BOOK REVIEWS

HELMBERG, G. M., *Introduction to Spectral Theory in Hilbert Space* (North Holland Publishing Company, Amsterdam, 1969), xiii+346 pp., 140s.

This book provides a gentle but thorough introduction to the spectral theory of a single linear operator on a Hilbert space. The definitions and elementary properties of inner-product and Hilbert spaces are given in chapter I. This chapter contains a section on normed linear spaces, which includes a brief discussion of the various topological notions required, and sections on the Hilbert spaces l^2 and $L^2[a, b]$. Chapter II is concerned with the geometry of Hilbert space. Subspaces, orthogonality and bases are discussed here. The classical bases for various L^2-spaces (Legendre, Hermite and Laguerre functions) are fully dealt with. The chapter concludes with a proof of the important theorem that there is an isometric isomorphism between any two separable Hilbert spaces over the same field. Chapter III is devoted to the theory of linear operators on a Hilbert space. There are sections on bounded linear operators, bilinear forms, adjoint operators and projection operators. The chapter concludes with a thorough discussion of the Fourier-Plancherel transform on $L^2(-\infty, +\infty)$. The first half of chapter IV continues with the theory of linear operators. There are sections on adjoint operators, closed linear operators, differentiation and multiplication operators on L^2-spaces. In the remainder of the chapter the basic ideas of spectral theory are introduced—invariant subspaces, eigenvalues and spectra. The chapter concludes with a detailed study of the spectrum of a self-adjoint operator. Chapter V is concerned with the spectral theory of a compact linear operator. There is a section on Fredholm integral equations, and the spectral decomposition theorem for a compact self-adjoint operator is proved. In chapter VI, there is proved in turn the spectral theorems for a self-adjoint, unitary and normal operator. The author's development of this theory is close in spirit to the treatment in " Functional Analysis " by Riesz and Sz-Nagy. The final chapter is devoted to the spectral theory of an unbounded self-adjoint operator. There is a section on the Cayley transform, and the spectral theorem for an unbounded self-adjoint operator is proved. The book concludes with two appendices, the first on the graph of a linear operator and the second on the Riemann-Stieltjes and Lebesgue integration theory used in the main text. There are numerous exercises throughout the book, which serve to illustrate the theory, and a comprehensive bibliography. The classical motivating examples for the abstract theory are fully worked out in the text. This is an admirable work on which to base a graduate course on Hilbert space.

H. R. DOWSON

LUKACS, EUGENE, *Stochastic Convergence* (Heath Mathematical Monographs, 1968), viii+142 pp.

The description of a book as a monograph tends to convey the impression that the author has allowed himself more licence than would have been the case had he set out with the purpose of writing a textbook. In this case the book itself confirms this impression. Very markedly it reflects the personal tastes and interests of the author rather than the requirements of a defined class of readers. Because of this, it is difficult to describe briefly the content and level of the book, and a more detailed account than is normal in a review becomes necessary.

Chapter 1 states, rather than discusses, basic ideas and theorems, knowledge of which is a prerequisite. Most of these are just what one might expect—ideas such as those of probability space, random variable, independence, characteristic function; theorems such as Boole's Inequality, Chebyshev's Inequality, the uniqueness theorem and continuity theorem for characteristic functions. However it is somewhat surprising to discover the Lévy-Khinchine canonical representation of infinitely divisible distributions as one of the prerequisites for reading a book with this title. Here is the first indication of the influence of the author's interests on the content.

Chapter 2 is concerned with convergence concepts and relations among them. Again much is as to be expected—the notions of almost certain convergence, convergence in probability, in quadratic mean, in law (though the author emphasizes that the last mentioned is only of marginal interest in the book). Less familiar modes of convergence are also introduced here, such as almost uniform convergence, complete convergence, Λ-convergence and information convergence.

The third chapter again exemplifies the licence which the author has permitted himself. It departs from the main thread of the theory of stochastic convergence and is concerned with the question of the existence of metrics on spaces of random variables which are compatible (in the obvious sense) with different modes of convergence. This chapter might well be of more interest to functional analysts than to statisticians.

Chapter 4 returns to territory more familiar to statisticians, and here we find the celebrated convergence theorems: zero-one laws, Kolmogorov's Inequality, the three-series theorem, laws of large numbers, the law of the iterated logarithm, the Glivenko-Cantelli theorem, etc.

Each of the remaining three chapters is short (about ten pages)—one on the definition of stochastic integrals and derivatives and the final two on somewhat esoteric characterizations of the normal distribution and the Wiener process respectively.

The difficulty facing a reviewer of giving a brief description of the content of this monograph will now be apparent. The style is easier to describe: a typical concise, pure mathematical style of definition followed by lemma, followed by theorem, all in logical order but somewhat lacking in motivation to those with a more applied outlook.

The reader who hopes for a useful intermediate textbook on stochastic convergence will be disappointed. However the book will satisfy those who wish a reference book containing clear definitions of concepts and lucid proofs of theorems. Some of these concepts and theorems are standard. Others, as indicated in the description of the content above, provide an introduction to less familiar theory.

A. D. SILVEY

HANSEN, E. (Editor), *Topics in Interval Analysis* (Oxford University Press, 1969), vi+130 pp., £2·50 or 50s.

In January 1968, a symposium on interval analysis was organized by L. Fox of the University of Oxford and held at the Culham Laboratory. This book contains the notes, amplified in some cases, of the lectures which were presented at it, together with a research paper by the editor.

Interval analysis, a recent development in numerical analysis, is an attempt to provide realistic error bounds on the result of a sequence of arithmetic computations. The subject springs from the recognition that although any number stored in a computer is not often known exactly close lower and upper bounds for it can be calculated easily. The proposal is therefore that a different type of arithmetic, called *interval arithmetic*, in which the arithmetic operations are performed on the intervals defined by these bounds, should be used. If this is carried out naively the results are disappointing for it is then a crude form of forward error analysis. When it is used