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ARTICLE

A building block approach to retirement income design

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Abstract

This paper addresses the retirement income planning problem from the perspective of the four main building blocks of retirement income: state pension, mortality credits, investment strategies, and drawdown schedules. We detail how these building blocks interact to form a retiree's overall retirement income portfolio, and what trade-offs and interactions must be considered. We find that while access to each building block increases the retiree's certainty equivalent consumption, the most substantial contributor to this increase is from utilization of the mortality credit building block (i.e., annuities).

Keywords: building blocks; default design; mortality credits; retirement income; retirement portfolio design

JEL classification: C61; D14; D15

The shift away from defined benefit toward defined contribution retirement income provision leads to a transfer of longevity risk and investment risk from pension providers onto retirees. Longevity risk describes the risk that a retiree lives longer than expected and exhausts wealth that generates retirement income. Investment risk describes the uncertain nature of investment returns, and the fact that they can differ from expectations, hence affecting retirement income. Many products exist to address these risks, but are only useful if utilized by retirees. Longevity risk can be managed via products that hedge or spread longevity risk, such as annuities. Investment risk can be managed by reducing exposure to risky asset returns or through options, at the cost of lower expected returns.

The aim of retirement savings should be to generate an income (Van Wyk, 2012) and to provide income certainty (Merton, 2014) rather than wealth maximization. The disparity between the complexity of the retirement income planning problem and retirees' capacity to make optimal decisions makes it unlikely they will behave optimally. Much of the existing literature investigates the retirement income planning problem by prescribing retirees certain income products or investment strategies, or by comparing products in isolation. The limitation here lies in a lack of clarity regarding how such products can fit together to create an overall retirement income portfolio, and what interactions and trade-offs must be considered. In the event such trade-offs and interactions are addressed, results are seldom presented in a manner that conveys the main sources of income for the retiree, and the value offered by incrementally increasing flexibility in decision making.

The contribution of this paper is a unique approach to the retirement income planning problem from the perspective of income building blocks. By doing so, we explicitly convey which building blocks are most important, and hence how different retirement income products can fit together to create an optimal investment strategy and consumption profile for the retiree, given their individual characteristics. We demonstrate this approach for a representative individual with simple power utility preferences, although the approach can easily be repeated for other settings. This understanding of the impact of building blocks can help inform the design of retirement income strategies and policy.

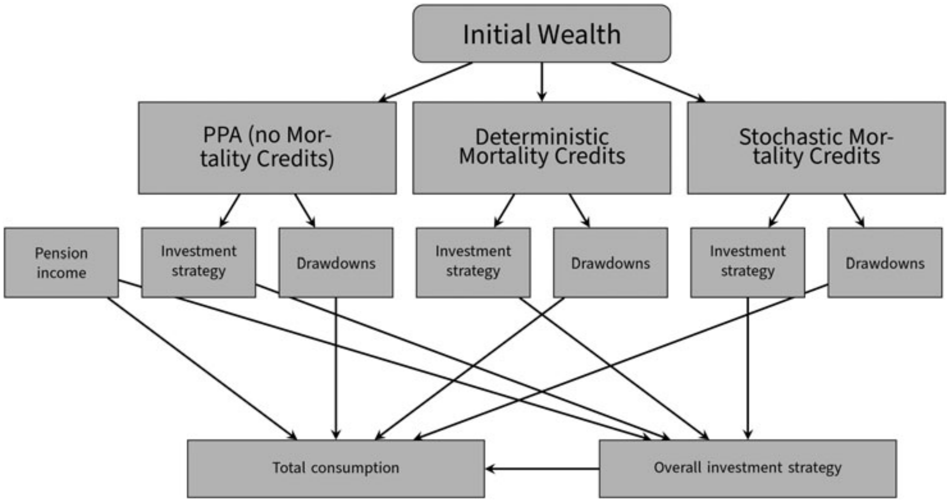

Figure 1. The interactions between the building blocks of retirement income.

A retiree's investment strategy and consumption profile can be attributed to four interacting building blocks: state pension, mortality credits, investment strategy, and drawdown schedule. The state pension (pension, hereafter) income is usually exogenous and can be treated as an input in the retirement planning problem (any defined benefit pension would also be a part of this building block). The remaining three building blocks represent the decisions that convert wealth saved for retirement into an income, and are integral to products available to the retirees. Each represents a dimension along which decisions must be made to achieve a preferred exposure to longevity and investment risk. Mortality credits represent a transfer of wealth from investors who die to investors who remain alive and assist in managing longevity risk. Investment strategy refers to the mix of assets in the portfolio to manage investment risk. The drawdown schedule is the amount or proportion of funds withdrawn to generate a retirement income for consumption, and interacts with both longevity and investment risk.

Linking this to products, retirees may have access to three types of products: private pension accounts (PPAs), fixed and variable annuities, and group self-annuitization (GSA) or pooled mortality products. PPAs provide no mortality credits but allow for full flexibility in investment strategy and drawdown schedule as well as catering to any bequest motives. Annuities provide guaranteed (deterministic) mortality credits but generally no or only partial flexibility in the other two building blocks. GSAs offer stochastic mortality credits based on the mortality experience of the group and usually offer some flexibility in investment strategy and drawdown schedule. PPAs and GSAs are typically cheaper (i.e., have lower fees) relative to annuities since the provider of these products does not need to hold capital against longevity risk (Zhou, 2020; Weinert and Gründl, 2021). All may also have add-on features that provide optionality in investment strategy, and by extension drawdown schedule.

In this paper, we consider the retirement income problem at the time of retirement in a utility maximization framework. Figure 1 shows the overall model structure, highlighting how the different building blocks interact to create the retiree's overall investment strategy and consumption profile. At retirement, the retiree allocates their liquid wealth across three products offering different mortality credits (none, deterministic, and stochastic). Then, an investment strategy and drawdown schedule is considered for each product. Alongside pension income received, the selected investment strategies and drawdown schedules will give the retiree's overall investment strategy and consumption profile.

The existing literature on this topic typically considers only a subset of the range of building blocks and products discussed in this paper. The seminal works of Yaari (1964), Merton (1969, 1971), and

Samuelson (1969) provide the foundation for the analysis of retirement consumption, wealth allocation, and investment decisions when faced with an uncertain future lifetime when individuals have access to PPAs alone. MacDonald *et al.* (2013) review retirees' drawdown behavior when faced with longevity and investment risk. They note that the higher flexibility in drawdowns from a PPA is one reason why they can be preferred over traditional annuity products. Milevsky (1998) finds that the higher fees in traditional annuities can be a deterrent for retirees, since they can achieve higher net returns via the PPA with a high probability until the annuity's mortality credits become materially large. Hence, there are trade-offs in allocating wealth across a PPA and a traditional annuity, with both products having attractive features which suggest some combination is optimal. Andréasson *et al.* (2017) analyze PPAs in the presence of the Australian means-tested pension system while Iskhakov *et al.* (2015), Milevsky and Huang (2018), and Butt *et al.* (2022) consider a combination of PPAs, pension income, and fixed annuity products.

In this paper, we separate the investment strategy decision from the longevity insurance (access to mortality credits) decision. A substantial amount of existing literature does not make this separation (see, e.g., Davidoff *et al.*, 2005; Lockwood, 2012; Horneff *et al.*, 2014), partly reflecting the trade-off imposed by traditional annuities. For example, life annuities offer no exposure to risky assets, which suggests that if available alongside a PPA, they will present retirees with a trade-off between risky assets and longevity insurance. A lack of separation means the retiree's preference to insure longevity risk or access risky assets is not clear. Rather, results only indicate how retirees handle the trade-off between risky assets and longevity insurance.

Variable annuities, which allow freedom to choose the investment strategy, remove the trade-off between risky assets and longevity insurance. Steffensen and Søe (2023) suggest that these help better convey a retiree's preference for risky assets, given they no longer come at the cost of longevity insurance. Horneff *et al.* (2010) consider access to a PPA, a fixed pension, and variable annuities and find that full annuitization is not optimal for specific drawdown schedules. Milevsky and Young (2007), conversely, find that self-annuitization of PPAs can dominate income portfolios. Ai *et al.* (2023) allow the retiree access to PPA, means-tested pension, and variable annuities with a selectable drawdown schedule. However, none of the aforementioned literature considers a stochastic mortality credit product.

The stochastic mortality credit products commonly covered in existing literature are GSA products, pooled annuity funds, and tontines. For our paper, the important difference between deterministic and stochastic mortality credit products is that a deterministic mortality credit product provides a complete hedge against longevity risk. Conversely, a stochastic mortality credit product spreads longevity risk across an investor pool (Zhou, 2020), thereby offering a partial hedge. We define a GSA (i.e., our chosen stochastic mortality credit product) in the same way as Piggott *et al.* (2005). This means it spreads longevity risk across the investor pool, and the investment risk is borne by the pool.

Stamos (2008) compares the attractiveness of GSA against PPAs and traditional annuities. However, they do not allow the retiree to spread their wealth across products. Chen *et al.* (2020) find that assuming reasonable loadings, a retirement income portfolio consisting of an annuity and a GSA is optimal relative to only investing in one. However, the preference across an annuity and a tontine does depend on the embedded loadings/fees, as suggested by Milevsky and Salisbury (2015).

Overall, the existing literature is subject to at least one of the following limitations. Some compare the value of different income products, but do not consider wealth allocation across products, and the trade-offs this presents the retiree. Other works do not separate the retiree's investment strategy and longevity insurance decisions. Some papers separate these decisions alongside pension income, but do not offer a stochastic mortality credit product. Overall, the literature does not approach the retirement income problem with the flexibility possible using the four building blocks, nor does it explicitly show how these building blocks and their interactions provide value during retirement.

Thus, we contribute to the existing literature by considering three distinct types of mortality credit products, and how they fit together to create an overall portfolio alongside pension income. We start with a PPA with a typical/default investment strategy and drawdown schedule, allowing the retiree to choose the optimal age for commencing the pension under the United States system. We then incrementally allow flexibility to the retiree across the three building blocks of mortality credits, investment strategy, and drawdown schedule. Within the investment strategy, we also find the value provided to retirees when the risky asset returns are bounded through options, similar to the commonly seen variable annuity riders across all products. Our modeling approach allows us to quantify the value of each building block to the retiree as well as ascertain which building blocks add the most value to the retiree.

We find that if the retiree only has access to a PPA alongside their fixed pension, being able to choose an investment strategy away from the default provides more value than being able to choose a drawdown schedule. However, the reverse holds if the retiree can access products offering mortality credits, regardless of whether these mortality credits are deterministic or stochastic. Being able to use options to smooth returns provides material value similar to that of being able to choose investment and drawdown strategy. In cases where the retiree has access to products offering mortality credits and can choose their drawdown schedules, a material proportion of wealth is kept in the PPA and is used as a tool to boost consumption in the initial periods of retirement. By doing so, the retiree is able to delay receipt of and hence increase their pension income. Overall, we find that the retiree obtains most value from products offering mortality credits alongside their fixed pension income, and always allocates a majority of their wealth to these when they are available.

1. Methodology

1.1 Model setup and dynamics

We model a single male who owns their home. The individual is aged $x = 66$, has reached full retirement age (FRA) as defined in the United States setting (Social Security Administration, 2023b), and has fully retired and does not earn any labor income in retirement. All modeling is assumed to be in real terms to account for inflation. No tax implications of different products or cash flows are modeled.

As per the structure outlined in Figure 1, at age 66 (time, $t = 0$), the retiree allocates their wealth, W_0, across three products offering different mortality credits: a private pension account offering no mortality credits (denoted *PPA*), a deterministic mortality credit product (denoted *DM*), and a stochastic mortality credit product (denoted *SM*). We implement a static optimization approach under the expected utility framework, that is, the retiree makes decisions at time $t = 0$ only, such that their expected lifetime utility is maximized. Calculations are performed in discrete annual time intervals, with t denoting the number of years after retirement and $x + T$ is the maximum age the retiree can attain. The wealth in product $i \in \{PPA, DM, SM\}$ at time $t = 0$, denoted $A_{0,i}$, is given by

$$A_{0,i} = \omega_i W_0$$

where ω_i denotes the proportion of initial wealth W_0 allocated to product i. The wealth in product i at time $t + 1$, denoted $A_{t+1,i}$, is given by the recursive equation

$$A_{t+1,i} = A_{t,i} - D_{t,i} + I_{t,i} + M_{t,i} \tag{1}$$

where $D_{t,i}$ is the drawdown from product i at time t, $I_{t,i}$ is the investment income earned by product i from time t to time $t + 1$, and $M_{t,i}$ is the mortality credits earned by product i from time t to time $t + 1$. We now describe our modeling approach for each of these three components of the recursive equation.

1.1.1 Investment returns

The retiree's investment strategy is confined to a risky asset and a risk-free asset. The risk-free asset's price process $(B_t)_{t\geq 0}$ satisfies the following differential equation

$$\frac{dB_t}{B_t} = r_f dt$$

where r_f denotes the annual risk-free rate. The risky asset's price process $(S_t)_{t\geq 0}$ is modeled as Geometric Brownian motion. Under the real-world measure $\mathbb{P}$, the risky asset satisfies the following stochastic differential equation (SDE) $(dS_t/S_t) = \mu dt + \sigma dZ_t$ where μ denotes the expected annual growth rate of S_t, σ denotes the annual volatility of returns for S_t, and Z_t is a Weiner process. The risky asset's realized return from time t to time $t+1$ is denoted $\hat{r}_{r,t}$. Where the retiree has access to options, we assume that the only strategy available to the retiree is a self-financing collar strategy. In this strategy, one purchases a put option whose strike corresponds to a return of $K_{p,i}$ and sells a corresponding call option whose strike corresponds to a return of $K_{c,i}$, for product i on the risky asset such that the net cost of the strategy is zero. Let $c_t(r_f, K_{c,i}, \sigma_{IV})$ and $p_t(r_f, K_{p,i}, \sigma_{IV})$ denote the prices for the call and put options at time t which both expire in one year. σ_{IV} denotes the volatility used in option price evaluation and is calculated as $\sigma_{IV} = \sigma + VRP$, where VRP is the constant volatility risk premium to distinguish between implied and realized volatility. With a chosen lower bound $K_{p,i}$ for the risky asset's annual returns in product i, $K_{c,i}$ is the value that satisfies the equation

$$c_t(r_f, K_{c,i}, \sigma_{IV}) - p_t(r_f, K_{p,i}, \sigma_{IV}) = 0. \quad (2)$$

The $K_{c,i}(>K_{p,i})$ that solves equation (2) ensures the option trading strategy is self-financing. Both c_t and p_t are calculated using the Black–Scholes model (Black and Scholes, 1973). Then, the retiree's bounded risky asset return for product i from time t to time $t+1$, denoted $r_{r,t,i}$, is $r_{r,t,i} = \hat{r}_{r,t} + \max(K_{p,i} - \hat{r}_{r,t}, 0) - \max(\hat{r}_{r,t} - K_{c,i}, 0)$. The choice of $K_{p,i}$ (and AIR_i, see Drawdowns below) may allow the retiree to construct a structure similar to that of a guaranteed minimum withdrawal benefit (GMWB) or other similar riders available in the marketplace. Given the above, the investment return for product i from time t to time $t+1$, denoted as $r_{t,i}$, is

$$r_{t,i} = \pi_{r,i} r_{r,t,i} + (1 - \pi_{r,i}) r_f - f_i, \qquad \pi_{r,i} \in [0, 1]$$

where $\pi_{r,i}$ is the proportion of wealth in product i invested in risky assets and f_i is the annual fee for product i. The asset allocation in each product i is rebalanced at every time t to maintain the proportion $\pi_{r,i}$ across time.

Drawdowns from each product i at time t, $D_{t,i}$, take place at the beginning of each year. Investment income is earned over each period after drawdowns occur and is credited to account balances at the end of each period. Thus, the investment income earned by product i from time t to time $t+1$ is given by

$$I_{t,i} = (A_{t,i} - D_{t,i}) \times r_{t,i}.$$

1.1.2 Mortality credits

We use the Cairns–Blake–Dowd (CBD) model proposed by Cairns *et al.* (2006), and mortality data from the United States, sourced from the Human Mortality Database (2023), to project mortality rates for the purposes of calculating deterministic and stochastic mortality credits. We assume that the modeled individual is healthy at age 66 and experiences lighter mortality rates than that of the general population. Hence, mortality rates compared to the general population are reduced by 48 percent at age 66, 46 percent at age 67,…, 2 percent at age 89, and 0 percent from age 90 onwards (see Graph 9 of Institute of Actuaries of Australia, 2008). By extension, we also assume that mortality rates of those purchasing *DM* and *SM* products, and by extension *DM* and *SM* mortality credits, also follow this structure. The CBD model is briefly described below.

For all times $0 \le t \le T$, the probability of an individual aged $x+t$ at time t dying before age $x+t+1$, denoted Q_{x+t}, is given by the logit function

$$\ln\left(\frac{Q_{x+t}}{1 - Q_{x+t}}\right) = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x})$$

where $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ is the average of the ages used to fit the CBD model, and n is the number of distinct ages used to fit the CBD model. $\vec{\kappa_t} = (\kappa_t^{(1)}, \kappa_t^{(2)})^\intercal$ follows a bivariate random walk with drift, such that

$$\vec{\kappa_t} = \begin{pmatrix} \kappa_t^{(1)} \\ \kappa_t^{(2)} \end{pmatrix} = \begin{pmatrix} \kappa_{t-1}^{(1)} \\ \kappa_{t-1}^{(2)} \end{pmatrix} + \begin{pmatrix} \mu^{(1)} \\ \mu^{(2)} \end{pmatrix} + \begin{pmatrix} \epsilon_t^{(1)} \\ \epsilon_t^{(2)} \end{pmatrix}$$

where $(\mu_t^{(1)}, \mu_t^{(2)})^\intercal$ is the drift vector, $(\epsilon_t^{(1)}, \epsilon_t^{(2)})^\intercal \sim N(0, \Sigma)$, and Σ denotes the variance–covariance matrix for $\vec{\kappa_t}$. The equivalent expected mortality rate, q_{x+t}, is equal to $E(Q_{x+t})$, or equivalently, is found by removing the $(\epsilon_t^{(1)}, \epsilon_t^{(2)})^\intercal$ vector in projections.

Expected mortality rates q_{x+t} are synonymous with deterministic mortality credits, and are known with certainty at $t = 0$. Actual mortality rates Q_{x+t} are synonymous with stochastic mortality credits, and are unknown at $t = 0$. We assume that the investor pool for the SM product is sufficiently large that the idiosyncratic component of longevity risk is diversified away, and is hence not modeled. The return from time t to time $t + 1$ that determines the mortality credits earned by product i, denoted $\theta_{t,i}$, is

$$\theta_{t,PPA} = 0, \qquad \theta_{t,DM} = \frac{q_{x+t}}{1 - q_{x+t}}, \qquad \theta_{t,SM} = \frac{Q_{x+t}}{1 - Q_{x+t}}.$$

The mortality credits are earned and credited to account balances only after investment income has been credited to account balances, this gives

$$M_{t,i} = (A_{t,i} - D_{t,i} + I_{t,i}) \times \theta_{t,i}.$$

The balance for each product i in equation (1) can hence be rewritten as

$$A_{t+1,i} = (A_{t,i} - D_{t,i})(1 + r_{t,i})(1 + \theta_{t,i}).$$

1.1.3 Drawdowns

Drawdowns are made from the DM and SM products following an annuitization-based drawdown strategy. We also impose the same strategy on the PPA for comparison purposes.

Let tpx denote the probability of an individual at exact age x surviving to exact age $x + t$, then

$$tp_x = \prod_{s=0}^{t-1} (1 - q_{x+s})$$

where $0p_x = 1$. We denote the assumed interest rate (AIR) for product i as AIR_i. v_i is defined as the one-period discount factor corresponding to AIR_i, which is

$$v_i = (1 + AIR_i)^{-1}.$$

Let $\ddot{a}_{x+t,i}$ denote the expected present value of an immediate annuity paying \$1 annually to a retiree currently aged exactly $x + t$, with an AIR of AIR_i. The value of this annuity is

$$\ddot{a}_{x+t,i} = \sum_{s=0}^{T-t} v_i^s sp_{x+t}.$$

The drawdown at time t from product i is given by

$$D_{t,i} = \frac{A_{t,i}}{\ddot{a}_{x+t,i}}. \tag{3}$$

Equation (3) implies that $D_{T,i} = A_{T,i}$ for all products i. We note that, given a mortality schedule of an individual (or group), the AIR_i completely characterizes the drawdown schedule of product i.

The annuity factors $\ddot{a}_{x+t,i}$ are assumed to always be based on the expected mortality rates as at $t = 0$. Therefore, the annuity factors do not allow for systematic mortality changes. This means for the SM product, all volatility in the mortality credits is assumed to be driven by the uncertainty of Q_{x+t}, and not by repricing the annuity factors at each time t.

1.2 Pension income

Pension income is assumed to be received annually at the same time as drawdowns are made. In the United States context, pension income received is dependent on an individual's primary insurance amount, which is the amount of pension an individual receives if they choose to access pension income at their FRA. Additionally, individuals are rewarded for delaying receipt of their pension beyond their FRA.

Assuming an FRA of x, the benefit of delayed receipt of pension income terminates after age 70. Therefore, letting x_p denote the age at which the retiree begins receiving pension income, we have $x_p \in \{x, \ldots, 70\}$. Let P_{t,x_p} denote the pension income received by the retiree at time t, given they begin accessing pension income at age x_p. We find

$$P_{t,x_p} = \begin{cases} 0 & t < x_p - x \\ P_{t,x} \times (1 + 0.08 \times \min(x_p - x, 70 - x)) & t \geq x_p - x, \end{cases}$$

where $P_{t,x}$ is the pension received at FRA, which is x. Note, we assume that the individual is currently aged x and has not yet started receiving any pension income.

1.3 Utility structure

At each time t, the retiree's drawdown from each product alongside any pension income received gives total consumption C_t as

$$C_t = D_{t,ABP} + D_{t,DM} + D_{t,SM} + P_{t,x_p}.$$

We use constant relative risk aversion (CRRA) preferences to describe the retiree's preferences over consumption (see, e.g., Koo *et al.*, 2022). Under CRRA preferences, the retiree's utility from consuming C_t at time t, denoted $U_t(C_t)$, is

$$U_t(C_t) = \frac{C_t^{1-\rho}}{1-\rho}$$

where ρ describes the retiree's level of risk aversion. A higher ρ indicates a higher level of risk aversion. No bequest motive is assumed.

1.4 Optimization procedure

Let δ denote the retiree's constant annual inter-temporal discount factor. Then, the retiree's lifetime utility, denoted V, is given by

$$V = \sum_{t=0}^{T} \delta^t \times tp_x \times U_t(C_t).$$

We assume the mortality component of utility is again always based on the expected mortality rates as at $t = 0$. We calculate the retiree's lifetime utility over 10,000 Monte Carlo simulations (Weinert and

Gründl, 2021) over multiple sets (see Analysis structure below) that grant the retiree varying levels of flexibility in their decision making. Let V_n denote the retiree's lifetime utility for the n^{th} Monte Carlo simulation, then the objective function to optimize is

$$V^{\star} = \max_{\omega_i \in \Omega, >\pi_{r,i} \in \Pi, gtAIR_i \in AIR, >K_{p,i} \in K, x_p \in X_p} \left(\frac{\sum_{n=1}^{10000} V_n}{10000} \right) \tag{4}$$

for $i \in \{PPA, DM, SM\}$, subject to the following constraints

$$\begin{aligned} \Omega &= \{(\omega_{PPA}, \omega_{DM}, \omega_{SM}) | 0 \leq \omega_{PPA}, \omega_{DM}, \omega_{SM} \leq 1, \omega_{PPA} + \omega_{DM} + \omega_{SM} = 1\} \\ \Pi &= \{(\pi_{r,PPA}, \pi_{r,DM}, \pi_{r,SM}) | 0 \leq \pi_{r,PPA}, \pi_{r,DM}, \pi_{r,SM} \leq 1\} \\ AIR &= \{(AIR_{PPA}, AIR_{DM}, AIR_{SM}) | AIR_{PPA}, AIR_{DM}, AIR_{SM} \geq 1\} \\ K &= \{(K_{p,PPA}, K_{p,DM}, K_{p,SM}) | -1 \leq K_{p,PPA}, K_{p,DM}, K_{p,SM} \leq r_f\} \\ X_p &= \{66, 67, 68, 69, 70\} \end{aligned}$$

where $V^{\star}$ is the maximized expected lifetime utility for a given set.

1.5 Comparison metric

We calculate a certainty equivalent consumption (CEC) for the retiree's maximized expected lifetime utility, $V^{\star}$, under each set as

$$V^{\star} = \sum_{t=0}^{T} \delta^t \times tp_x \times \left(\frac{CEC^{1-\rho}}{1-\rho} \right).$$

The CEC reflects the constant level of consumption in retirement which gives the retiree a utility equal to their maximized expected lifetime utility. The changes in CEC across different sets indicate the value provided to the retiree by varying levels of flexibility in building blocks available.

1.6 Analysis structure

The pension building block and choice of age X_p is assumed to be available in all sets. In order to align building blocks with product availability, we treat the mortality credit building block (PPA, DM, and SM) as being the product spaces. The investment strategy and drawdown schedule building blocks are called decision sets. The sets over which optimization takes place make up the intersection of the decision sets and product spaces. Table 1 describes the five decision sets considered which grant the retiree different levels of flexibility in decision making.

Decision set 1 provides a baseline for comparison. Set 2 shows the isolated impact of allowing choice of risky asset allocation. Set 3 shows the isolated impact of allowing choice of drawdown schedule. Set 4 shows the interactive impact of sets 2 and 3. Set 5 shows the impact of using options to bound the risky asset return. Within each decision set, there are three product subsets, as described in Table 2, which show the impact of allowing different mortality credit structures.

The combination of a decision set and product subset produces what we call a 'set'. In each decision set above, the vectors Ω, Π, K, and AIR are adjusted as necessary to reflect the available decision variables within that set. For example, in set 5.2, the SM product is unavailable but all investment strategy and drawdown schedule building blocks can be freely chosen, so Ω represents $(\omega_{PPA}, \omega_{DM})$, Π represents $(\pi_{r,PPA}, \pi_{r,DM})$, AIR represents (AIR_{PPA}, AIR_{DM}), and K represents $(K_{p,PPA}, K_{p,DM})$.

In some sets, some optimization variables may not be freely chosen. For example, in set 2.2, the allocation to risky assets in both PPA and DM may be chosen, but options are not available and

Table 1. Decision sets

Decision set	Optimization variables
1	Ω, X_p
2	Ω, X_p, Π
3	Ω, X_p, AIR
4	Ω, X_p, Π, AIR
5	Ω, X_p, Π, AIR, K

Notes: The optimization variables are defined in equation (4). The table provides the optimizations variables that are analyzed in each of the decision sets.

the drawdown schedule is fixed. In the Initialization procedure section below, we provide a discussion of the default value for variables that are not freely chosen.

1.7 Parameter calibration

Table 3 states the parameter values used along with their references. The parameters without stated references are assumed.

We assume $T = 38$ because sufficient mortality data are not available for ages beyond 104.

The DM product's annual fee f_{DM} captures the product provider's requirement to hold capital against longevity risk. The fee of $f_{DM} = 0.5$ percent per annum, combined with the difference between population and the modeled individual's mortality rates, is equivalent to an approximately 11 percent initial loading, which is broadly consistent with the findings of Ganegoda and Bateman (2008). Neither the PPA nor the SM product requires capital to be held against longevity risk. Furthermore, other fees common across the three products are assumed to be zero. As described by Ai *et al.* (2023), we make this assumption because only the relative value of the fees matters for comparison purposes.

The use of $P_{t,66} = \$\ 21,552$ is consistent with the monthly pension data provided by the Social Security Administration (2023a), which is first rounded down to the nearest dollar (Social Security Administration, 2003).

1.8 Initialization procedure

This section outlines the initialization procedure used before optimization to establish the value of W_0. For all sets except set 5.3, some of the variables in equation (4) are set to default values rather than optimized. Using the superscript *def* to denote a default variable, the structure is as follows.

$$\begin{aligned} \pi_{r,i}^{def} &= 0.5 \\ AIR_i^{def} &= \pi_{r,i}\mu + (1 - \pi_{r,i})r_f - f_i - \frac{1}{2}(\pi_{r,i}\sigma)^2 \\ K_{p,i}^{def} &= -1. \end{aligned}$$

The use of $\pi_{r,i}^{def} = 0.5$ as the default investment strategy is appropriate for our chosen $\rho = 4$, following Khemka *et al.* (2021) and is similar to the Vanguard Target Retirement Fund in the

Table 2. Product subsets for optimization

Product subset	PPA product	DM product	SM product
*.1	✓		
*.2	✓	✓	
*.3	✓	✓	✓

Table 3. Parameter values

Parameter	Description	Value	Reference
x	Age at retirement	66	(Social Security Administration, 2023b)
T	Maximum years lived in retirement	38	See Mortality credits discussion
ρ	CRRA risk-aversion	4	(Khemka *et al.*, 2021)
δ	Inter-temporal discount factor	0.977	(Kudrna *et al.*, 2022)
μ	Expected risky asset return	0.065	(Dimson *et al.*, 2023)
σ	Risky asset volatility	0.174	(Dimson *et al.*, 2023)
r_f	Risk-free rate	0.015	(Dimson *et al.*, 2023)
VRP	Volatility risk premium	0.0403	(Gibson, 2024)
f_{PPA}	PPA fee	0	
f_{DM}	DM fee	0.005	(Ganegoda and Bateman, 2008)
f_{SM}	SM fee	0	
$P_{t,66}$	Pension at FRA	\$21,552	(Social Security Administration, 2023a)
CEC_0	Initial CRRA certainty equivalent consumption	\$50,000	(Guzman and Kollar, 2023); (Whitehouse and Queisser, 2007)
W_0	Initial wealth	\$617,762	See Initialization procedure discussion

United States (Morningstar, 2018). The form of AIR_i^{def} matches the geometric average return of the default investment strategy (less any fees), given it reflects the annualized average return in a deterministic projection (McCulloch, 2003). This gives an expected flat drawdown profile for a chosen investment strategy. The choice of $K_{p,i}^{def} = -1$ ensures that options have no impact on returns.

We set W_0 to be that required for the retiree to achieve a $CEC = \$\ 50{,}000$ for set 1.1. In set 1.1, the only decision variable for the retiree is age to access pension, x_p, and so in addition to above, we impose $\omega_{PPA}^{def} = 1$. The value of \$50,000 is chosen as an approximation of applying a 70 percent replacement rate (Whitehouse and Queisser, 2007) to the median income for males with no spouse in the United States, which was \$73,630 in 2022 (Guzman and Kollar, 2023). The retiree's objective function in set 1.1 is therefore

$$\max_{x_p \in \{66,67,68,69,70\}} \left(\frac{\sum_{n=1}^{10000} V_n}{10000} \,|\, \omega_{PPA}^{def}, \pi_{r,PPA}^{def}, AIR_{PPA}^{def}, K_{p,i}^{def} \right). \tag{5}$$

The initialization procedure yields $W_0 = \$625{,}912$ with the retiree choosing to optimally begin accessing pension income at age $x_p = 68$.

2. Results

The optimization results are presented in Table 4. Following this, the analysis of results is partitioned based on the level of flexibility provided to the retiree in their decision making, where we analyze the results for sets 1–5 respectively. For some sets, we use graphical illustrations of the median consumption profiles, attributing consumption to its component building blocks. The components are pension, capital drawdowns from the initial wealth (account balance), drawdowns from investment income (net of fees) earned since retirement, and drawdowns from mortality credits earned since retirement. Details on the construction of the graphs are provided in the Appendix, including all plots for all sets. We conclude this section with a brief discussion of the results.[1]

[1]In these plots, allocations to products <1 percent of wealth are excluded. Noting that such allocations are always accompanied with an extremely negative AIR, this effectively creates a deferred annuity that finances consumption materially only in the last years of retirement. Graphing such results created consumption plots whose scales were significantly impacted by consumption at time $T = 38$.

Table 4. Fixed pension optimization results

Set	ω_{PPA}	ω_{DM}	ω_{SM}	$\pi_{r,PPA}$	$\pi_{r,DM}$	$\pi_{r,SM}$	$K_{p,PPA}$	$K_{p,DM}$	$K_{p,SM}$	AIR_{PPA}	AIR_{DM}	AIR_{SM}	x_p	%ΔCEC from set 1.1	%ΔCEC from previous set
Decision set 1 – no decision variables															
1.1	*100.0%*			*50.0%*						*3.6%*			68		
1.2	0.0%	100.0%		–	*50.0%*					–	*3.1%*		66	13.6%	13.6%
1.3	0.0%	0.0%	100.0%	–	–	*50.0%*				–	–	*3.6%*	66	16.6%	2.6%
Decision set 2 – access to investment strategy															
2.1	*100.0%*			94.9%						*4.9%*			69	3.1%	
2.2	0.0%	100.0%		–	68.1%					–	*3.7%*		66	14.5%	11.0%
2.3	0.0%	0.0%	100.0%	–	–	66.2%				–	–	*4.1%*	66	17.3%	2.4%
Decision set 3 – access to drawdown schedule															
3.1	*100.0%*			*50.0%*						6.3%			70	1.9%	
3.2	11.9%	88.1%		*50.0%*	*50.0%*					74.9%	2.3%		68	14.8%	12.7%
3.3	0.2%	11.8%	88.0%	*50.0%*	*50.0%*	*50.0%*				78.1%	75.8%	2.5%	68	17.8%	2.6%
Decision set 4 – access to both investment strategy and drawdown schedule															
4.1	*100.0%*			89.3%						6.9%			70	4.1%	
4.2	21.4%	78.6%		4.4%	85.0%					40.2%	2.5%		69	16.5%	11.9%
4.3	11.2%	0.8%	88.0%	0.7%	5.8%	78.1%				75.2%	76.7%	2.8%	68	19.4%	2.5%
Decision set 5 – access to options in the investment strategy and including drawdown schedule															
5.1	*100.0%*			100.0%			−16.9%			6.8%			70	6.3%	
5.2	12.1%	87.9%		100.0%	100.0%		−5.6%	−15.2%		75.5%	3.0%		68	19.8%	12.7%
5.3	11.4%	0.3%	88.3%	84.4%	99.3%	100.0%	−5.9%	−16.7%	−15.1%	82.3%	−30.8%	3.4%	68	22.9%	2.6%

Notes: The description of the decision sets and product spaces are provided in Tables 1 and 2, respectively. The column headings represent the optimization variable as discussed in equation (4). CEC is described in the comparison metric discussion. The dark-shaded blocks represent decisions that were imposed at default values in a set. Cells with '–' represent decisions that were not used as the allocation to the product was 0%.

2.1 Set 1: no decision variables

In set 1.1, the retiree only has access to a PPA which does not provide any mortality credits, and the investment and drawdown strategies are set to their defaults. They delay pension receipt by two years to age $x_p = 68$ which increases their pension income to $P_{t,68} = \$24{,}996$ from $P_{t,66} = \$21{,}552$. Since the retiree has a fixed drawdown schedule, this reduces their consumption in the first two years of retirement, which the retiree is willing to endure for a higher consumption safety net.

With access to a DM product in set 1.2, the retiree allocates $\omega_{DM} = 100$ percent of their wealth to the DM product, consequently increasing their *CEC* by 13.6 percent relative to set 1.1. This conveys the value provided by deterministic mortality credits to the retiree, despite the annual fee of $f_{DM} = 0.5$ percent. This gain from annuitization is lower than the 22 percent quoted by Steffensen and Søe (2023), as the presence of the pension income provides some effective annuitization even in set 1.1. The retiree also opts to receive pension income from age 66, since the DM product offers the retiree longevity insurance, thereby reducing the incentive to delay pension income receipt to obtain a higher pension. In set 1.1, the retiree faces a trade-off between adequate longevity insurance and initial consumption. This trade-off is dampened in set 1.2 as deterministic mortality credits are available to the retiree.

In set 1.3, the retiree allocates $\omega_{SM} = 100$ percent of their wealth to the SM product, leading to a further 2.6 percent increase in *CEC* from set 1.2. This improvement is attributable to the lack of fees in the SM product though the effect is dampened due to the uncertain nature of its mortality credits in the SM product. The pension access age remains at 66, and we note that the retiree is willing to absorb the volatility of the SM product's payouts in exchange for its 'free' mortality credits.

Figure 2 shows the attributed median consumption profiles for set 1. The different subsets show the overall shape of the retiree's median consumption profile and its changes as different mortality credit structures are accessible to the retiree.

Set 1.1 displays a decreasing consumption profile, attributable to the lack of mortality credits provided by the PPA, which incentivizes the retiree to delay pension income receipt until age $x_p = 68$ in order to achieve a sufficiently high level of consumption in later years. Consumption declines because the PPA's lack of mortality credits means that drawing down in accordance with its geometric mean return is not sustainable.

In set 1.2, the consumption level is flat indicating the stable payouts from the pension and (annuity-like) DM product. Here, the shape of the drawdowns from account balance and investment returns is identical to the PPA product, but the stability in later years is achieved through the mortality credits available to the retiree. In set 1.3, the pattern is similar to set 1.2, but at a slightly higher level due to the lack of fees. This higher level is reflected in the higher 'investment income' component, as the DM fee is included in the net investment return.

2.2 Set 2: access to investment strategy

In set 2, we see the improvements in the retiree's lifetime expected utility when they can set the risky asset allocation for each product. The shapes of the median consumption profiles are similar to those presented in Figure 2 and are presented in the Appendix.

In set 2.1, we see that the retiree chooses a risky asset allocation of $\pi_{r,PPA} = 94.9$ percent in their PPA. This improves their *CEC* by 3.1 percent from set 1.1 where the risky asset allocation was fixed at $\pi_{r,PPA}^{def} = 50$ percent. This higher risky asset allocation is driven by the retiree's receipt of fixed pension, which incentivizes them to increase their exposure to risky assets in hopes of higher returns, knowing they have a consumption safety net. This strategy also leads to an increase in the default AIR_{PPA} to 4.9 percent meaning their initial drawdowns from the PPA are higher relative to set 1.1. Therefore, the retiree is willing to delay pension receipt even more than in set 1.1, to age $x_p = 69$.

In set 2.2, the retiree allocates $\omega_{DM} = 100$ percent of their wealth to the DM product, with an increase in *CEC* of 11 percent from set 2.1. The increase is a modest 0.8 percent compared to set 1.2, indicating relatively little improvement from choosing investment strategy. This is despite the

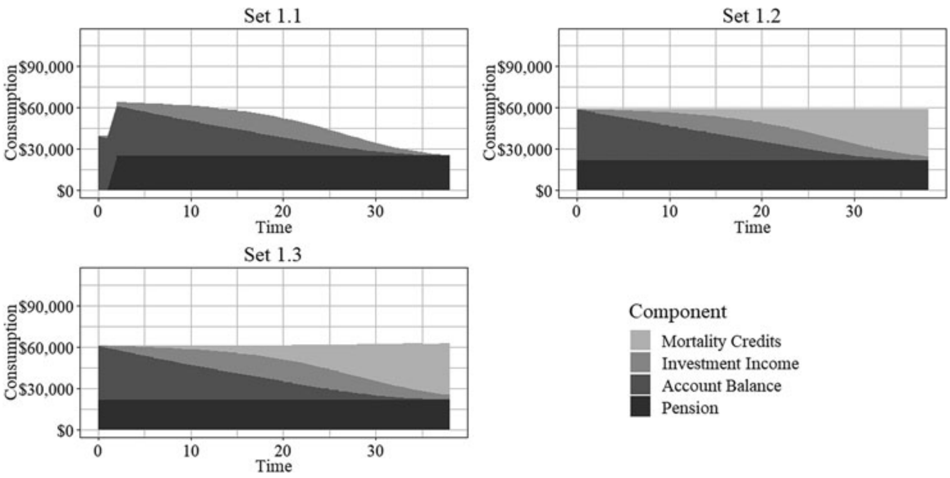

Figure 2. Median consumption profiles under decision set 1.
Notes: The description of the decision sets (sets) and product spaces (subsets) are provided in Tables 1 and 2, respectively.

change in risky asset allocation from 50 to 68 percent, indicating that even moderate changes in risky asset allocation away from 50 percent have minimal impact on outcomes, whereas the use of the DM product has a much more substantial impact. Furthermore, the risky asset allocation for the DM product is much lower than for the PPA. This is because the addition of mortality credits reduces the relative importance of the pension safety net over the whole of retirement, thus necessitating a more conservative strategy in the DM product.

Set 2.3 results largely mirror the findings from sets 1.3 and 2.2 on the impact of SM and choice of risky asset allocation.

2.3 Set 3: access to drawdown schedule

Set 3 allows the retiree to choose an *AIR*, that is, drawdown schedule, but the investment strategy is set at the default as per set 1. The median consumption profiles for this set are presented in Figure 3.

In set 3.1, we see an increase in *CEC* of 1.9 percent compared to set 1.1., driven by the retiree choosing to increase AIR_{PPA} to 6.3 percent. The consumption profile is similar to Butt *et al.* (2022), reflecting that a model with a single *AIR* decision variable on drawdown can achieve similar results to a model using dynamic programming. The retiree also chooses to access the maximum pension possible of $P_{t,70}$ = \$28,452 by drawing from age $x_p = 70$. The retiree essentially draws down on PPA faster in the earlier years to delay and increase pension receipt. From the top-left panel of Figure 3 we see that the shape is similar to the corresponding panel of Figure 2 but at a higher level of consumption throughout. We note that for the PPA account, access to investment strategy is of more benefit than drawdown schedule, in isolation.

Unlike sets 1.2 and 2.2, in set 3.2, the retiree maintains an allocation to the PPA of $\omega_{PPA} = 11.9$ percent and delays pension receipt to age $x_p = 68$. The retiree funds consumption in the early years from the PPA as reflected through $AIR_{PPA} = 74.9$ percent. This leads to a 12.7 percent increase in *CEC* above set 3.1, which also means set 3.2 provides slightly greater value beyond set 1.1 relative to sets 1.2 and 2.2. In other words, choice of drawdown strategy provides less value than choice of risky asset allocation to the retiree when only a PPA is available, but more value when DM is available as well. From the top-right panel of Figure 3 we can see that the high *AIR* of the PPA account leads to a dip in consumption at age 67 as very little funds are left in the PPA account by the second year, but this is more than compensated with pension receipt at age 68. Further, the low $AIR_{DM} = 2.3$ percent leads to an increasing overall consumption profile as the retiree ages.

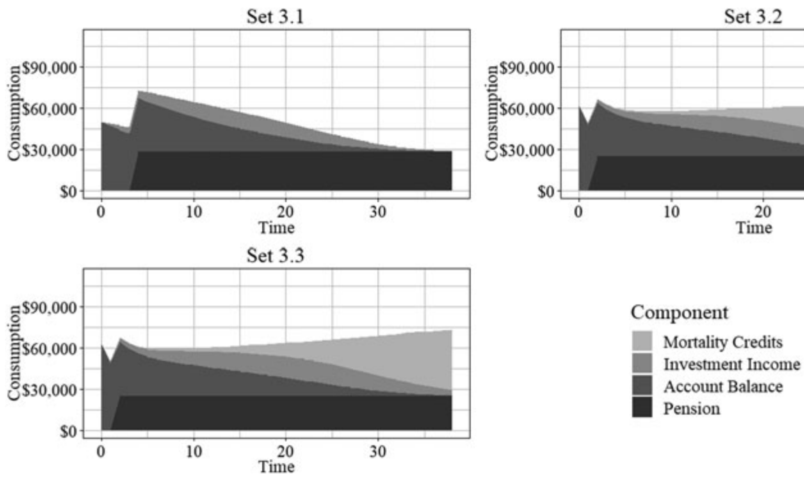

Figure 3. Median consumption profiles under decision set 3.
Notes: The description of the decision sets (sets) and product spaces (subsets) are provided in Tables 1 and 2, respectively.

In set 3.3 interestingly, the retiree allocates very little to the PPA product (0.2%), with allocations of 11.8 and 88.1 percent to the DM and SM products, respectively. The findings largely mirror sets 1.3 and 3.2 on the impact of SM and choice of AIR. The AIR_{SM} = 2.5 percent is higher than the AIR_{DM} = 2.3 percent in set 3.2, although the difference is smaller than the DM fee f_{DM} = 0.5 percent and so the bottom-left panel of Figure 3 shows a stronger increasing consumption profile than set 3.2. The high AIR for both the PPA (78.1%) and DM (75.8%) products indicate that they are largely used to fund initial consumption when the pension is absent with the SM product and pension providing consumption from age 68 onwards.

2.4 Set 4: access to both investment strategy and drawdown schedule

In set 4, the retiree can choose both investment strategy and drawdown schedule. The shapes of the median consumption profiles are similar to those presented in Figure 3 and are presented in the Appendix.

Rather than commenting on each set individually, we note that the retirees in set 4 utilize a combination of the strategies observed in sets 2 and 3. In all cases, the improvement in *CEC* is greater than the equivalents in sets 2 and 3, as expected.

We are now also able to comment on the relative impact of the various building blocks. Clearly, the addition of mortality credits has the biggest individual impact on *CEC*, with increases of 13.6 and 16.6 percent for *DM* and *SM* in sets 1.2 and 1.3, respectively; whereas the individual addition of investment strategy and drawdown schedule decisions leads to *CEC* improvements of only 3.1 and 1.9 percent, in sets 2.1 and 3.1, respectively. Collectively, the impact of investment strategy and drawdown schedule improves *CEC* by only 4.1 percent in set 4.1. This relative impact is despite the presence of the pension, which provides some longevity protection on its own. Including investment strategy and drawdown schedule decisions in addition to access to *SM* improves *CEC* by 19.4 percent in set 4.3, compared to 16.6 percent in set 1.3 where investment strategy and drawdown schedules are imposed at the default. This represents only a 1.0 percent improvement in CEC when allowing a choice in investment strategy and drawdown schedules when SM is available.

2.5 Set 5: access to options in the investment strategy and including drawdown schedule

In set 5, the retiree has access to options in the investment strategy, in addition to the choice of risky asset allocation and drawdown schedule. The shapes of the median consumption profiles are similar to those presented in Figure 3 and are presented in the Appendix. We note that in the presence of options, the retiree has the choice of bounding the risky returns through the use of the collar strategy. By adopting the collar strategy, the retiree manipulates the risky asset's return distribution and increases its risk-adjusted return, thereby making it a more attractive investment asset. This implies an increase in the risky asset allocation in set 5.

When the retiree only has access to PPA (set 5.1), they allocate 100 percent of the wealth to the risky asset (up from 89 percent in set 4.1) but truncate the return distribution to (–16.9, 27.0 percent) through the use of options. This truncation allows the retiree to minimize the impact of severely negative risky asset returns and increases *CEC* by 2.1 percent compared to set 4.1. This is a substantial outcome given that the *CEC* impact of choosing risky asset allocation in set 2.1 is 3.1 percent.

In set 5.2, the retiree's allocation across PPA and DM is almost identical to set 3.2 as is the pension access age of $x_p = 68$. However, the retiree allocates 100 percent of the wealth in both products to the risky asset. They truncate the PPA return distribution to (–5.6, 10.4 percent) to ensure funds to finance early consumption. The DM return distribution is much wider at (–15.2, 24.3 percent). The ability to choose investment strategy and an appropriate return range increases *CEC* by 4.4 percent relative to set 3.2 and by 2.8 percent relative to set 4.2. The choice of truncation for the DM return distribution indicates that it is not optimal for the retiree to seek a GMWB structure, as the lower bound of −15.2 percent provides ample opportunity for the income from the DM product to decrease between years, even accounting for the fact that we are modeling in real rather than nominal terms.

In set 5.3, the retiree has access to all products (including options), and to all decision variables. Decisions are broadly similar to those noted for set 5.2 but with lower allocation to the risky asset for PPA at 84.4 percent. Also, the small $\omega_{DM} = 0.3$ percent allocation to the DM structure, coupled with its $AIR_{DM} = -30.8$ percent, suggests it is mostly being used to increase consumption in the final few years of retirement (like a deferred annuity). The retiree's *CEC* increases by 2.9 percent above set 4.3, compared to the increase of 2.1 percent for set 5.1 above set 4.1 noted earlier, indicating that the use of options provides greater value when mortality credits are available than if they are not. Furthermore, the increase of 2.9 percent is greater than the increase of 1.0 percent observed when including both risky asset and drawdown decisions, indicating that the use of options improves outcomes more than both of these decisions.

2.6 Discussion

Here, we further discuss the optimization results presented earlier in this section, emphasizing the implications of our results. We also discuss some additional robustness tests performed and address the key limitations of our analysis.

Our analysis allows for an explicit identification of the value provided by each building block of retirement income to the retiree. This also delivers insights into the interactions between the building blocks, and how the retiree behaves in light of this. While mortality credits provide the most value, this result is also driven by the fact that all products had a default investment strategy balanced between risky and risk-free assets. That is, when mortality credits are offered to the retiree, in the most restrictive settings they came with a 50 percent allocation to risky assets. As stated previously, this default is based on the literature and defaults available in the marketplace. Had we instead defined the default investment strategy as being risk-free, then the value added by allowing risky investment (whilst still imposing the drawdown schedule) would be 14.5 percent with PPA only (i.e., set 2.1), and 11.8 percent with PPA, DM, and SM (i.e., set 2.3). This improvement is closer to, but still below the 16.6 percent improvement noted for set 1.3 for the use of SM in isolation.

In sets where the retiree is not able to select an *AIR* for the products, we see a clear dominance by the SM product, and the DM product (in sets where the SM product was not available). The dominance of the SM product stems from lower fees relative to the DM product. Upon further investigation, we find that the annual DM fee f_{SM} must be no more than 0.1 percent (compared to the selected value of 0.5 percent) for the DM product to be competitive with the SM product. This implies that the retiree's 'value' of the greater stability of the DM product is less than 0.1 percent per annum. Part of the reason for this low value is likely due to volatility in the SM product being overwhelmed by the larger scale of investment volatility. We note, however, the simplistic approach used in this paper to model the SM mortality credits, and leave for future research to incorporate idiosyncratic volatility and repricing of the SM product into the model (see, e.g., Qiao and Sherris, 2013).

The dominance of SM and DM products over the PPA holds partly due to the lack of a bequest motive in our analysis. We are assuming a retiree whose aim is to provide an income in retirement, ignoring any goals for inter-generational wealth transfer and/or contingent asset provision. Consideration of these motives in this setting will likely lead to different trade-offs and interactions and is left as an area for future research.

In sets where the *AIR* of products could be chosen, the retiree uses the PPA as an immediate consumption tool when only DM is available, and both PPA and DM when SM is also available. By doing so, the retiree is willing to delay pension income receipt, thereby increasing the pension they eventually receive. This highlights the interaction between the ability to choose the drawdown schedule of the PPA (and the DM), with the decision to delay receipt of pension income in order to receive a higher pension income. This result is observed despite the model limitation of static optimization at time $t = 0$ instead of a more comprehensive dynamic programming method. The inflexibility of the static optimization approach leads to large changes in median consumption in early retirement, as the use of the PPA (and the DM) transitions to the commencement of the pension. The static approach was chosen given the complexity of the problem addressed. By set 5.3, the retiree has 12 decision variables to consider. We hypothesize that the impact of the drawdown decision would be substantially greater if a dynamic model was possible. Further, a byproduct of the static structure is that optimal investment strategy decisions at time $t = 0$ can drift over time. For example, if the retiree implements different investment strategies for each product at time $t = 0$, even if the investment strategy is rebalanced each year within each product, their overall investment strategy will drift according to changes in the relative size of each account balance. Hence, we also hypothesize that the impact of the investment strategy decision would also be greater in a dynamic model.

This paper demonstrates that differences in optimal allocation to risky assets and use of options are significantly different across the products, which we can see as our model explicitly separates the investment strategy and longevity insurance decisions. The use of options does not replicate the riders widely available in the marketplace, instead seeking to avoid only the most severe market downturns, rather than providing a guarantee on returns. This result is dependent on the CRRA utility structure used in the analysis, we leave to future research to investigate the impact of alternative preference structures such as cumulative prospect theory (Tversky and Kahneman, 1992), habit formation (Pollak, 1976), and Epstein-Zin (Epstein and Zin, 1989).

We now turn to the implications of our work. Our results show that sensible default investment strategies and drawdown schedules can largely replicate optimal outcomes. Whilst the use of options can add some value, their relative complexity means they will likely be of interest to only a small cohort of retirees. Mortality credits are the most significant generator of added value in retirement; however, we note that in 2017, only 7.9 percent of retirees received any form of annuity income (Congressional Research Service, 2022). This 'annuity puzzle' has been widely researched (Ramsay and Oguledo, 2018); however, our research shows the importance of extending access to, and utilization of, products including mortality credits, particularly those that do not incorporate significant fees. Providers and policymakers should be cognizant of this outcome in seeking to improve the market penetration of these products. As part of this goal, we note that the fee charged for the DM product is transparent in our model, whilst in practice transparency of fees for these products is often problematic and can distort the true value of the product.

In addition to those described earlier in this section, other simplifications were made in the analysis, and could be relaxed for future research. This could include analysis of couples in addition to singles. Tax and minimum drawdown rules could also be incorporated. Additionally, no allowances were made for increasing healthcare needs, or the need for aged care, as the retiree ages. These can be incorporated via utility functions that incorporate a non-constant consumption target (which can be incorporated under prospect theory). Housing wealth, and the ability to use a reverse mortgage to further increase consumption, was also not considered. Including these will likely require consideration of bequest motives, as discussed earlier. Further, these alternatives may lend themselves to alternative defaults to those used in this research.

3. Conclusion

We explore the retirement income problem from the perspective of four interacting building blocks. These four building blocks are pension income, mortality credits, the investment strategy, and the drawdown schedule. The modeled retiree is assigned the task of maximizing their expected lifetime utility by considering the decision variables they have available at the time of retirement, which pertain to the aforementioned building blocks.

Over the course of many sets of decision variables, the flexibility of the retirement income problem increases, thereby reducing its parsimony. This is done by offering the retiree more mortality credit products to invest in, and more accompanying investment and drawdown decisions to make. The retiree's utility under different decision variable sets is then compared using a CEC for each set. We consider the problem primarily under the United States fixed pension system.[2] Utilizing this breakdown of the retirement income problem, we achieve an understanding of the dynamics and interactions between these building blocks, and what trade-offs the retiree faces. The main contribution of this paper to existing literature is its unique approach to the retirement income problem from the perspective of these four building blocks, which allows for a quantification of the value provided by each. For our model retiree, we find that utilization of mortality credits provides the largest increase in welfare of the availability building blocks and that, given sensible defaults, investment strategy and drawdown schedule offer far less scope for welfare improvement.

We acknowledge that the retirement income problem is not simple. For most retirees, the simplest, and subsequently default option, is keeping wealth in a PPA and making drawdowns as necessary. However, this reduces the overall income delivered during retirement. Through its unique representation of the retirement income problem, this paper shows that spreading wealth across products, some of which offer longevity insurance via mortality credits, can greatly improve retirement outcomes. The approach used in this paper can be utilized across other retiree characteristics and preferences, and can be utilized by providers and policymakers to assist in the retirement planning problem across cohorts.

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Competing interests. None.

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[2]The authors also perform sensitivity analyses for a means-tested pension and no pension. These are available upon request.

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Appendix

Graphical results

This appendix describes the graphical representation of the retiree's median consumption profile, along with presenting the plot for each set. Note that in all graphs, allocations to products <1 percent of wealth were excluded (see footnote 1). The full set of figures is provided in Figure A1.

Table A1 defines the necessary notation to explain the underlying mathematical formulation of the graphs.

For product $i \in \{DM, SM\}$, at every t we partition its account balance into three components: the underlying account balance ($A_{UAB,t,i}$), the investment income balance ($A_{IIB,t,i}$), and the mortality credits balance ($A_{MCB,t,i}$). Then, the values of $A_{UAB,t+1,i}$, $A_{IIB,t+1,i}$, and $A_{MCB,t+1,i}$ for $t \geq 0$ are found using

$$A_{UAB,t+1,i} = \text{med}(A_{t,i}) - \frac{\text{med}(A_{t,i})}{\ddot{a}_{x+t,i}} \tag{A1}$$

$$A_{IIB,t+1,i} = \left(\text{med}(A_{t,i}) - \frac{\text{med}(A_{t,i})}{\ddot{a}_{x+t,i}}\right) \times \text{geo}(r_{t,i}) \tag{A2}$$

Table A1. Graph generation notation used in the appendix

Term	Definition
$\text{geo}(r_{t,i})$	Geometric average investment return for product *i* from time *t* to *t* + 1 across all 10,000 simulations
$\text{med}(A_{t,i})$	Median account balance for product *i* at time *t* across all 10,000 simulations
$\text{med}(C_t)$	Median total consumption at time *t* across all 10,000 simulations
$A_{UAB,t,i}$	Underlying account balance for product *i* at time *t*
$A_{IIB,t}, i$	Investment income balance for product *i* at time *t*
$A_{MCB,t,i}$	Mortality credits balance for product *i* at time *t*
$D'_{UAB,t,i}$	Amount of drawdown from product *i* at time *t* attributable to underlying account balance
$D'_{IIB,t,i}$	Amount of drawdown from product *i* at time *t* attributable to investment income balance
$D'_{MCB,t,i}$	Amount of drawdown from product *i* at time *t* attributable to mortality credits balance

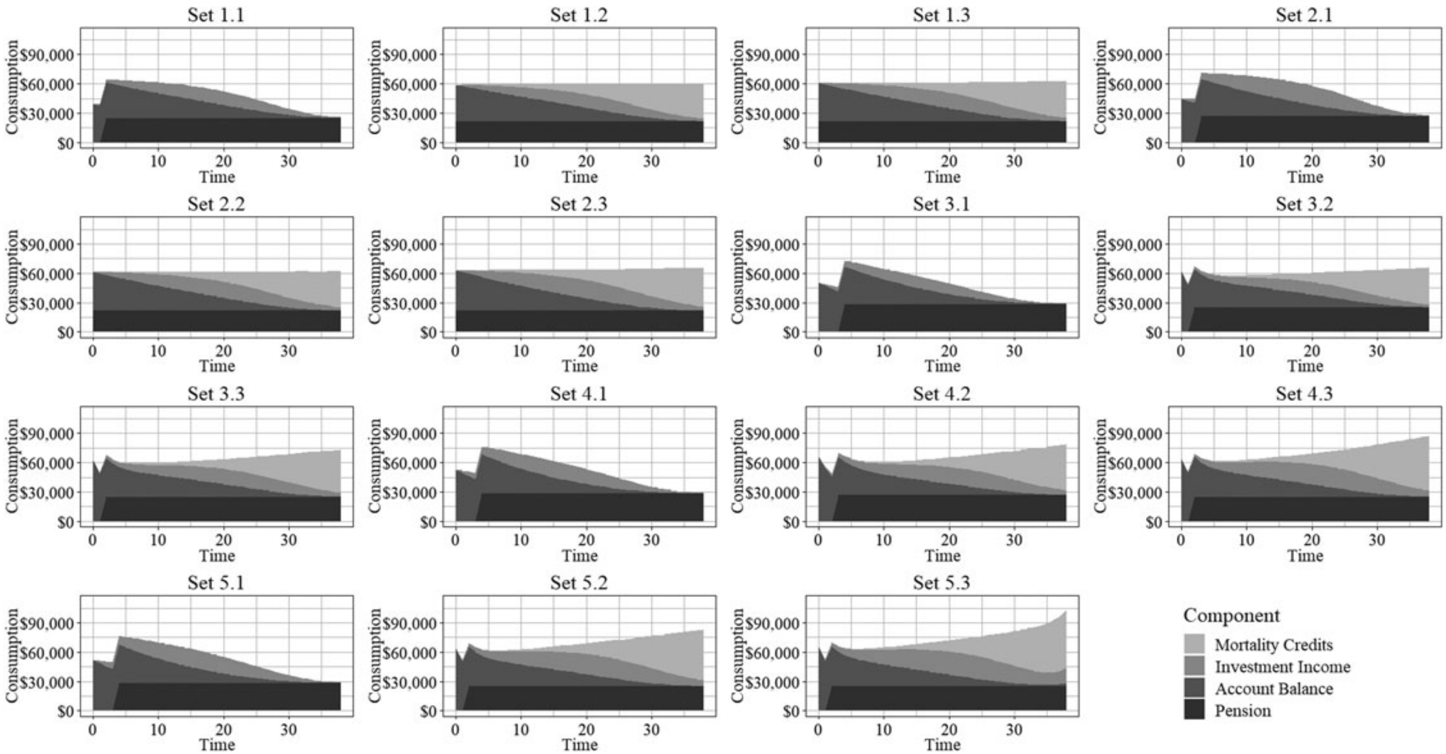

Figure A1. All median consumption profiles.
Notes: The description of the decision sets (sets) and product spaces (subsets) are provided in Tables 1 and 2, respectively.

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$$A_{MCB,t+1,i} = \left(\text{med}(A_{t,i}) - \frac{\text{med}(A_{t,i})}{\ddot{a}_{x+t,i}}\right)(1+\text{geo}(r_{t,i}))\left(\frac{q_{x+t}}{1-q_{x+t}}\right) \quad \text{(A3)}$$

where $A_{MCB,T,i} = 0$. The use of q_{x+t} rather than Q_{x+t} for the SM product is because the median stochastic mortality rates are equal to the deterministic mortality rates.

For product $i \in \{DM, SM\}$, at each time t, the component of its drawdown $(\text{med}(A_{t,i})/\ddot{a}_{x+t,i})$ attributable to $D'_{UAB,t,i}$, $D'_{IIB,t,i}$, and $D'_{MCB,t,i}$ is given by

$$D'_{UAB,t,i} = \left(\frac{A_{UAB,t,i}}{\ddot{a}_{x+t,i}}\right)\left(\frac{\text{med}(A_{t,i})}{A_{UAB,t,i}+A_{IIB,t,i}+A_{MCB,t,i}}\right) \quad \text{(A4)}$$

$$D'_{IIB,t,i} = \left(\frac{A_{IIB,t,i}}{\ddot{a}_{x+t,i}}\right)\left(\frac{\text{med}(A_{t,i})}{A_{UAB,t,i}+A_{IIB,t,i}+A_{MCB,t,i}}\right) \quad \text{(A5)}$$

$$D'_{MCB,t,i} = \left(\frac{A_{MCB,t,i}}{\ddot{a}_{x+t,i}}\right)\left(\frac{\text{med}(A_{t,i})}{A_{UAB,t,i}+A_{IIB,t,i}+A_{MCB,t,i}}\right) \quad \text{(A6)}$$

respectively. The expressions above involve a scaling of $(\text{med}(A_{t,i}))/(A_{UAB,t,i}+A_{IIB,t,i}+A_{MCB,t,i})$ to ensure consistency between the account balances used for graph generation and the median account balances obtained after optimization.

Once the terms above have been found for the DM and SM product, and noting that for $i \in \{DM, SM\}$ that $D'_{t,i} = D'_{UAB,t,i} + D'_{IIB,t,i} + D'_{MCB,t,i}$, the remaining consumption required to attain the median consumption level (i.e., $\text{med}(C_t) - D'_{t,DM} - D'_{t,SM} - P_{t,x_p}$) is found. Again to ensure consistency between the median optimized results, we attribute this remaining required consumption entirely to the PPA. This means at each time t we set $\text{med}(A_{t,PPA})$ equal to the value that satisfies the equation

$$\text{med}(A_{t,PPA}) = (\text{med}(C_t) - D'_{t,DM} - D'_{t,SM} - P_{t,x_p}) \times \ddot{a}_{x+t,PPA}.$$

Following this adjustment to $\text{med}(A_{t,PPA})$, the values of $A_{UAB,t+1,PPA}$, $A_{IIB,t+1,PPA}$, $A_{MCB,t+1,PPA}$, and the corresponding attribution of drawdowns across $D'_{UAB,t,PPA}$, $D'_{IIB,t,PPA}$, and $D'_{MCB,t,PPA}$ are found by repeating equations (A1) through (A6) whilst setting $i = PPA$. The exception to this is that $A_{MCB,t,PPA} = D'_{MCB,t,PPA} = 0$ for $t \geq 0$. For completeness, we note that $D'_{t,PPA} = D'_{UAB,t,PPA} + D'_{IIB,t,PPA}$.

Finally, for each time t, to obtain the shaded area attributable to 'account balance', we evaluate $\sum_i D'_{UAB,t,i}$. To find the shaded area attributable to 'investment income' we evaluate $\sum_i D'_{IIB,t,i}$. To find the shaded area attributable to 'mortality credits', we evaluate $\sum_i D'_{MCB,t,i}$. Alongside the shaded area for pension income (derived directly from P_{t,x_p}), this generates the graphs presented.

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