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Corrigendum

Phragmèn–Lindelöf theorems in slabs for some systems of non-hyperbolic second-order quasi-linear equations

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In the proof of theorem 2.6, the demonstrations that u_k is a supersolution (e.g. (3.11)) and v_k is a subsolution are incorrect; the dependence of G_k and J_k on parameters which are not constant in Ω_1 was overlooked. If assumption 2.5 is replaced by the following language and certain minor modifications to the proof are made, then the argument given in the paper becomes correct.

Assumption 2.5*. *Given $N_0, N_1, N_2 > 0$, there exist non-negative constants α_{ij}, $1 \leqslant i, j \leqslant m$, with $\alpha_{ii} = 0$, and $\beta_0 > 0$ such that for each $k = 1, \ldots, m$ and $n_1, \ldots, n_m \in [0, N_0]$, there exist G_k and J_k (depending on $n_1, \ldots, n_m$) in $C^2(I_M) \cap C^0(\bar{I}_M)$ satisfying*

$$G_k''(y) + E_k(\omega, y, \boldsymbol{z} + \boldsymbol{t}, q + G_k'(y)) - E_k(\omega, y, \boldsymbol{z}, q) \leqslant 0,$$
$$J_k''(y) + E_k(\omega, y, \boldsymbol{z} + \boldsymbol{t}, q + J_k'(y)) - E_k(\omega, y, \boldsymbol{z}, q) \geqslant 0,$$
$$-\sum_{j=1}^{m} \alpha_{j,k} n_j \leqslant J_k(y) \leqslant 0 \leqslant G_k(y) \leqslant \sum_{j=1}^{m} \alpha_{j,k} n_j,$$

and

$$|G_k'(y)| \leqslant \beta_0, \qquad |J_k'(y)| \leqslant \beta_0,$$

for each $y \in (-M, M)$ whenever $q \in \mathbb{R}$ with $|q| \leqslant N_2$, $\boldsymbol{z} = (z_1, \ldots, z_m) \in \mathbb{R}^m$ with $|z_j| \leqslant N_1$ for $j = 1, \ldots, m$, and $\boldsymbol{t} = (t_1, \ldots, t_{k-1}, 0, t_{k+1}, \ldots, t_m) \in \mathbb{R}^m$ with $|t_j| \leqslant n_j$ for $j = 1, \ldots, m$. Further, the $m \times m$ matrix $C = (\alpha_{ij})$ has the property that

$$\lim_{p \to \infty} C^p = 0 \quad (\textit{the zero matrix}).$$

The sentence which includes inequality (3.6) should be replaced by the following.

We claim that if $(\boldsymbol{x}_0, y) \in \bar{\Omega}$, $\boldsymbol{x}_0 \in W$ and

$$d_j(x_0, H) = \max\{|f_j(\boldsymbol{x}, y) - F_j(y)| : (\boldsymbol{x}, y) \in \bar{\Omega}, |\boldsymbol{x} - \boldsymbol{x}_0| \leqslant A(H)\mathrm{e}^{\chi(H)}\},$$

then

$$|f_k(\boldsymbol{x}_0, y) - F_k(y)| \leqslant \sum_{j=1}^{m} \alpha_{j,k} d_j(x_0, H) + 4\epsilon. \tag{3.6}$$

After equation (3.7), set $n_j(\boldsymbol{x}_0, H) = \max_{(\boldsymbol{x},y)\in\bar{\Omega}_1} |f_j(\boldsymbol{x}, y) - F_j(y)|$, $1 \leqslant j \leqslant m$, and $N_2 = \max_{y\in\bar{I}} |U_k(y)|$. Let $G_k(y)$ and $J_k(y)$ be as given in assumption 2.5* with parameters N_0, N_1, N_2 and $(n_1, \dots, n_{k-1}, 0, n_{k+1}, \dots, n_m)$. Then define u_k and v_k as in the original article. We observe that inequality (3.14) will be modified in a similar manner to that of (3.6). The remainder of the proof remains unchanged. It is unclear if there is any important difference between the original statement of assumption 2.5 and the modified statement given here; we observe that theorem 2.8 and the argument preceding it on pp. 1160 and 1161 correspond equally well to the language of assumption 2.5* or assumption 2.5.

The conclusions of example 4.1 are correct; however, some of the justification provided is incorrect. In the second complete paragraph on p. 1172, the last sentence should be replaced by the following.

Since $M^2 < \frac{1}{\sqrt{3}}$, we may select $\delta_* > 0$ such that $(2 + \frac{1}{8}\delta_*)\frac{1}{2}(3 + \frac{1}{3}\delta_*)M^4 < 1$. Further, assumption 2.4 is satisfied, since for each $\alpha \in (0, 1)$, the choices $\delta \in (0, \min\{2\alpha, \delta_*\})$, $L_1(y) = 4 - \frac{1}{3}\delta$, $L_2(y) = 3 - \frac{1}{2}\delta$, $U_1(y) = 4 + \frac{1}{4}\delta$, and $U_2(y) = 3 + \frac{1}{3}\delta$ satisfy the required conditions.

In the next paragraph, we need to define

$$G_1(y) = \tfrac{1}{2}(M^2 - y^2)(4 + \tfrac{1}{4}\delta)n_2, \qquad G_2(y) = \tfrac{1}{2}(M^2 - y^2)(3 + \tfrac{1}{3}\delta)n_1$$

and

$$J_1(y) = \tfrac{1}{2}(y^2 - M^2)(4 + \tfrac{1}{4}\delta)n_2, \qquad J_2(y) = \tfrac{1}{2}(y^2 - M^2)(3 + \tfrac{1}{3}\delta)n_1$$

with $n_1 = n_1(\boldsymbol{x}_0, H)$ and $n_2 = n_2(\boldsymbol{x}_0, H)$ as above. Then in assumption 2.5* we have $\alpha_{21} = (2 + \frac{1}{8}\delta_*)M^2$ and $\alpha_{12} = \frac{1}{2}(3 + \frac{1}{3}\delta_*)M^2$. The remainder of example 4.1 is correct.

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