The final chapter, entitled 'The Turn of the Century', essentially considers the years up to the end of the first decade of the 20th century, where the story presented in this book ends. This covers the development and progress in function theory by Landau, Jensen, Lindelöf and other well-known historical figures who have had such a big influence on analytic number theory. The work of Landau, in particular, crops up everywhere: from the classical explicit formula for $\psi(x)$ through the initial applications of tauberian theorems and finally to the zeros of the zeta-function on the critical line. After a brief discussion on the sign changes of $\pi(x) - li(x)$, the chapter (and the book) ends with the celebrated conjectures of Hardy and Littlewood (extended by Bateman-Horn and Schinzel) and a look at how far we are from reaching these.

I can thoroughly recommend this entertaining and very informative book to any active analytic number theorist who has, in addition, more than a passing interest in how the subject has evolved in the way it has. The author is to be congratulated for having dedicated such a considerable effort to providing a really useful addition to the existing literature.

M. NAIR

Breuer, T. *Characters and automorphism groups of compact Riemann surfaces* (LMS Lecture Note Series 280, Cambridge, 2000), xii+199 pp., 0 521 79809 4 (paperback), £24.95 (US$39.95).

In the last 20 years, finite group theory has been revolutionized by the classification of finite simple groups, which allows proofs by inspection, by the development of powerful computational techniques, which are beginning to turn the subject into an experimental science, and by the increased willingness of its practitioners to look outside their subject area for applications. One particularly fruitful area is the relationship between groups and Riemann surfaces, based in part on Schwarz's theorem that a compact Riemann surface X of genus $g \geqslant 2$ has a finite automorphism group G. Since there are induced linear actions of G on various modules associated with X (homology, cohomology, differentials, etc.), there are natural roles for representation theory and character theory in this relationship. These are powerful techniques, now enhanced by rich databases such as the ATLAS, and sophisticated group theory programs such as GAP.

Motivated by these developments, Breuer considers two dual objectives in this book: first, to classify all groups of automorphisms of compact Riemann surfaces X of fixed genus $g \geqslant 2$, up to equivalence of the action on the space $\mathcal{H}^1(X)$ of holomorphic abelian differentials on X; secondly, to classify those characters of a given finite group which can arise from its action on such a module. The first problem is solved, with the aid of GAP, up to genus 48; this is a great advance on previous knowledge, which was complete only for $2 \leqslant g \leqslant 5$. Concerning the second problem, necessary conditions on the character are found, and in many cases these are shown also to be sufficient; these results are illustrated with a detailed study of several classes of groups, such as the linear fractional groups $L_2(q)$ and the Suzuki groups $Sz(q)$, again taking us well beyond what was previously known. These are very difficult problems, and it is unreasonable at this stage to expect complete solutions; by bringing together a number of very effective techniques, and by presenting a mass of specific evidence, Breuer has done the mathematical community a considerable service in directing attention towards some interesting and challenging open problems in this area.

Chapter 1 gives a clear and concise account of the theory of compact Riemann surfaces, often referring to easily obtainable standard textbooks (such as Farkas and Kra) for full details. Chapter 2 does the same for character theory, first in general, and then concentrating on the character afforded by $\mathcal{H}^1(X)$. Chapter 3 gives a detailed examination of how this particular character is related to the fixed points of elements of G on X, for instance through the Eichler trace formula. These three chapters give an excellent quick guide to the general theory of compact Riemann surfaces and their automorphisms and holomorphic differentials, though the reader

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would need to look elsewhere for full proofs. My only real criticism here is that Breuer's focus is rather strongly centred on $\mathcal{H}^1(X)$, the subject matter of the later chapters: for the general reader, it would enhance the usefulness of the book to mention (if only briefly) a few other important G-modules associated with X, such as the first homology and cohomology modules, and their decompositions in terms of holomorphic and antiholomorphic differentials. Examples of such applications, which might be mentioned here, include Macbeath's investigation of the action of G on homology, Sah's use of modular representations to construct abelian coverings, and the recent use by Streit and Wolfart of the action on differentials to study fields of definition.

Chapter 4 concerns the question of whether (and, if so, in how many ways) a finite group can be generated by elements with specific properties, usually as an epimorphic image of some Fuchsian group. Breuer brings together a number of methods which have recently been developed for studying surface coverings, the inverse Galois problem, and the symmetric genus problem. Although much of this material is readily available in research papers, it is very useful to have it collected, unified and further developed here.

In Chapters 5–7, with the preliminaries disposed of, Breuer makes a detailed attack on the two problems stated earlier. Much of this is his own recent research, using algorithms specifically developed for the purpose; I found it rather impressive, though significantly harder going than the more familiar material in the first part of the book. Some of the results consist of long and detailed tables, obtained by careful case-by-case analysis, frequently assisted by GAP. A typical example is the classification of the groups G which act irreducibly on $\mathcal{H}^1(X)$: there are 36 of these, the largest group (and genus) being $L_2(13)$ for $g = 14$; in each case Breuer gives the signature of the corresponding Fuchsian group and the matrices representing a set of generators of G on $\mathcal{H}^1(X)$.

The text is well organized, and (given the technical nature of some of the subject matter) not too hard to read; a well-motivated postgraduate student should have no great difficulty in making progress through it, especially in the more expository first half. The presentation of the material is excellent, with attractive layout and very few misprints. There is a very useful and up-to-date bibliography, and the index is also good. Taken together, the clear expository Chapters 1–4 and the interesting new algorithms and results in Chapters 5–7 make this book a valuable addition to the literature, which should be essential reading for anyone interested in the connections between Riemann surfaces and finite groups.

G. JONES

McLean, W. *Strongly elliptic systems and boundary integral equations* (Cambridge University Press, 2000), xiv+357 pp., 0 521 66375 X (paperback), £20.95 (US$32.95).

The book offers a concise and self-contained study of distributional solutions of linear, strongly elliptic three-dimensional systems of partial differential equations, which covers the existence of such solutions and their integral representations in terms of single- and double-layer potentials.

After a brief review of some fundamental elements of functional analysis on normed spaces, distributions and their Fourier transforms are discussed succinctly and the full range of Sobolev spaces is introduced and described in some detail, with particular reference to Lipschitz domains. This is followed by a study of the Dirichlet, Neumann and mixed boundary value problems for strongly elliptic systems, in which variational formulations are derived and the existence, uniqueness, regularity and continuous dependence of their solutions on the data are established. The main tool in this analysis is the Fredholm Alternative.

Next, single-layer and double-layer potentials are defined, and the mapping properties of the boundary operators generated by these potentials are investigated. By having their solutions sought in the form of potentials, the above boundary value problems in both interior and exterior