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Critical Mass for Affirmative Action: Dispersing the Critical Cloud

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The concept of critical mass has been invoked by social scientists and the Supreme Court in affirmative action decisions as a solution to problems related to underrepresentation of minority students in institutions of higher education. Little distinction is made by scholars between the Court's use of critical mass as a metaphor and its application in research as a mathematical concept. I use Agent-Based Modeling—a simulation technique in which systems are modeled through repetitive interaction of autonomous decision-making "agents" to observe the complex dynamics that emerge from interaction—to investigate the Supreme Court's conception of the relationship between student-body composition and student isolation and stereotyping. Findings demonstrate that the relationship between student body representation and the educational outcomes of interest as detailed by the Court, specifically minority students' feeling of isolation and majority students' retention of negative stereotypes, does not exhibit a specific threshold or tipping point as we would expect from a system that has a critical mass at which sudden and sustainable change in the state of the system occurs. Simulations of student interactions show there is not one definable threshold or critical mass of minority students that achieves educational goals of reducing either the isolation felt by minority students or the negative stereotypes held by majority students about their minority peers. Instead, greater minority representation is consistently associated with better outcomes for students in all contexts.

> I understand my job [is] to determine if your use of race is narrowly tailored to a compelling interest. The compelling interest you identify is attaining a critical mass of minority students at the University of Texas, but you won't tell me what the critical mass is. How am I supposed to do the job that our precedents say I should do?

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Chief Justice John Roberts, Oral Arguments for Fisher v. University of Texas, October 10, 2012

> We should probably stop calling it critical mass then, because mass, you know, assumes numbers, either in size or a certain weight … So we should stop calling it mass … Call it a cloud or something like that.

Justice Antonin Scalia, Oral Arguments for Fisher v. University of Texas, October 10, 2012

1. Introduction

During oral arguments in *Fisher v. University of Texas at Austin* in October 2012, Justice Antonin Scalia and Chief Justice John Roberts expressed confusion over the concept of critical mass. Both perceived that it implied numbers and found definitions of critical mass without reference to numbers dissatisfying. Scalia suggested that the description, established in the *Grutter v. Bollinger* (2003) majority opinion, described an amorphous cloud more than a definable mass. Roberts expressed frustration that he was asked to rule on the merits of an insufficiently explained concept. Their frustration illustrates a problem with the use of critical mass in both legal discourse and in social science. Reaching a critical mass of underrepresented students and employees has been hailed as a cure for problems related to stereotype and social identity threat,[1] unconscious bias,[2] lack of diversity, and climates of exclusion in universities and organizations. However, what remains unclear is how organizations should determine what number constitutes a critical mass of students or employees, or whether indeed there even is a particular critical mass that will solve institutional issues related to a lack of diversity and inclusion. In this paper, I survey the history and use of the term critical mass in the social sciences and in affirmative action jurisprudence, then, I use Agent-Based Models (ABMs) to instantiate the Supreme Court's assumptions as testable models of the relationships between student body composition and isolation and stereotyping to evaluate whether those relationships really have the properties the Court assumes.

[1] Stereotype threat refers to an individual's fear that they will confirm a negative stereotype about their group. That fear can negatively affect performance and test scores (see Steele 2011); social identity threat comes from a broader theory that posits people will feel fear or threat of being negatively evaluated based on the groups to which they belong (see Turner and Tajfel 1986).

[2] Unconscious bias is a belief an individual holds about a particular group that they are unaware they hold but which may impact their behavior towards members of that group (see Greenwald and Krieger 2006).

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Unfortunately, critical mass is simultaneously underdefined and overdefined within the social sciences. While it is invoked across a broad scholarly literature, there is no consensus on what it is and how it can be accomplished. Its use by scholars and lawyers—simultaneously as a metaphor, mathematical threshold, and social scientific relationship—creates confusion, particularly within the legal discourse around affirmative action (see Garces and Jayakumar 2014; Terrell 2011 for more details). Recent invocations of the concept by the Supreme Court highlight the need for clarification. For example, in *Grutter v. Bollinger*, decided in 2003, the Court accepted the University of Michigan Law School's use of race as a "plus factor"[3] in admissions decisions as a means to create a "critical mass" of minority[4] students. In Justice O'Connor's majority opinion, which shaped the terms of the court's future debate about the constitutionality of affirmative action programs used by institutions of higher education, critical mass has less a concrete definition than a list of conditions it is intended to prevent. The Court fought over this nondefinition in both rounds of *Fisher v. University of Texas at Austin*, which provoked the aforementioned frustration on the part of Justices Scalia and Roberts. Critical mass—as it is used both in the physical and social sciences—implies a specific mathematical form, where, at a specific value of one variable (in this case proportion of minority students in the student body population), there will be a sudden and sustainable change in the state of another variable (in this case the number of students who feel isolated in their universities). Starting from the Court's list of goals for affirmative action, the aim of this paper is to explore whether the processes implied by the Court's discussion of critical mass actually exhibit this form, and therefore, whether critical mass is the appropriate mathematical concept for policies linking student body composition to the outcomes of interest to the Court within the system the Court describes.

I begin by discussing the evolution of the concept of critical mass from the physical to the social sciences. Then, I discuss critical mass as it is used in affirmative action jurisprudence. I then present two ABMs that test the relationship implied in the Court's

[3] This means that the university can use race as an additional positive factor in admissions decisions but not as the sole factor.

[4] The term minority began use as a numerical description but has taken on a racial meaning in the United States, signifying individuals who are not white. Grouping all racial/ethnic minorities together as if homogeneous is problematic, however, it is the term the Court uses throughout its opinion in Grutter v. Bolinger (2003). Accordingly, it is the term I use in the present paper, while testing the Court's assumptions about the relationship between student body composition and student isolation and retention of negative stereotypes.

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definition of critical mass between student body composition and institutional outcomes, specifically reduction of minority student isolation and majority student retention of negative stereotypes. While some may argue critical mass is a useful metaphor in affirmative action jurisprudence because its ambiguity accounts for the variability of the institutional contexts the law must accommodate, I find that the relationship between student body composition and the institutional outcomes cannot be mathematically described as a specific point at which there is a sudden shift in the outcomes once a particular number of minority students has been reached. Rather, increasing representation of minority students is consistently associated with better outcomes, and the relationship is complex and dependent on specific institutional context (Garces and Jayakumar 2014). The findings demonstrate a need for further empirical research to (1) differentiate between mathematical concepts and useful metaphors and (2) more concretely define the relationship between student diversity, institutional context, and the social and educational goals of affirmative action programs.

2. From Fission to Fisher

2.1 Evolution of a Concept

The critical mass concept originated in the physical sciences. It appears in print as early as 1880 with reference to solar and stellar heat (Chapman 1880). In Hugo Benioff's 1919 review of Arthur Eddington's *The Interior of a Star*, it is clear that Eddington used critical mass to refer to a threshold mass in the interior of a star at which a "vital change of condition takes place" (Eddington 1918; Benioff 1919: 213). In nuclear physics, critical mass refers to the "minimum quantity of uranium or other fissionable material" necessary to start and sustain a nuclear chain reaction (Logan 1996). Across the physical sciences, critical mass consistently refers to a minimum quantity necessary to induce an irreversible change of state.

By analogy, it has been applied to social questions to indicate the number of people or actions necessary to create and sustain changes in group behavior. This use follows two traditions: critical mass for collective action and critical mass for representation and diversity. The use of critical mass in these two strands of research is conceptually very different. While both assume change will occur at some threshold, they are interested in different types of change. Conceptually, critical mass in collective action is about how accumulated individual actions lead to changes in goal-oriented group behavior, opinion, and conformity in face-to-face

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interactions. In the diversity and representation context, critical mass is about an accumulation of different types of people whose presence in an organization causes changes in attitudes, perceptions, and culture.

In the 1980s, Critical Mass Theory emerged from research on collective action to explain the necessary conditions for spontaneous group action. Oliver and Marwell's theory of critical mass for the production of collective action posited that there were a minimum number of individual actions required to spur group action (Marwell et al. 1988; Marwell and Oliver 1993; Oliver et al. 1985; Oliver and Marwell 1988). They assumed that individuals take into account other's actions when making decisions (Oliver et al. 1985). Using analytic modeling, which is a type of mathematical model with a closed-form solution, and ABMs to simulate interaction within groups, they determined that the way in which input of resources is related to output of collective goods in a particular group, described as either decreasing or increasing marginal returns, the heterogeneity of the group, and the size of the group are all important factors in determining the amount (or critical mass) of individual action necessary to produce collective action (Oliver and Marwell 1988). Other research on face-to-face interactions has found important effects of majority group size when social influence from members of the group is part of the mechanism for eliciting individual conformity to group beliefs and behaviors (Bond 2005).

The other common use of the concept of critical mass in social science appears in work on representation of women and minorities in education, government, and employment. Empirically, critical mass has been explored with regards to the number of women and minorities required to change organizational outcomes and culture. Much of this work stems from Kanter's (1977) research on tokenism and its consequences. Kanter outlined four types of groups: the uniform group, where membership is virtually homogeneous; the skewed group, comprised of 85% members of the dominant group and 15% members of other groups, creating a situation where minorities are "token," understood by majority group members as representative of their group as a whole; the tilted group, where the ratio is closer to 65:35, with enough of each group to form alliances and affect organizational culture; and the balanced group, where interaction and organizational culture reflect the even distribution of types in the workplace (Kanter 1977). Multiple studies of critical mass have used Kanter's skewed and tilted ratios to define and test critical mass as a means of enacting change in organizations (see Etzkowitz et al. 1994; Grey 2002; Joecks et al. 2013; Saint-Germain 1989).

Kanter focused on "token" women within a skewed workplace to determine the dynamics of interaction and the consequences

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both for the group as a whole and for the "tokens" individually. She found that being a "token" causes (1) higher visibility, which creates performance pressures; (2) polarization, wherein dominant group members emphasize their commonalities and exaggerate differences from the "token"; and (3) assimilation pressures, which cause the "token" to become trapped in stereotypical characterizations of their token role (Kanter 1977). Following Kanter, other work has confirmed the psychological and performance implications of tokenism.

A study testing Kanter's theory among elite Black professionals found race and gender tokenism were related to increased work-related stress and decreased psychological well-being (Jackson et al. 1995). Being a "racial token" was associated with higher levels of "token stress," which included "a loss of black identity, multiple demands of being black, having to demonstrate more competence than [non-black] peers, and a sense of isolation." Being a token by gender was associated with higher levels of "role overload": "too many time demands, juggling private and work life, and too many responsibilities" (Jackson et al. 1995: 550).

A study of black students at three majority white, elite universities in the United States found that "token" black students' experiences of racial microaggressions[5] from their white classmates led to several negative outcomes (Solorzano et al. 2000). In addition to increased stereotype threat, these students experienced stress from feeling like the sole spokesperson for their race in classroom and social settings. They felt the need to build counter-spaces where they could feel comfortable and safe on campus. This stress negatively impacted their academic achievement and they had increased feelings of hopelessness and the desire to leave the campus for someplace more hospitable (Solorzano et al. 2000). Many of the students said the stress and pressure would be alleviated if they were not so frequently the only one of their race in their classes (Solorzano et al. 2000). In another study, black students at an elite white institution expressed feelings of sociocultural alienation that they attributed to their group's insufficient representation on campus (McClelland and Auster 1990).

Education research has focused on increased diversity as a solution to the problems of tokenism. Efforts to increase diversity in education at all levels have been motivated by the educational benefits that a diverse student body may bring to all students. For example, more frequent cross-racial interactions are associated

[5] Racial microaggressions are regular "verbal, behavioral, or environmental indignities, whether intentional or unintentional, that communicate hostile, derogatory, or negative racial slights and insults" (see Sue et al. 2007).

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with increased student openness to diversity, improved cognitive development, and increased self-confidence (Misa et al. 2006). In addition, increased campus diversity has a small, but positive and significant impact on the atmosphere of inclusion on campus that results in increased likelihood of forming interracial friendships, talking about race and diversity, higher student retention, greater overall college satisfaction, and increased self-perceptions of intellectual and social self-confidence among all students (Chang 2001). Participation in programs designed to promote diversity and cross-group interactions is associated with increased perspective taking (the ability and incidence of taking someone else's perspective) by students, greater recognition by students of group similarities rather than differences, and increased rates of interaction between students from different cultural, racial, or ethnic groups over the course of a college career (Gurin et al. 2004). Cross-racial contact between students is also associated with higher levels of positive affect toward black people among white students, as well as positive changes in this affect over time and decreases in beliefs associated with modern forms of racism, such as opposition to policies aimed at increasing racial equity (McClelland and Linnander 2006). It is important to note, however, that most studies of cross-racial contact find it is beneficial only when it is meaningful rather than superficial, like close friendships, intimate relationships, and cooperative contact. These studies also find that the benefits of such contact may not be equally distributed among all parties (Chang et al. 2004, 2006; McClelland and Linnander 2006; Slavin and Oickle 1981).

In this context, critical mass is promoted by many social scientists and diversity and inclusion advocates as a cure for the ills of tokenism and racial and gender homogeneity in institutional settings. The promise of critical mass—reaching a threshold number of underrepresented individuals—is that it will spur institutional change that increases institutional performance and eliminates negative effects of tokenism. Critical mass has also been proposed as part of strategies to reduce social identity threat and discrimination related to unconscious bias. The presence of a sufficient number of people from one group can demonstrate significant heterogeneity within that group, thereby dispelling stereotypes (Steele 2011). Exposure to a sufficient number of people who defy racialized or gendered stereotypes may provide the necessary input to change generalized associations between race or gender and individual traits, such as competence, that may lead to unconscious discriminatory action or create individual stress from internalization of stereotypes and fear of confirming them (Banaji and Greenwald 2013). The concept of critical mass enters the legal debate on affirmative action programs in higher education against this backdrop of hope for its transformative power.

2.2 Critical Mass in Affirmative Action Jurisprudence

Affirmative action originates in Executive Order 10925, signed by President Kennedy in 1961. The order stated that government funded projects must take affirmative action to ensure there is no racial discrimination in hiring and employment practices, and created the President's Committee on Equal Employment Opportunity, which led to the establishment of the Equal Employment Opportunity Commission (EEOC) with the passage of the 1964 Civil Rights Act (Exec. Order No. 10925 1961). While it does not use the words "affirmative action," the Act prohibits discrimination based on race, color, religion, or national origin (Civil Rights Act of 1964). The EEOC's mission was to end discrimination in employment and to deploy programs to help realize the promise of equal opportunity (EEOC 2016). As President Johnson stated during his 1965 commencement speech at Howard University, the rationale behind affirmative action was to attack a problem that could not be overcome purely through legal change. Recognizing that centuries of systematic disadvantage could not be erased only with a declaration of rights, affirmative action was meant to "open the gates of opportunity" and give all U.S. citizens "the ability to walk through those gates" where equality was not just "a right and a theory" but a "fact" and a "result" (Johnson 1966). Thus, affirmative action was a proactive tool to level the playing field with equity in both process and outcomes.

Affirmative action entered jurisprudence on discrimination in higher education with the 1978 Supreme Court ruling on *Regents of the University of California v. Bakke*. The school admitted students from separate pools, dividing the competition into a general track and one for minority and economically disadvantaged students. They reserved 16 spots out of 100 for the latter (*Regents of Univ. of Cal. v. Bakke* 1978). In its decision, the Court struck down the use of quota programs, while simultaneously affirming that it was constitutional to consider race in admissions (*Regents of Univ. of Cal. v. Bakke* 1978). The true legacy of the *Bakke* opinion is that it established diversity as a "compelling interest"[6] for institutions of higher education.

There are several mentions of "critical mass" in the pre-*Grutter* legal discourse on affirmative action. For example, in *Oliver v. Kalamazoo Board of Education* (1980), the decision stated the school district could not fire the most recently hired employees as directed by a collective-bargaining agreement until the district

[6] "Compelling interest" indicates that the stated interest (for Affirmative Action this is the intended outcome of race-based admissions initiatives) is of sufficient importance to justify race-based measures and that the interest satisfies some legal standard for action (see Liu 1998).

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reached a "critical mass" of 20% African-American teachers and administrators. Based on a past finding that the district "engaged in racial discrimination against students," this critical mass was necessary because of "the need for role models … [to] encourage minority students to higher aspirations and at the same time work to dispel myths and stereotypes about their race" (*Oliver v. Kalamazoo Board of Education* 1980). In other words, affirmative action was a means of shaping employment by race in order to remedy a constitutional injury inflicted on students by the school board and district through past segregation and discriminatory hiring.[7]

Critical mass was also invoked in *United States v. Virginia* (1996) when evidence was presented that 10% female enrollment at the Virginia Military Academy—previously an all-male institution—would create a critical mass of female cadets to provide a positive educational environment, and in *Comfort v. Lynn School Committee* (2005) when the District Court opinion, citing experts, said a critical mass of minority students is necessary to gain the benefits of diversity, reduce racial stereotyping and tensions, and increase racial harmony and understanding. While these cases included the idea of critical mass in both evidence and reasoning, *Grutter v. Bollinger* (2003) marked the beginning of the idea of critical mass as a driving rationale for affirmative action. *Grutter* also signaled a move away from affirmative action as a remedy for historical discrimination toward a new goal of "diversity for diversity's sake" (Coleman 2012; Schneider and Segura 2014).

In *Grutter* v. *Bollinger* (2003), following *University of California Regents v. Bakke* (1978), the court affirmed that there is a compelling interest in using race as a "plus factor" in admissions decisions in order to achieve a critical mass of minority students and reap consequent educational benefits, as long as the affirmative action program is narrowly tailored to meet that interest.[8] The majority opinion refers to critical mass as "meaningful numbers," "meaningful representation," "a number that encourages underrepresented minority students to participate in the classroom and not feel isolated," and "numbers such that underrepresented students do not feel isolated or like spokespersons for their race" (*Grutter v. Bollinger* 2003). The opinion also states that the benefits of critical mass are that it creates "a mix of students with varying backgrounds and experiences who will respect and learn from each other"; it creates "diversity which has the potential to enrich

[7] Interestingly, the court did not seem similarly concerned with past discrimination against minority teachers, but the outcome effectively protected them, as well.

[8] E.g. specifically designed to meet only that interest.

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everyone's education"; students with diverse interests and backgrounds "enhance classroom discussion and the educational experience both inside and outside the classroom"; and "when a critical mass of underrepresented minority students is present, racial stereotypes lose their force because nonminority students learn that there is no "minority viewpoint' but rather a variety of viewpoints among minority students" (*Grutter v. Bollinger* 2003).

Bringing all the components together in his dissent, Justice Rehnquist summarizes the Court's understanding of critical mass as "a sufficient number of underrepresented minority students to achieve several objectives: To ensure that these minority students do not feel isolated or like spokespersons for their race; to provide adequate opportunities for the type of interaction upon which the educational benefits of diversity depend; and to challenge all students to think critically and reexamine stereotypes" (*Grutter v. Bollinger* 2003). Importantly, the majority opinion makes clear that critical mass is not a quota, which was ruled unconstitutional in *Bakke*. A quota is a fixed number or proportion reserved for a particular type of student. Since there is no stated percentage, range, or number for what constitutes a critical mass, it should not be considered a quota.

Following *Grutter*, critical mass played a major role in oral arguments for *Fisher v. University of Texas Austin* in both (2013 and 2016) appearances before the Supreme Court, including significant debate over critical mass's definition and whether it was a quota. In particular, in 2012, Justices Scalia and Alito both expressed confusion regarding its scope—was it meant to be a critical mass for the whole institution or within each classroom, and was it meant to be of "minority" students or for each type within the population? Counsel for the university stated they were interested in a university-wide critical mass, but took classrooms into account when judging if the goal had been reached, and they were looking for a critical mass of "minority students" rather than in each subgroup. In addition, they asserted that they were not attempting to reach proportional representation based on state demographics, but that they wanted to achieve a mix of students with a variety of experiences. Beyond a racially diverse student body, the university had an interest in diversity within racial groups and determined that race was a necessary plus factor in admissions decisions to achieve that goal.

The Court's definition assumes a relationship between population makeup in the form of "critical mass" and institutional outcomes. This paper asks, given the relationship between population makeup and institutional outcomes stipulated by the Court, whether there is a point in the growth of the minority population at which there is a sudden and sustainable shift in the state

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of the organization. While critical mass may be a useful legal metaphor to support affirmative action, we should not assume the metaphor translates literally into social processes described by the mathematical concept that shares its name. The purpose of the following analysis is to investigate the relationship the Court describes between population composition and the outcomes of interest to determine if critical mass is indeed the correct mathematical concept to explain the relationship between the two.

3. An ABM Approach

A review of previous research that tests for processes that are assumed to have a critical mass confirms there is no consensus on what it is or how it should be operationalized. The point at which sudden change in an organization should be expected to occur because of sufficient minority representation has been variously stated as 10% (*United States v. Virginia* 1996), 15% (Etzkowitz et al. 1994; Grey 2002; Saint-Germain 1989), 20% (Oliver v. Kalamazoo Board of Education 1980), 30% (Joecks et al. 2013; Norris and Lovenduski 2001), 3 or more individuals (Konrad et al. 2008; Konrad and Kramer 2006; Torchia et al. 2011), and increasing representation on a continuous scale (Hagedorn et al. 2006). One thing all these operationalizations of critical mass have in common is an assumption that there is a literal critical mass, or tipping point, at which the number of minority students or employees in an organization triggers a sudden and sustainable change in the state of that organization.

In the following analysis, I use ABMs to test models implied in the Court's definition linking critical mass to educational outcomes. ABMs are a simulation technique in which systems are modeled through repetitive interaction of autonomous decision-making "agents." ABMs allow us to observe the complex dynamics that emerge from interaction and which would otherwise be obscured by pure mathematical modeling. They are an appropriate choice for this analysis because they provide an efficient and effective way to test theoretical models of interaction across many combinations of parameter values to demonstrate the properties of the model across the range of imaginable scenarios.

The Court's decisions consistently focus on three outcomes: reducing minority student isolation; achieving educational benefits from diversity for all students; and combating negative stereotypes held by majority students about minority students. The first outcome is related to the ways increased representation affects experiences of underrepresented students, the third outcome is related to the ways increased diversity impacts the perceptions of

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majority students about those who are underrepresented, and the second outcome is assumed to affect students of all types. With regard to critical mass as a mathematical property, the Court conflates a process with its outcomes, which are themselves differently relevant to distinct populations within the same institution. Empirically, we may find an effect for one or two but not all three of the outcomes. Similarly, the actual level and type of minority composition necessary to achieve the effect may differ for each. Given the variety of specifications in previous research, the abstract nature of the Court's definition, and the assumption that there is a critical mass, I focus here on the form of the relationship between population makeup and the outcomes of interest to determine whether the Court's assumptions are correct.

I restrict this investigation to two of the outcomes—minority student isolation and majority student retention of negative stereotypes about minority students—because they are easily operationalized in ways consistent with the Court's discussion.[9] I rely on ABMs to investigate the relationships between student body composition and these outcomes. While social reality is complex, for the purposes of testing theory it is useful to strip nuance away for the sake of generality (Healy 2017). The models will not represent the full complexity of real life, but they are useful for determining if the models implied by the Court's discussions exhibit a critical mass, or tipping point. The goal is not to imitate reality precisely, but rather to capture complex phenomena with the simplest and most parsimonious means possible.

ABMs were developed to model interactions when analysis of groups at the aggregate or of individuals without consideration of broader group dynamics is unable to capture the complexity of nonlinear systems (Macy and Willer 2002). The behavior of human groups can often be "highly complex, nonlinear, path-dependent, and self-organizing" with dynamics better understood by modeling them "not at the global level but instead as emergent properties of local interaction among adaptive agents who influence one another in response to the influence they receive" (Macy and Willer 2002: 144). Put differently, social phenomena may be best understood as something greater than an aggregation of individual behaviors, but rather as something that emerges from individual and group interactions and therefore cannot be captured either by investigating at the level of the individual or at the level of the group. Rather, such phenomena can only be captured by looking at what emerges out of individuals' interactions with each other.

[9] The other outcome, overall educational benefits of diversity, is so vague as to be impossible to operationalize in a way that is justifiably consistent with the Court's intended meaning.

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An ABM is a computer simulation where autonomous actors, called agents, are coded to have certain characteristics, follow behavioral rules, and interact with each other in a simulation space. The simulation space can be defined in terms of spatial characteristics, as is the case in ABMs of segregation; in terms of organizational characteristics, such as the number of agents; or in terms of time constraints, such as the number of interactions the model will simulate before the program stops. While the simulation is running, agents interact with each other within the programmed constraints of the space while obeying their assigned behavioral rules. For example, in models of residential segregation (see Schelling 1969), agents might be programmed to belong to a set number of groups and the simulation space might be programmed to resemble a grid with different group proportions in each cell of the grid, like neighborhoods with different demographics. Agents then are programmed to move around in the space and interact both with the space and other agents. Over the course of the simulation time the agents would evaluate each cell, based on the characteristics of the cell and the other individual agents they encounter, against their programmed preferences and make decisions about whether they would choose to live in that cell. The results of the interactions are recorded to create data for analysis. This type of modeling relies on the assumption that whatever interdependence exists among agents is fully specified by the rules of the model and that agents are reactive to both present and past interactions and circumstances (Macy and Willer 2002). Through this method, it is possible to observe and measure phenomena that emerge from iterated interactions as properties of the system (rather than simple aggregations). Such emergent properties would potentially be missed in analyses conducted at either the microlevel or macrolevel that do not bridge the two (Macy and Willer 2002).

ABMs are particularly useful for analyses of the outcomes of social interactions. In the present case, whether or not a student feels isolated, or holds onto a stereotype, is shaped through interaction with other students within the constraints of the university. Therefore, it is necessary to investigate the outcomes of interest in relation to the interactions that produce them. As this is an investigation of fundamental models implied in the Court's definition linking student body composition to isolation and stereotyping by means of a critical mass, ABMs provide an effective way to evaluate those models to determine if there is indeed evidence of a critical mass effect that links an input (here, additional students from underrepresented populations) to outcomes (reduced isolation and stereotyping) through interaction. The ABMs presented here are designed to replicate the Court's implied models of the relationship between minority composition and isolation and

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stereotyping. Where the Court's statements lack necessary detail, I have relied on existing research in sociology and social psychology on isolation and stereotyping to inform modeling decisions.

All models presented below were run in Python 3.5.3 using the Mesa module for Agent-Based Simulation. Each model was run for 500 iterations of each combination of parameters and all presented results are proportions averaged across all 500 iterations. In evaluating the resulting data, I look for evidence of a critical mass or tipping point at which there is a sudden change of state in the outcomes of interest—in other words, a point at which most agents in the minority group go from feeling isolated to not feeling isolated or at which most agents in the majority group go from holding a negative belief about members of the minority group to discarding that belief. While one can argue that a tipping point need not induce an immediate change of state, this is an assumption implied by the Court's language around critical mass, especially given that there is no prescription beyond reaching a "critical mass" such that the outcomes of interest can be achieved. This is, therefore, the standard for critical mass against which I compare my models.

4. Agents-Based Models

4.1 Isolation

In social psychology, objective and perceived social isolation are understood as separate phenomena with different risks and potential outcomes (Cacioppo and Hawkley 2009). Beyond the negative health consequences of objective social isolation, perceived social isolation is associated with poor cognitive performance and accelerated cognitive declines, perceptions of danger in surroundings, "poor executive function, increased negativity and depressive cognition, [and] heightened sensitivity to social threats" (Cacioppo and Hawkley 2009). This type of perceived social isolation mirrors feelings expressed in a study of "token" black students in an elite white institution (Solorzano et al. 2000). Despite the fact that "token" students were not objectively socially isolated from other human beings, they experienced perceived social isolation because they felt isolated from others who were meaningfully like them. Perceived social isolation maps closely onto descriptions of tokenism and has been shown to influence students' decisions to exit early from secondary education, among other negative outcomes (Frostad et al. 2015; Kanter 1977).

Given that the Court describes isolation in terms related to tokenism as described in the literature on educational outcomes, the Court's description of isolation is appropriately conceptualized as perceived social isolation. Realistically, isolation in organizations is

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likely much more complicated. Minority students with strong in-groups and many in-group friends may, nevertheless, feel isolated due to repeated rejection and exclusion by members of the majority group. Out-group friendships could potentially alleviate isolation in ways not accounted for by the Court (Pettigrew & Tropp 2006). Perceived isolation may also stem from other institutional factors, such as lack of support from faculty, advisers, and administrators. Research with younger students shows that sense of belonging in school is related to both their relationships with teachers and personal characteristics such that immigrant and disadvantaged students experience a lesser sense of belonging than their peers (Organisation for Economic Co-operation and Development [OECD] 2014). In addition, acute issues, such as overt discrimination and bullying, have serious effects on student well-being (Elamé 2013). The Court's discussion of isolation, however, focuses solely on minority student representation. Therefore, for the purposes of this analysis, I limit the conceptualization of perceived social isolation to numeric representation within the university, or existence of "like-others."

I operationalize isolation as a binary state—a student either feels isolated in a given situation, or not—and therefore, students will need a particular number of others like them in the population to not feel isolated. This number is a "like-others" threshold, which is a measure of the amount of social representation a student needs in order to not feel isolated within the larger social context. I model the relationship between diversity and isolation as a function of group size, group heterogeneity, and this "like-others" isolation threshold.

The Isolation Model has two types of agents: *A* agents are the traditional majority group in the population, those who generally have more social power and privilege; *B* agents are the traditional minority group in the population, those who generally have less social power and privilege. On most university campuses in the United States, white individuals are in the statistical majority and hold social power and privilege within the institution. I limit the model to small groups (between 5 and 30 agents) because small groups are the most likely to produce meaningful interaction that would affect students' perceptions of their school environment. Large class sizes often do not allow for any real student–student interaction and potentially produce negative effects in first-year students beyond any issues related to student representation (Cuseo 2007; Wulff et al. 1987).[10] The model thus simulates small

[10] The results are easily mathematically extrapolated to larger groups with no difference in outcome, though it is likely there are other mechanisms at play in groups where meaningful interaction is improbable, such as large lecture classes with several hundred students.

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group interactions within a larger population, such as small classes, recitations and labs, or club meetings within a university. Agents divide into groups where each *B* agent counts how many other *B* agents there are in the group and either maintains a feeling of isolation or stops feeling isolated depending on whether the tally is above or below their individual isolation threshold.

The model is instantiated with several predefined parameters: population, group size, isolation threshold, and proportion of *B* to *A* agents. *Population* is the simulation population, that is, the total number of agents. *Group Size* is the size of the small groups that agents are sorted into for interaction at each step of the model. *Isolation Threshold* is the number of other *B* agents each *B* agent needs to not feel isolated, or like a token representative of all other *B* agents, in small group interactions. *Proportion of B to A Agents* is the proportion of *B* agents in the total population. Over multiple runs of the models, I vary the values for *Group Size*, *Isolation Threshold*, and *Proportion of B to A Agents*. *Group Size* varies from groups of 5 to 30 in intervals of 5 and the *Proportion of B to A Agents* varies from 0.05 to 0.5 in intervals of 0.05.[11] The nature of the *Isolation Threshold* varies over several treatments, which I discuss in further detail below.

Figure 1 provides a visual flow diagram of how the Isolation Model works. The program begins by generating the *A* and *B* agents based on the specified population size and proportion.[12] At each time step, all agents are randomly sorted into groups of *Group Size n*. *B* agents then count the number of other *B* agents in their group and, if the total number of other *B* agents in that group is greater than or equal to the *Isolation Threshold*, the *B* agent sets its *Isolation* variable to 0 (where 0 means that the *B* agent does not feel isolated). If there are fewer *B* agents than the *Isolation Threshold*, the value of *Isolation* is set to 1. The program repeats this procedure 50 time steps and at each step it calculates the proportion of the *B* agents who feel isolated (have *Isolation* set to 1).[13] Finally, the program averages the proportion of isolated *B* agents at each step to provide an overall average proportion of *B* agents who feel isolated after 50 steps. Due to the fact

[11] As the assumptions of the model only hold when *B* agents are in the minority, I cap the proportion of *B* agents at 0.5.

[12] Where ***Population* = N** and ***Proportion of B to A Agents* = P**:

Number of *B* agents = N * P and

Number of *A* agents = N * (1 – P)

[13] The reader may note that the following analyses of isolation could be achieved through Monte Carlo Simulation. I have chosen to use ABMs rather than Monte Carlo Simulation because it achieves the same result while providing a useful framework for considering the student as an agent in group interactions in the university environment and allows for methodological consistency across analyses.

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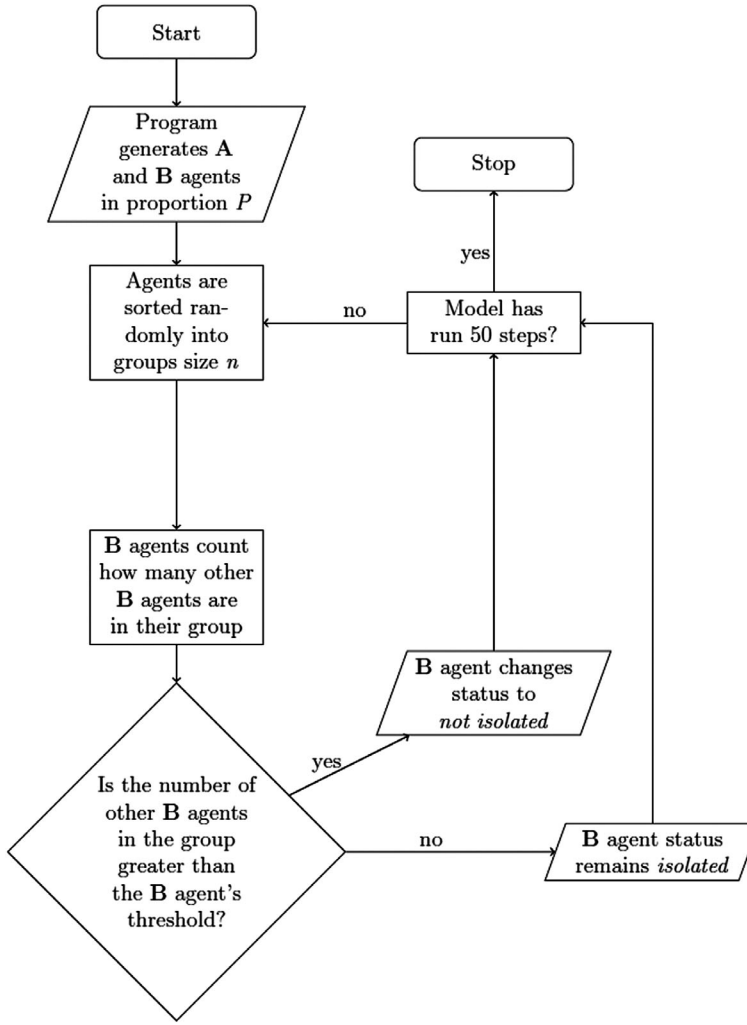

Figure 1. Flow Diagram of Isolation Model.

that *B* agents' isolation is not cumulative over time, but is evaluated separately at each interaction, we expect the isolated proportion to modulate around a true mean that remains constant over time. Therefore, the final output of the model approximates the true mean derived from an average proportion across all time steps. The output of this model is not dependent on the number of time steps—rather, the number of steps the model runs only serves to make the estimate of the true mean more precise. Below I discuss several variations of the Isolation Model (see Table 1 for comparison). While they differ in the construction of the threshold variable, they all follow the same procedure, as described in

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Table 1. Comparison of Isolation Models by Threshold Type

Model	Threshold Type	Threshold Range	Evaluation
Isolation 1	Same for all	1–10	Number of other Bs
Isolation 2	Random Normal	0–14.4	Number of other Bs
Isolation 3	Random Proportion	0.05–0.5	Proportion B

the flow diagram. All of the Isolation Models are run for all combinations of parameter values.

In Isolation Model 1, *Isolation Threshold* is the number of like-others each *B* agent needs to not feel isolated, and all *B* agents have the same threshold for each run of the model. I run the model with all combinations of group sizes, population distributions, and thresholds. Figure 2 shows the results from Isolation Model 1 with a *Population* of 1,500 agents[14], *Group Sizes* of 5 to 30 in intervals of 5, *Proportion B to A* from 0.05 *B* agents to 0.5 *B* agents in intervals of 0.05, and for values of *Isolation Threshold* of 2, 4, 6, 8, and 10 "like-others." It could be argued that the group sizes I chose for this simulation are too small—in fact, I also tested groups of 50, 75, and 100. Although the results are not shown, they remain consistent and can be mathematically extrapolated—the larger the group the less isolation on average as the *B* agent population grows.

Examining the output for evidence of a critical mass, it is important to consider what critical mass might look like in the context of a university population. As mentioned in the previous section, if we take the metaphor of critical mass literally in terms of its scientific origin, and following the Court's language about the concept, then we are looking for a point at which there is a sudden shift from one state to another. For example, we might be looking for a proportion of *B* agents in the population where adding one more *B* agent, or increasing the *B* population by one percentage point, will cause most of the *B* agents to go from feeling isolated to not feeling isolated. This is an unlikely finding in a social context. Instead, if we see any "tipping point" characteristics, we might expect to find a range rather than a specific point because social processes are more likely to develop gradually over a range of values. Under the conditions of this model, the relationship between

[14] I set the population to 1,500 agents for several reasons. First, I chose a number that is divisible by all the group sizes for ease of coding, computation, and interpretation. Second, I wanted a population that allowed for a sufficiently large number of groups at all group sizes, but not so large as to create computational problems when running the models over multiple sets of parameters, with 500 iterations of each parameter combination, and up to 80 times steps each. Finally, while a population size of 1,500 may seem arbitrary, the results of the model are scalable and can be extrapolated to other population and group sizes.

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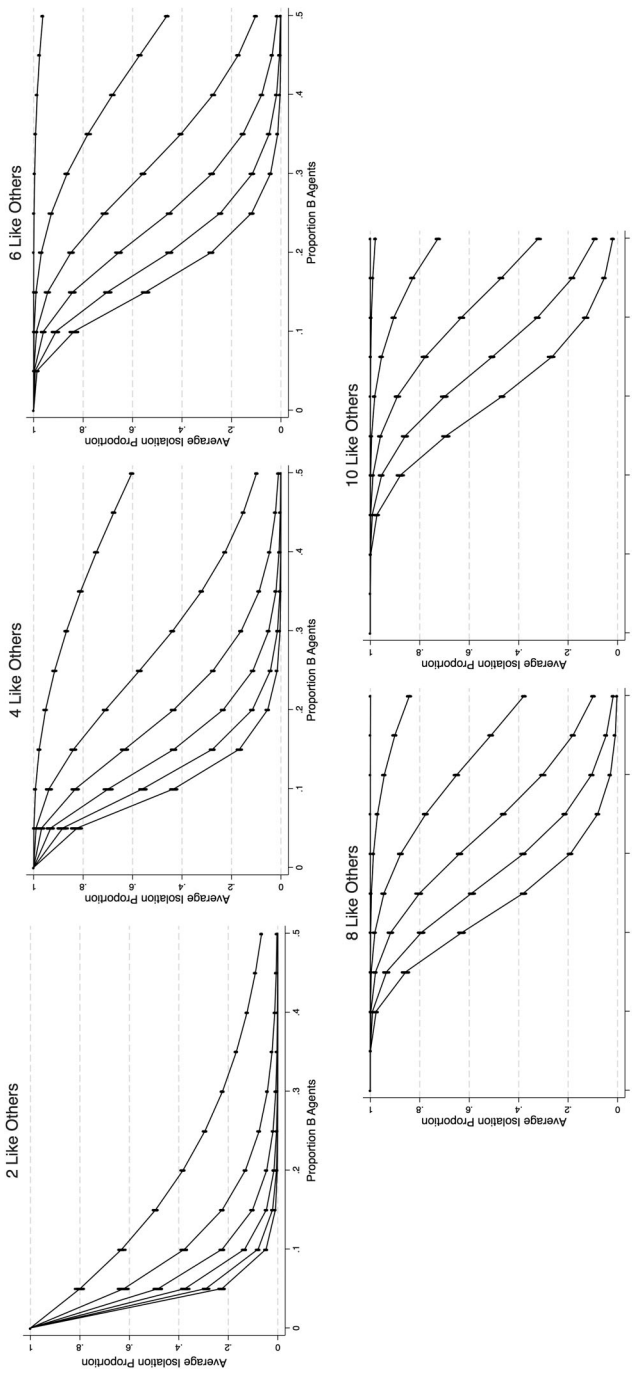

Figure 2. Isolation Model Output for Thresholds 2–10. *Note*: In each group, from top to bottom, lines represent group sizes 5, 10, 15, 20, 25, and 30. Points represent output from 500 runs for each combination of parameters. Lines are lowess smoothers

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population composition and isolation looks like a logistic curve, indicating a potential "critical range" within which there will be the greatest return to satisfaction through increase in *B* agent representation, although the exact range is highly specific to the different combinations of parameters. Therefore, Isolation Model 1 shows some evidence of a critical range.

It is, however, highly unlikely that all students from marginalized groups share identical thresholds for how many like-others they need to not feel isolated in group interactions. It is more likely that individuals have specific thresholds that are related to individual identity and experience. Taking this into account, in Isolation Model 2, *B* agents' thresholds are again a number of "like-others," but, rather than being the same value, each individual *B* agent is randomly assigned a threshold from a normal distribution[15] with mean of 7 and s.d. of 1.96. Figure 3 shows the output from Isolation Model 2. Again, the relationship between population composition and isolation has logistic properties. However, there is no salient point in the distribution at which there is a sudden shift of state when all *B* agents go from isolated to not isolated, and the range of values of *Proportion of B to A Agents* over which the change in *Isolation* occurs is much larger than in Isolation Model 1. Isolation Models 1 and 2 demonstrate that there appear to be monotonically increasing positive returns to more minority agents and that the most efficient means of reaching organizational goals will be highly context (population size, group size, individual threshold) specific. For example, there is a noticeable difference in the impact of increased *B* agent representation on isolation by group size. When agents meet in groups of 5, even at 50% *B* agent population representation, more than 90% of the *B* agents remain isolated. When agents meet in groups of 30, almost all *B* agents are satisfied when they make up 50% of the population by virtue of increased probability of encountering other *B* agents in larger groups.

The previous two models assume that an individual's threshold for feeling isolated in a given group is defined as a number of like-others. It may be more realistic to assume this threshold is a proportion of "like-others" in a group, which would make it scalable by both group size and size of the total institutional population. Figure 4 shows the results for Isolation Model 3, which is

[15] A distribution with this mean and standard deviation gives values within a similar range of the other Isolation Model formulations, while insuring that there are no values less than zero as such a threshold would be meaningless.

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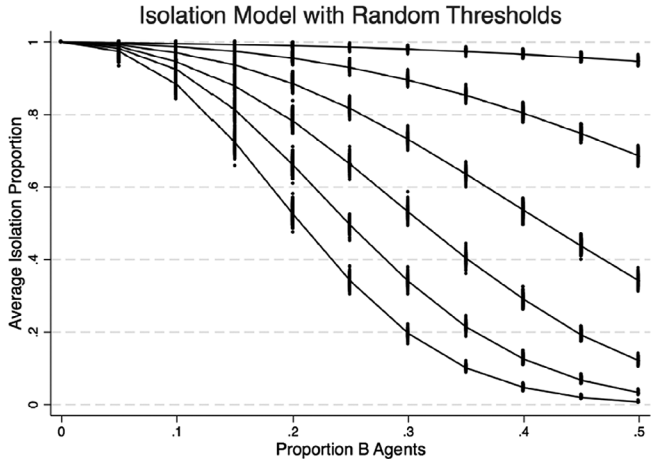

Figure 3. Isolation Model with Random Thresholds from a Normal Distribution (mean = 7, s.d. = 1.96). *Note*: From top to bottom, lines represent group sizes 5, 10, 15, 20, 25, and 30. Points are from 500 runs for each combination of parameter settings. Lines are lowess smoothers.

identical to Isolation Model 2 except the *Isolation Threshold* is a randomly assigned proportion ranging between 0.05 and 0.5, and *B* agents evaluate the proportion of their group that is other *B*s rather than the number of other *B*s.

The relationship between population composition and isolation is still curvilinear but does not exhibit the logistic properties

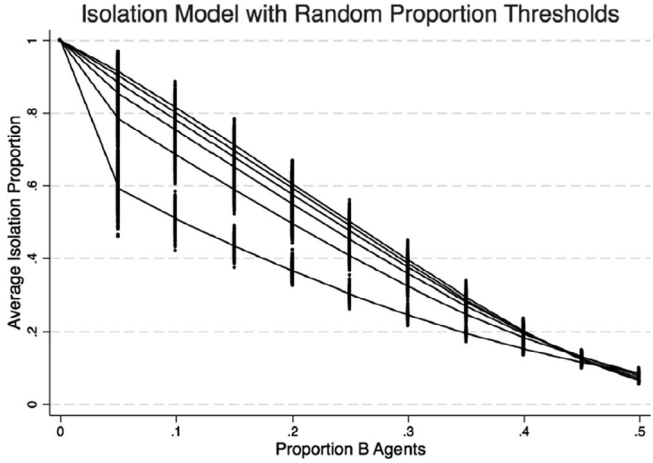

Figure 4. Isolation Model with Random Proportion Thresholds from 0.05 to 0.5. *Note*: From top to bottom, lines represent group sizes 30, 25, 20, 15, 10, and 5. Points are from 500 runs for each combination of parameter settings. Lines are lowess smoothers.

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seen in the output from Models 1 and 2. Additionally, as we might expect from proportions, the relationship is similar across all tested group sizes.[16] More clearly than in the previous models, there is no discernible tipping point where an additional *B* agent, or increase in *B* agents by one percentage point, leads to a sudden shift away from isolation, nor is there a critical range in which there is a rapid change from one state of the outcome to the other. Instead, increased population representation is associated with consistently decreasing isolation among *B* agents. Finally, comparing the results of the three Isolation Models make clear that the more realistic the assumptions of the model the less evidence there is for a critical mass or range.

4.2 Stereotypes

The Court's conception of the relationship between minority representation in a university and reductions in stereotyping implies that stereotypes can be combatted over time with disconfirming information. The Court stated that one goal of affirmative action was for a critical mass of minority students to create the kind of diversity in the classroom necessary for students in the majority to "think critically and reexamine stereotypes" and where those stereotypes further "lose their force because nonminority students learn that there is no 'minority viewpoint'" (*Grutter v. Bollinger* 2003).

Stereotypes are "beliefs about the characteristics, attributes, and behaviors of members of certain groups" and "theories about how and why certain attributes go together" (Hilton and von Hippel 1996). They have been described both in terms of individual schemas that help us simplify and interpret reality and as products of collective ideologies that justify existing social power structures (Augoustinos and Walker 1998). While stereotypes are part of human cognitive function, they are problematic due to their potential to influence behavior. The link between implicit stereotypes/biases and discriminatory behavior is not fully understood; however, it is generally believed that implicit biases can negatively impact behavior toward members of the stereotyped group, even without conscious awareness on the part of the actor that he or she holds biases or has discriminated (Blair 2001). In the university context, implicit biases among students from the majority group that lead to differential treatment of minority students may contribute to minority students' feelings of social isolation. Additionally, these feelings of isolation may impact students' academic

[16] Group size 5 produces a different line due to its small size and the difficulty of realistically defining the range of proportions in terms of indivisible agents.

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performance, emotional well-being, and desire to remain at institutions where they feel out of place (Loo and Rolison 1986; Robinson 2013).

Evidence from social psychology suggests that stereotypes are difficult to change (Johnston and Macrae 1994). Several studies have shown that when subjects are given disconfirming information and asked to process it, there is a subsequent reduction in stereotype beliefs (Johnston and Hewstone 1992; Weber and Crocker 1983). However, when subjects are given both confirming and disconfirming information, they are more likely to consume the confirming information and retain their stereotype beliefs (Johnston and Macrae 1994). While, realistically, stereotypes are likely influenced by a variety of factors, including both confirming and disconfirming information, social influence of peers, and dynamics of social identity construction, the Court's discussion of critical mass centers around the idea that exposing majority group students to a wider variety of minority students will trigger stereotype reduction by combating the misconception that there is no intragroup variation within minority groups. Therefore, to test if there is a tipping point in the relationship between minority group representation and stereotypes as the Court has described it, I model the link with the assumption that disconfirming information is the key mechanism. Furthermore, I assume the strength of an individual's prejudice, or how strongly they believe a stereotype, will be correlated with the amount of disconfirming information they will need to change their belief. In other words, an agent who has a strongly held belief in a negative stereotype about another type of agent will need more disconfirming information to change its belief than an agent who holds that belief less strongly.

Several assumptions are built into the model. First, based on the Court's formulation, the model assumes that exposure to disconfirming information over time will cause a change in held beliefs. It also assumes that holding a stereotype is a binary state—either an agent holds a particular belief about another group or does not—that cannot revert once changed. In reality, it is more likely that disconfirming information can change a belief, confirming information can change it back, and social influence from other actors exerts influence in both directions. Furthermore, it is possible that holding a stereotype is continuous rather than binary. However, the assumption of a binary state that can only change once is useful for investigating the relationship between the variables of interest in its simplest form to determine if, underlyingly, there is a critical mass or tipping point.

The model is based on several further assumptions. I posit individuals fall on a scale of how much or how little their

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individual characteristics confirm negative stereotypes, regardless of whether those connections are justified. In the university context, it is possible that minority students who are admitted tend to conform less to negative social stereotypes about their group than those who do not gain acceptance to institutions of higher education. I assume, for the purposes of this model, that those who hold stereotypes may react to perceivable characteristics of minority students, such as appearance or accent, and assume they are indicative of underlying characteristics based on stereotypes about that group. Additionally, I assume the range of those perceivable characteristics will be distributed across the minority student population so that each minority student falls somewhere on a scale of stereotypicality. I also assume that individuals have different thresholds for what they will consider to be confirming or disconfirming of stereotypes and how many disconfirming encounters they need to stop holding a particular stereotype, and that the latter will be dependent on the former.[17] So, for example, an individual from Group *A* may believe that everyone in Group *B* is less intelligent. All members of Group *B* will have different levels of intelligence and different perceivable characteristics that members of Group *A* will assume indicate something about intelligence, which either confirm or disconfirm a member of Group *A*'s belief. Member 1 of Group *A* may have a high individual threshold for how perceptibly intelligent someone from Group *B* must be for the experience to disconfirm the belief that a particular *B* is less intelligent, and that same individual may need to have 20 such encounters with *B*s they perceive as intelligent in order to completely change the belief that all *B*s are less intelligent. Each member of Group *A* may have a different threshold and every member of Group *B* will have different perceivable characteristics that suggest intelligence to Group *A*. The question is whether or not there is a particular number of *B*s that are necessary in a population to get all or most of the *A*s to change their beliefs about the *B*s.

Once again, we must address the problematic grouping of "minority" students into one category. In reality, stereotypes vary based on the target group, the background of the stereotype holder, and the sociocultural context. It is, however, useful to begin with two types of students—the stereotyped and the stereotype holders—for the sake of modeling simplicity, and it is consistent with the Court's discussion of "minority" students as one group of interest, which is what we are investigating. We may even interpret the *A* and *B* agents not as representative,

[17] While beyond the scope of this analysis, more work is needed on the relationship between level of prejudice and opinion change.

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necessarily, of majority and minority, but relative to each other as in-group and out-group, where the *A* agents are those with social power relative to the *B* agents in each set of interactions.

Figure 5 provides a flow diagram of the Stereotype Model. As in the Isolation Model, the Stereotype Model is populated with *A* agents and *B* agents. While the Isolation Model simulated *B* agent population representation in order to affect *B* agent outcomes (whether or not *B* agents feel isolated), the stereotype model simulates *B* agent population representation in order to affect *A* agent outcomes.[18] In the Stereotype Model, agents sort into small groups where *A* agents evaluate *B* agents and tally disconfirming encounters until they reach a certain number necessary to change a negative belief about the *B* agents as a group.

As in the Isolation Model, the Stereotype Model has the preset parameters of group size, population size, and proportion of *B* to *A* agents. The Stereotype Model also relies on several other parameters: stereotypicality index, belief threshold, and belief strength. *Stereotypicality Index* indicates the extent to which a *B* agent confirms or disconfirms a negative stereotype. In the real world, this may be the extent to which a student from a particular group has more or fewer stereotypical features of that group, which are related to negative stereotypes about that group. *Belief Threshold* is assigned to *A* agents and it is their threshold for the value of *Stereotypicality Index* a *B* agent must have for the *A* agent to consider them to be disconfirming of the stereotype. For example, an *A* agent with a *Belief Threshold* of 10 would consider a *B* agent with a *Stereotypicality Index* of less than 10 to be disconfirming of the stereotype. This would be a particularly open-minded *A* agent as it would consider a *B* agent disconfirming of a stereotype who many other *A* agents would not accept as disconfirming. An *A* agent with a *Belief Threshold* of 1, on the other hand, would only consider a *B* agent disconfirming if that *B* agent had a *Stereotypicality Index* of 0. This would be a particularly prejudiced *A* agent. *Belief Strength* is related to *Belief Threshold*, on the assumption that an individual's level of prejudice will determine the amount of disconfirming information needed to change a stereotype belief, and is the number of disconfirming encounters an *A* agent needs to change their belief about *B* agents. When *A* agents encounter *B* agents, they evaluate the *B* agent's value on the *Stereotypicality Index* and, if that value is less than their *Belief Threshold*, they count it as a disconfirming encounter. As in the

[18] *A* agent stereotype reduction may indirectly affect *B* agents through *A* agent action – to what extent *A* agents hold negative stereotypes about *B* agents and may or may not act on those stereotypes. While this indirect outcome is not the focus here, it should be taken up in future work.

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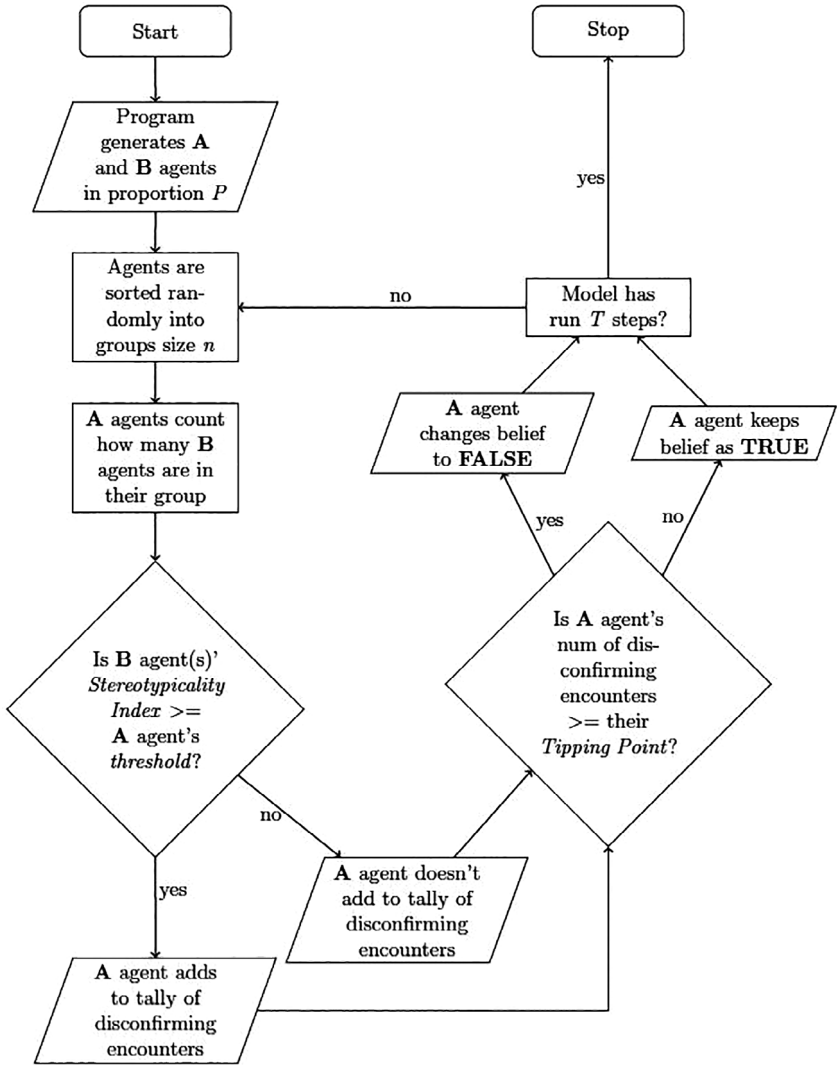

Figure 5. Flow Diagram of Stereotype Model.

Isolation Model, *Group Size* is the size of the small groups agents are sorted into for interaction, *Population* is the total size of the agent population, and *Proportion of B to A Agents* is the proportion of *B* agents in the total population. I vary the values of *Group Size* and *Proportion of B to A Agents* over multiple runs.

At setup, the program generates the *A* and *B* agents based on the specified population size and proportion, using the same formula described for the Isolation Model. All *A* agents begin with a *Belief Held* parameter set to "yes"—in other words, all *A* agents

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start out holding a negative stereotype about *B* agents. At each time step, all agents are randomly sorted into groups of *Group Size n*. *A* agents then count the number of *B* agents in their group, determine how many of those *B* agents qualify as disconfirming encounters, and add the number of disconfirming encounters to a list that serves as their memory. At the next step, agents are sorted into different groups, perform the same counting procedure, and append the number of disconfirming encounters from that step to their memory. Once an *A* agent's total count of disconfirming encounters is equal to or greater than their *Belief Strength* value, they change *Belief Held* to "no"—that is, they drop the negative stereotype about *B* agents. The program calculates the proportion of *A* agents that continue to hold their belief at each step, as well as a mean proportion of *A* agents still holding the belief after the specified number of steps.

Unlike the Isolation Model, where *B* agents' isolation at one time step is independent of their isolation at the next time step, in the Stereotype Model, *A* agents' belief at one time step is directly related to their interactions at previous steps. Therefore, the number of steps the model runs has a direct impact on the outcome of the simulation. Figure 6 shows the decline in the proportion of *A* agents who hold the belief over one run of the model for 100 time steps when the group size is 15. As we would expect, the longer the model runs the lower the proportion of *A* agents who continue to hold the belief.

We cannot, however, just let the model run until there are no *A* agents with the negative belief and then claim victory over stereotyping. Realistically, students in a university will not have an unlimited number of different group interactions over the course of their college careers. Rather, the goal of stereotype reduction through disconfirming encounters must be accomplished in a finite timeframe; otherwise students may graduate before they have accrued enough disconfirming encounters to change their beliefs. Since it is impossible to accurately quantify the number of institutional interactions students will have over the course of their college years, I run the Stereotype Model across all combinations of parameters for 20, 40, 60, and 80 time steps. Substantively the results are the same, and as we would expect, the longer the model is allowed to run, the greater the reduction of stereotypes as agents have the opportunity to gather a greater number of disconfirming encounters. Furthermore, note the expected interaction between time, minority agent representation, and group size. The more minority students there are, the larger the group size, and the greater the number of interactions (facilitated by more time steps) the faster the proportion of *A* agents holding the belief moves toward zero. This is consistent with

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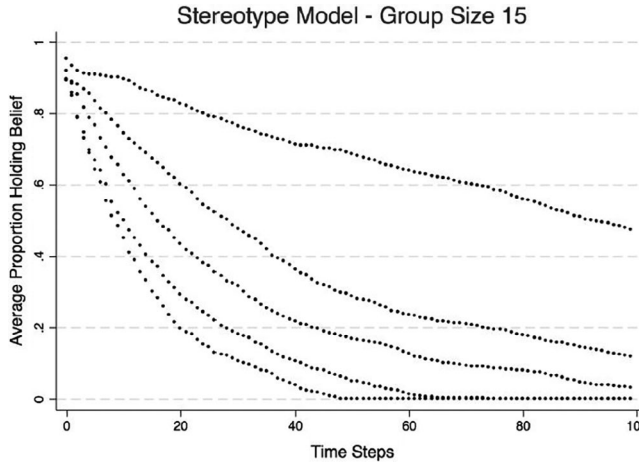

Figure 6. Stereotype Model for One Run of Group Size 15. *Note*: From top to bottom, lines represent 0.05, 0.15, 0.25, 0.35, and 0.45 *B* agent population proportion—100 time steps each.

studies showing positive effects of increased interpersonal contact between white students and students of color (McClelland and Linnander 2006).

Figure 7 shows the results from this model run for 20, 40, 60, and 80 time steps with *Population* set to 1,500; *Group Size* from 5 to 30 in intervals of 5; for values of *Proportion of B to A* from 0.05 to 0.5 in intervals of 0.05; *Stereotypicality Index* a value from 0 to 10 randomly assigned to each *B* agents, where 0 is not at all stereotypical and 10 is the most stereotypical; *Belief Threshold* a value from 1 to 10 randomly assigned to each *A* agent, where 1 indicates that an *A* agent would only consider disconfirming a *B* who is not at all stereotypical and 10 indicates an *A* agent that is much more tolerant of stereotypicality in *B* agents; and *Belief Strength* assigned to each *A* agent based on their value of *Belief Threshold* such that:[19]

$$Belief\ Strength = 0.5\left(Belief\ Threshold^2\right) + 51.$$

The output from the Stereotype Model demonstrates that the relationship between minority agent representation and stereotype reduction is not characterized by either a critical mass or

[19] I chose this equation based on the assumption that those who would not consider someone with the lowest value on the *Stereotypicality Index* as disconfirming of that stereotype would need significantly more disconfirming encounters to change their belief than someone who was willing to take as disconfirming an individual who scored much higher on the index. Based on this assumption, I related the two variables with a quadratic function. Given the inputs 1 through 10, the function cannot yield values less than 0, as an agent needing fewer than no disconfirming encounters to change a belief is not logical.

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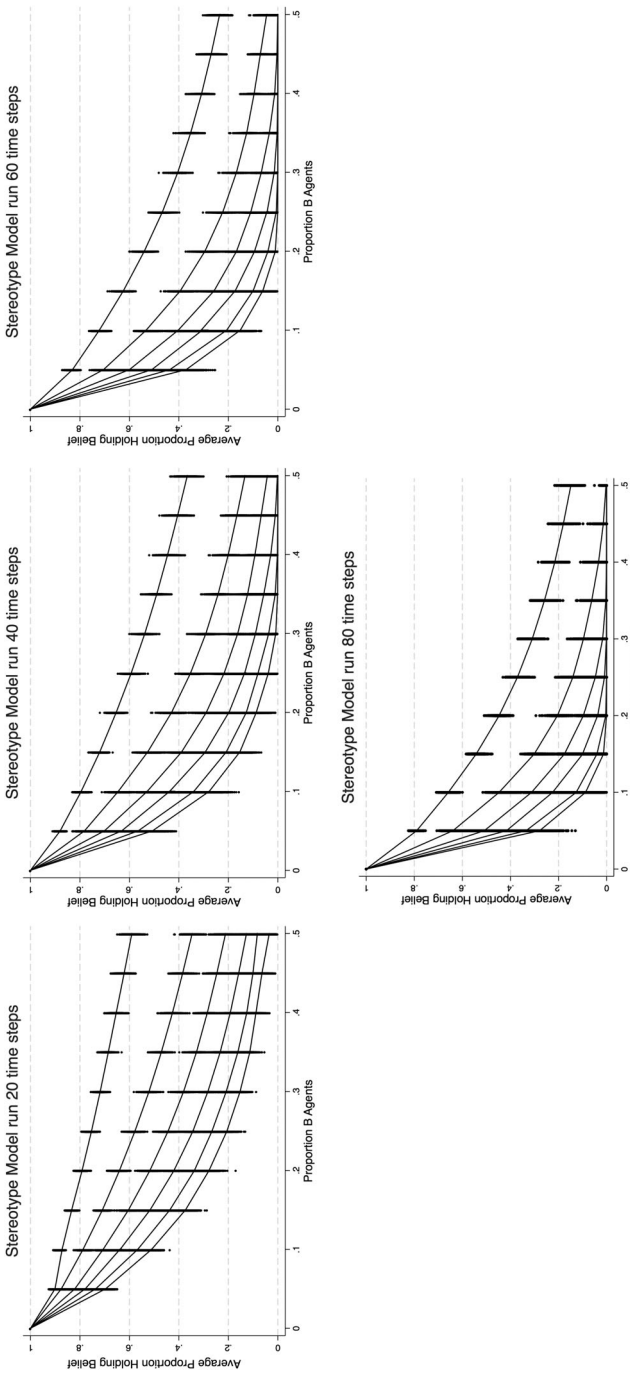

Figure 7. Stereotype Model run 20, 40, 60, and 80 time steps. *Note*: In each group, from top to bottom, lines represent group sizes 5, 10, 15, 20, 25, and 30. Points are from 500 runs at each combination of parameter settings. Lines are lowess smoothers.

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range. Instead, the relationship is curvilinear with no clear point at which a change of state, from all *A* agents hold stereotypes to no *A* agents hold stereotypes, suddenly occurs. As with the Isolation Model, more minority agent representation is generally associated with better outcomes for stereotype reduction. This model does not take into account the potential for agents to change their belief about a stereotype and then revert, or for their opinion to be influenced by confirming information in contradiction to the disconfirming information for reasons stated above. Additionally, it is likely that social influence from other like agents would affect the maintenance or dropping of stereotypes. However, what this model does demonstrate is that there does not appear to be a critical mass, threshold, tipping point, or range in which there is a sudden change of state related to stereotypes in the simplest conception of the mechanism between population representation and the retention of negative beliefs as implied by the Court. Furthermore, the factors that the model does not take into account are likely to dampen rather than promote sudden drastic changes of state.

5. Discussion

The models above test the assumptions of the Supreme Court in their definition of critical mass in *Grutter*. As such, they simplify the reality of student life in modern universities. They are, however, useful for investigating the form of underlying relationships. The premise of the Court's definition of critical mass for affirmative action, if taken literally, assumes there is one underlying mechanism connecting student body composition to a set of outcomes and that the relationship exhibits specific mathematical properties, with the potentially dangerous paired assumption that once a critical mass is reached no more action need be taken. These findings demonstrate that, within a system based on the Court's conception of student relations in the university, there is not a particular number of minority students that constitutes a critical mass that could cause a sudden and sustained change of state with regards to minority students' feelings of isolation or majority students' retention of stereotype beliefs, but that greater representation produces consistently better outcomes.

In the Court's formulation, the relationship between student body representation and isolation takes the form of a logistic curve, which has mathematical thresholds, only when we assume that all agents need the same number of "like-others" to not feel isolated. When we allow for agents to have individual thresholds, the suggestion of a tipping point disappears. Ultimately, the

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results from the Isolation Models demonstrate that more representation of an underrepresented group is better.

The relationship between student body and stereotypes, as described by the Court, shows no signs of a tipping point. The results demonstrate, as in the Isolation Model, that more representation is associated with greater stereotype reduction with benefits leveling off somewhat at higher levels of representation. Furthermore, both models demonstrate that any numbers, proportions, or ranges associated with the outcomes of interest are highly context specific. That is, there is no literal critical mass that will fully achieve all outcomes of interest to the Court, within the Court's own formulation. Instead, there are institution, group, and individual specific needs that do not cohere in an all-encompassing critical mass.

"Critical mass" for affirmative action, therefore, may be better understood as a metaphor than a mathematical mechanism (Addis 2007; Bowen 2011; Terrell 2011). Metaphors from physics can be useful in the social and legal realm, but only insofar as they are appropriately applied (Tribe 1989). As a mathematical representation of the way in which more diversity in the student body will affect the extent of isolation and stereotypes, critical mass does not fit. But as an idea, that more student diversity will positively impact the outcomes of interest, it is immediately easy to understand. Therefore, it may have more power as a legal argument than as a policy prescription. Rather, the policy prescription should be that more representation is necessary, but it alone is not sufficient. What the Court's discussion, and therefore these models, do not account for is the impact of other institutional factors, such as institutional climate (which may encompass such things as commitment of the administration and professors to inclusion, organizational structure and bylaws, and academic structure and curriculum) on the inclusion of minority students in the culture and social life of the university. The idea that there is a critical mass of minority students that will solve problems around isolation and bias fails to account for institutional and interpersonal factors that research shows are necessary for creating inclusive diverse student bodies (Garces and Jayakumar 2014; Gurin et al. 2002; Hurtado and Alvarado 2015; OECD 2014). Increased representation only provides the numbers to begin to combat these problems—systemic institutional change that works to eliminate institutional and social barriers to inclusion is necessary in order to unlock the benefits of increased diversity (Downes 2013; Downes 2014; Mor Barak 1999; Roberson 2006).

More empirical research is needed to better understand the relationship between student population and isolation and stereotyping in the university. There are several questions that

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must be addressed. First, if not critical mass, what best characterizes the relationship between population representation and the outcomes of interest? While the results here indicate more is generally better, additional empirical work is necessary to determine the details of the relationship. Second, at what levels of an institution must representation goals be met to affect the desired outcomes? For example, is adequate representation at the institutional level—rather than the classroom level—sufficient? Relatedly, is adequate representation of "minority" students sufficient, or can the reduction of isolation, tokenism, and stereotyping only be accomplished with adequate representation within each subgroup?

In *Fisher v. University of Texas*, the University argued that it did not seek classroom or subgroup level critical mass, despite the fact that the social science evidence thus far suggests that adequate representation at lower levels would meet stated educational and climate goals better than representation of minority students in general, as if they made a single, meaningful, homogeneous group. The University may have made the argument for a critical mass of minority students rather than a critical mass within each subgroup because advocating for critical mass by subgroup could be interpreted by the Court as serving the now-disfavored goal of righting prior injuries to historically disadvantaged groups. It may also have been a strategic decision on the part of the university because critical mass at the institutional level is sufficiently ambiguous as to accommodate for broad variation across institutions and to avoid characterization as a quota, whereas critical mass or adequate representation at the subgroup level may be more easily dismissed as quotas.

Third, what relationship does existing university climate, in terms of inclusion and respect in intergroup interactions, have to the type of representation of minority students needed to reduce minority student feelings of isolation and majority student retention of negative stereotypes? That is, do more inclusive campuses need less representation for minority students to feel comfortable? Or is increased representation a necessary condition for an inclusive environment? Answers to these questions will help determine the shape that university policies should take in order to best achieve the social and educational goals of affirmative action and diversity and inclusion programs.

6. Conclusions

Critical mass, while a potentially useful metaphor and legal tool for affirmative action advocates, does not mathematically describe the relationship between minority student body

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population and the outcomes of interest as conceived of by the Supreme Court. Rather than being linked by one underlying tipping point, even in the simplest formulation based on the Court's implied models, ABM simulations suggest that student body population relates to isolation and stereotyping in complicated ways that are heavily influenced by institutional context, consistent with other qualitative and quantitative research on diversity in organizations (see Garces and Jayakumar 2014; Hurtado and Alvarado 2015). Therefore, the metaphor's usefulness cannot be clearly extended beyond the legal realm to the development of policy. Instead, we should work toward better understanding the actual mechanisms that link student body representation to educational outcomes. This should begin with researchers being explicit about their assumptions regarding critical mass, its definition, and its use as a mathematical concept or as a metaphor.

As Terrell points out, "the concept [of critical mass] is so seductively intuitive that courts and commentators have used [it] without considering what it can sensibly mean in a social-science context" (2010: 236). Critical mass thus may be "too illusory to be a useful doctrinal tool" (2010: 234). What I have demonstrated here is that, when applied to the relationship between student representation and the outcomes of interest to the court within the confines of the Court's understanding of that relationship, critical mass can only be understood as a metaphor. Whether or not the metaphor is doctrinally useful is a matter I will leave to legal scholars. However, from a social scientific perspective, the usefulness of the concept of critical mass in determining how student body representation relates to isolation, stereotyping and other educational considerations is no longer so clear. What is clear is that the metaphor of critical mass, both as described by the universities in legal arguments and by the Justices in their majority rulings, does not equal a quota. We cannot determine one number or one range or one percentage of minority students that must be admitted in order to achieve the benefits of diversity. Instead, the evidence from this study and from the existing literature on diversity in education and educational psychology agrees that more representation is better, that numbers alone are not sufficient, and that other contextual factors are necessary in order to reap the benefits that constitute a compelling interest to institutions of higher education (Downes 2013, 2014; Garces and Jayakumar 2014; Gurin et al. 2002; Hurtado and Alvarado 2015; Mor Barak 1999; OECD 2014; Roberson 2006). These findings support the idea that increasing diversity in student body representation is necessary to meet that compelling interest.

The work presented here pares models down for the sake of parsimony, and to explore underlying relationships as described by the Court. Moving beyond these models of the Court's

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assumptions, further research should strive to determine the mathematical shape of the relationship(s) between student body representation, isolation, and stereotyping given what is already known about student needs, inclusionary contexts, and the impact of administrative policies on student interaction. Such studies should account for the inclusiveness of institutional cultures to explore how it affects the relationship between student representation and isolation.

Additionally, more research must be done to explore differences between classroom and social interactions to see if representation in private networks mitigates the negative impact of isolation in classroom interactions, as well as the impact of cross-racial friendships. Research on stereotyping and stereotype reduction should investigate the impact of social networks on belief retention, specifically the influence peers exert on one another's stereotype retention and reduction. Campus climate could also affect decisions among minority students on how to present themselves, therefore changing the extent to which they confirm or disconfirm stereotypes. One could imagine, however, that if the institutional culture made minority students feel the need to alter their self-presentation to change their classmates' beliefs about them, this might have a pernicious impact on their feelings of social isolation and stress. Beyond classrooms, students' university experiences are also influenced by social interactions in friend groups and peer networks, constructed counterspaces, club and extramural activities (Grier-Reed 2010; Wilcox et al. 2005), power dynamics within student groups and between students and administration (Downes 2014; Haller et al. 2000; Jost 1995; Neville et al. 2005; Valls and Kyriakides 2013), and representation among staff and faculty (Lee 1999). While the simulations presented above are not purely models of individual behavior but of individuals within a simply defined social system,[20] future research should consider other layers of social influence.

Finally, the most important lesson from this research is that increased minority student representation is associated with better outcomes for both isolation and stereotype reduction, other factors being equal. If there are more minority students, it is more likely that students from minority groups will feel less isolated in their universities. Simultaneously, increased diversity gives majority group students the opportunity to encounter a wide variety of types and viewpoints within groups that they may otherwise assume to be monolithic.

[20] The models are both collective and individual - they model individual decisions within the context of a social system and the decisions, which are bounded by common rules, are drawn from a distribution within a possible set of actions in the context of the system. The rules are representative of the collective constraints of the social world of the model.

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Essentially, increased representation raises the possibility of reaching the Court's goals for affirmative action—which is why critical mass has been such a "seductively intuitive" legal metaphor (Terrell 2011: 236)—even though there is no specific critical mass that will achieve both goals at the same time. Rather than extending the metaphor too far and assuming there is such a specific and definable critical mass, research should focus on understanding the mechanisms and complexities in the relationship between population representation and the outcomes of interest. Understanding those mechanisms is essential to understanding both how to structure policy for reaching the representational goals of affirmative action and how to structure institutional systems to best manage diversity within organizations to achieve the benefits that diversity is intended to foster.

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