet about graphs of uncountable chromatic number. The third example is a proof of Komjáth and Milner in [**87**] of a conjecture of Rödl and Voigt ⟨**21**⟩. This conjecture says that if $\lambda$ is an infinite cardinal number, $\lambda^+$ is the least cardinal greater than $\lambda$, and $T$ is a tree in which each vertex is incident with exactly $\lambda^+$ edges, then there exists a graph $H$ with exactly $\lambda^+$ vertices such that $H \to (T)^1_\lambda$. The statement $H \to (T)^1_\lambda$ means that for every colouring of the vertices of $H$ with $\lambda$ colours, there exist $\lambda^+$ vertices of the same colour which, together with all the edges joining pairs of them, form a graph isomorphic to $T$. In fact, Komjáth and Milner proved that, more generally, $T$ can be replaced by any graph with exactly $\lambda^+$ vertices which is the union of finitely many trees.

The volume of high-quality work produced by Milner during approximately three decades is impressive, and this short account can look at only illustrative samples of it, selected almost at random. Much important work remains completely unmentioned. However, personal interest prompts me to mention one more item. In ⟨**15**⟩, I proved that a graph $G$ is decomposable into circuits (that is, there exists a collection of circuits in $G$ such that each edge of $G$ is in exactly one of them) if and only if it has no finite cutset of odd cardinality. The proof is easy for finite or countably infinite graphs, but in the uncountable case very long and complicated arguments were used to prove this apparently simple statement. One naturally wondered whether any other approach might work. So far as I know, this matter was thereafter neglected until Polat ⟨**19**⟩ discovered a generalisation. He pointed out that the theorem could be reformulated by defining a *matching* in a family of sets $\mathscr{E}$ to be a subfamily $\mathscr{M} \subseteq \mathscr{E}$ whose members are disjoint. Let $\mathscr{E}$ be called *matchable* if $\bigcup \mathscr{M} = \bigcup \mathscr{E}$ for some matching $\mathscr{M} \subseteq \mathscr{E}$, and *finitely matchable* if for every finite set $F \subseteq \bigcup \mathscr{E}$ there exists a matching $\mathscr{M} \subseteq \mathscr{E}$ such that $F \subseteq \bigcup \mathscr{M}$. If the members of $\mathscr{E}$ are the sets of edges of the circuits in $G$, then the theorem of ⟨**15**⟩ says in effect that $\mathscr{E}$ is matchable if it is finitely matchable. Polat ⟨**19**⟩ showed that, more generally, the set of circuits (minimal dependent sets) of any binary matroid (that is, any matroid in which every symmetric difference of two circuits contains a circuit) is matchable if it is finitely matchable. However, both ⟨**15**⟩ and ⟨**19**⟩ used complicated *ad hoc* arguments, and one might wonder whether we could invoke the sort of 'compactness arguments' often used to deduce facts about infinite structures from corresponding facts about their finite substructures. In [**67**], Komjáth, Milner and Polat proved a generalisation of Polat's theorem to arbitrary matroids using (to quote the paper) 'very general compactness techniques of the kind first used by Shelah' ⟨**22**⟩. The theorem of ⟨**19**⟩ as stated above does not extend to general matroids, but a re-statement of it does. Specifically, call a family of sets $\mathscr{E}$ *finite matching extendable* if for every $x \in \bigcup \mathscr{E}$ and every finite matching $\mathscr{M} \subseteq \mathscr{E}$, there exists a matching $\mathscr{M}' \subseteq \mathscr{E}$ such that $\mathscr{M} \subseteq \mathscr{M}'$ and $x \in \bigcup \mathscr{M}'$. This property is equivalent to being finitely matchable when $\mathscr{E}$ is the set of circuits of a binary matroid, and in [**67**] the authors showed that the set of circuits of any matroid is matchable if it is finite matching extendable.

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