RADIO ARRAYS – mm to Meters

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1. Introduction

There are now approximately two dozen operating interferometers in astronomy, with more under construction or in planning. The wide distribution of interferometer arrays, and their acceptance as essential tools in astronomy are a result of the remarkable flexibility and imaging capabilities of modern arrays. Much of the success of these arrays is due to advances in data processing and imaging brought about by innovative and powerful algorithms.

This review briefly summarizes the state of the world's astronomical arrays. In keeping with the spirit of this conference, I will concentrate on evolving techniques and methods of interferometry, and on problems which limit the data taken by these instruments. Section 2 discusses the essentials of the main classes of arrays used in radio astronomy, while Section 3 reviews the thorny problem of array speed and image quality. A unique form of array is promoted as a means of maximizing the speed of array surveys. Sections 4 and 5 touch on the common problems of data processing, with a specific example given for illustration.

2. Three Types of Radio Interferometers

The world of radio interferometers can broadly be broken into three groups: (a) Instantaneous beam forming arrays, (b) Fourier synthesis arrays, (c) Imaging lens arrays. I will briefly discuss each in turn.

2.1 DIRECT BEAM-FORMING ARRAYS. This is the oldest array type, and is used only by a very few operating arrays. Typically, they consist of two continuously filled long rows ('arms') of array elements ('antennas') whose voltage outputs are summed. The two sums (typically, one from an E-W arm, and one from a N-S arm) are multiplied. The result is an instantaneous pencil beam which can be moved by insertion of appropriate phase gradients along the respective arms. Often, a phase-shifting network (a Butler matrix) will be used to form many simultaneous beams, thus increasing the speed of the instrument. Arrays of this type are extensively discussed in the book by Christiansen and Högbom (1972). Two of the remaining examples of this type of array are the UTR-2 (Kharkov), and Northern Cross (Bologna) instruments. The Guaribidanur and Molonglo arrays are discussed in Section 2.2, as they are actually hybrid instruments.

Most beam-forming arrays are now decommissioned – victims to the superior performance of synthesis arrays.

2.2 FOURIER SYNTHESIS ARRAYS. There is no need to review for this audience the techniques of Fourier synthesis arrays. But it may be useful to

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review why this technique has scored a near-total victory over beam-forming arrays. The dominance is due to the superior performance which results from recording individual Fourier components, thus allowing flexible beam formation to occur in a computer under human control rather than in real time in a correlator. The ability to perform post-observing calibration and data manipulation is enhanced by modern correlators which provide all possible products for the N antenna inputs. This enables the formation of closure loops which, either explicitly or implicitly, greatly assists the techniques of self-calibration. A necessary ingredient for successful use of a synthesis array is a modern high-speed computer. Really, it is the emergence of high speed computing which has permitted the Fourier synthesis technique to dominate. The only real advantage of beam-forming arrays is their low computing requirements.

There exist at least two instruments which work in a hybrid mode. The Guaribidanur array is a T instrument which works by Fourier synthesis. The summed voltage from the EW arm is correlated against each of the banks distributed down the NS arm, resulting in a Fourier series which is inverted to give the brightness distribution of a meridional strip. The entire sky is thus mapped in each sidereal day. The Molonglo array correlates the East half of a linear array with the West half, giving a fan-beam through which the sky moves. By repeatedly observing the same patch (accomplished by inserting phase gradients along the arm), a series of one-dimensional strip distributions are obtained, from which a two-dimensional image can be made using back projection techniques very similar to medical tomography.

2.3 IMAGING LENS ARRAYS. There is only a single example for this class of imaging array, which I will describe more fully in the next section. In essence, this class comprises a rectangular array of fundamental elements. The output of each of these is digitized, and the whole is Fourier transformed in real-time at the Nyquist rate to produce a time series of instantaneous images of the sky. If there are N elements in the array, there will be formed N beams in a regular array on the sky. There are no correlators in this system – the transform is the lens. This system can be thought of as N independent total-power 'single-dish' antennas, each having the sensitivity and resolution of a single aperture covering the same area as the array. The data products can be integrated upon output to reduce the archived data output. This imaging system has some distinct advantages for surveying the sky at high frequencies, a subject to be developed in the next section.

3. Surveys, Speed, and Fidelity

Astronomy depends upon surveys. Without surveys, there will be neither objects for observers to image, nor theorists to ponder. To some, it may come as some surprise that there are no all-sky surveys for radio sources at a wavelength shorter than 6 centimeters. * Indeed, only a small fraction of the sky at millimeter wavelengths has been surveyed – the most extensive survey known to me is the

* The COBE survey is for extended emission on very large spatial scales, not for small-diameter radio sources.

CO survey of the galactic plane that John Bally has conducted using a 7-meter antenna – covering less than 1 percent of the entire sky to an equivalent continuum sensitivity of a few Janskys.

The reason for the lack of any complete survey at these wavelengths survey is quite obvious – it takes a very long time. How long? This brings us to the question of speed. I will define T_{sur} as the time is takes to survey 1 steradian to an r.m.s. noise level of σ_{mJy}. This quantity can then be estimated using the standard formulae, remembering that it will be the time taken to reach that noise for a single field times the number of fields in one steradian. For a single antenna with a total power radiometer, the answer is very straightforward:

$$T_{sur} = \frac{3}{B_G A_e}\left(\frac{T_{sys}}{\lambda_{cm}\sigma_{mJy}}\right)^2 \text{ years,}$$

where the bandwidth B_G is in GHz, the antenna effective area, A_e, is in square meters, the wavelength λ_{cm} is in centimeters, and the rms noise σ_{mJy} is in milliJanskys. From this expression, and remembering that the system temperature T_{sys} is itself at least a linear function of frequency, it is clear that all-sky surveys at millimeter wavelengths to useful noise levels are not practical with a single dish equipped with a single feed. The time required can be reduced by using multiple feeds – the speed-up factor is obviously N_f, the number of feeds. However, the advantage remains small until focal plane arrays can be developed to provide more independent beams than can currently be obtained with multiple horns.

The same equations can be used for an interferometer, and it will be quickly found that for an interferometer with the same **total** collecting area the survey time is reduced by a factor $N_{el} - 1$, where N_{el} is the number of elements in the array. However, the apparent large advantage of the interferometer is somewhat reduced by dropping my hidden assumptions of equal system temperature, equal antenna efficiency, and a perfect correlator.

One should also notice that the speed-up factor for an interferometer also tells us that many small elements are better than a few big ones. But be aware that this result comes from analyzing the problem solely from the viewpoint of speed of surveying. It has not included the necessary argument of cost nor of accuracy of calibration. The latter clearly favors fewer, bigger antennas: For a given total collecting area, and given observation time on a calibrator, and presuming the atmospheric phase errors are uncorrelated between elements, the error in determining the gain of an antenna element in an array is proportional to $\sqrt{N_{el}}$, where N_{el} is the number of elements in the array. It should also be noted that I have implicitly assumed that the array collects enough information in the time observed to make a satisfactory image – an assumption which assumes a large N_{el}.

The question of fidelity is a difficult one, which has been touched on by a number of authors, but for which no simple analysis exists. I intend to say very little on the subject, preferring to make what is surely a self-evident point – that fidelity is improved both by use of better electronics (with modern correlators capabable of producing a true voltage product accurate to better than 1 part in 10^4 being a good example), and by having more elements in the array.

3.1 FFT IMAGING ARRAYS. I commented in the last section that all-sky source surveys have never been performed at high radio frequencies because of the inefficiency of single dishes. I have also shown that, at least in principle, arrays are more efficient in surveying than single antennas by a factor equal to the number of antennas. This factor encourages consideration of imaging arrangements with large numbers of receivers or elements. One can imagine then an arrangement in which a large aperture, sufficient for very sensitive observing, is subdivided into an extremely large number of subapertures, possibly as high as many hundreds to a few thousands, each of which receives radiation from all or most of the sky. One then must consider how to combine the information from these subapertures to mimic the response of the entire aperture, but without losing the field of view of the subaperture. A massive correlator will do this, but at the cost of producing a very large dataset. Another approach, which is not widely known even amongst the experts in this field, is to directly employ a Fourier transform to the datastream. In this section, I briefly discuss the interesting properties of the FFT imaging lens.

The direct imaging lens consists of a two-dimensional array of N_{el} equally spaced primary elements. (The requirement of equal spacing is set only to allow an FFT to be used. Similarly, the array need not be square, nor the number of elements on each axis a power of two. However, in doing so, the required computations are simplified). Rather than use a complex correlator to measure the coherence properties, the digital lens directly transforms the voltage outputs of the primary elements to form a two-dimensional array of N beams equally spaced on the sky. This is done by digitizing each of the element outputs, then feeding these data streams into an FFT engine. Those who recall what a Butler matrix is will recognize that the FFT is in essence forming the simultaneous beams by digital summation of the voltages with various phase slopes in each of the two orthogonal axes, just as a Butler matrix does with analog signals from a 1-dimensional array.

It should be obvious that each of the N beams is identical in form, and has the same sensitivity as the central beam, except for the effects of the element response. Since the number of beams is equal to the number of feeds, this arrangement will survey N times faster than a single antenna of the same aperture. The survey speed equation can then be inverted to give the number of subapertures required to survey one steradian to an rms noise level of σ_{mJy}:

$$N_{el} = \frac{3}{T_{yr} A_e B_G} \left(\frac{T_{sys}}{\lambda_{cm} \sigma_{mJy}} \right)^2 .$$

Since small horn antennas are relatively simple structures (compared to large parabaloids), we might conceive of arrays consisting of hundreds, or even thousands, of feeds, connected to specially constructed FFT machines – an arrangement which would offer drastic improvements in survey capability. It could even be easily imagined that each horn output goes first to a 1-dimensional (temporal) FFT routine, to obtain the spectrum, and the output then to the 2-d spatial FFT machine. In this way, full imaging over a large solid angle, and full spectral information will be gained.

However, there is a significant downside to this concept. The N_{el} data streams must each be sampled at the Nyquist frequency, at least, so the FFT engine must operate at this same rate, in order to preserve the information. Elementary considerations show that R, the rate of operations, in GOPS (Giga-Operations per Second), must be of order

$$R = 2B_G N_{el}(N_c \log_2 N_c + N_{el} \log_2 N_{el}),$$

where B_G is the bandwidth in GHz, N_{el} is the number of feeds, and N_c is the number of spectral line channels. The Table below roughly gives the number of elements, the required processing speed, and the size of each element required to survey 1 steradian to 1 mJy at 3 mm wavelength, with a bandwidth of 2 GHz, a total lens area of 2000 sq. meters, with a system temperature of 150K, in the time listed in the first column. The right-hand column gives $\sqrt{A_e/N_el}$, the subaperture size in centimeters.

Time	Number	R (GOPS)	L_{el}(cm)
1 day	70,700	3.2×10^{11}	17
1 month	2,280	5.7×10^{7}	94
1 year	190	1.1×10^{6}	324

I know of only one such instrument at this time – T. Daishido of Waseda University is developing a suitably modest version, consisting of 64 elements operating at 10.6 GHz. See Daishido *et al.* 1991, for more details and references.

4. Array Data Processing

Arrays make a lot of data. The rate of data production is proportional to the number of baselines time the number of channels, and inversely proportional to the integration time. In my survey of operating instruments, taken in preparation for this meeting, I asked about improvements being implemented, and being planned. Many instruments are increasing the number of antennas, and/or developing new correlators which without exception provide far more channels per baseline than before, and plan to record data with shorter integration times. Some of the new correlators being built will provide enormously greater spectral flexibility – the AT and the EVN have, or will have, correlators providing 4096 channels per baseline, while the VLBA's can give 2048 channels per baseline. A result of these improvements will be enormously greater data processing requirements. Below are listed the maximum data rates for a few instruments, in megabytes per hour:

VLA	150
AT	90
WSRT	75
BIMA,NRO	20
VLBA	1800

Although these values are maxima, and are not likely to be regularly maintained, even an average rate of 10% of these numbers will strain the capacity of many data reduction systems. Added to this enormous data flow are the processing

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requirements. Multichannel data usually require multichannel images. Certain VLA projects can usefully use 2048 x 2048 x 512 pixel images. Some VLBA afficionados can envisage even larger image cubes for their projects. Fortunately, the recent workstation explosion has greatly improved our prospects of keeping up with the data flow. While the types of projects listed above will require supercomputing of some type, the majority of projects to be scheduled on these machines can be accommodated in existing computers. This situation is an enormous improvement over the situation of two years ago.

In my view, the real challenge is in interpretation – the interface between the generated data cubes and the human brain. Both the NRAO and the AT are slowly developing visualization systems whose goal is to improve this interface. This is an area of critical need, and one which desperately needs more resources. We have succeeded wonderfully in making marvelous instruments. Now we must learn how to use them to their intended capacity.

5. The Non-Coplanar Array Problem

I will end this review with a few words on the 'non-coplanar array' problem. The wording suggests some terrible effect, but in truth, the only 'problem' here is in the computing. A proper solution to this 'problem' exists, but is computationally expensive.

The origin of the problem is in the simple fact that, unless special arrangements are made, arrays make their measurements of the coherence function in real three-dimensional space – the (u, v, w) volume. Yet we commonly treat the data as if they were functions only of two dimensions – u and v. Projecting the visibility data to the $w = 0$ plane is accomplished by multiplication by $\exp(2\pi i w)$. But it must be noted that this correction is only valid for one point on the sky – the phase tracking center. All other points, in general, are incorrectly adjusted. This is the geometric origin of the error. We grid the data as if they were taken on a plane, after having made a correction valid only for one direction.

Barry Clark, in an internal NRAO memorandum, first pointed out the origin of the problem, and also its general solution – a three-dimensional Fourier transform of the data. The result is a thre-dimensional volume whose basis vectors are the direction cosines, and in which the (dirty) sky is found on a hemispherical cap of unit radius. Those unfamiliar with this problem should refer to Perley (1989), and to Cornwell and Perley (1992) for a more complete description of the problem and its solution. Tim Cornwell has spent much time and effort finding better computational solutions to the problem, a recent memo (Cornwell, 1993) summarizes his progress to date.

The point of bringing this up now is two-fold. First: This is a general problem for most arrays. The only true exceptions are those for which the data collected are coplanar (on any plane). The best known examples are the E-W colinear interferometer, and the 2-d coplanar array used in a 'snapshot' mode. We might add, for insight, that a coplanar array at the North or South poles always takes all its data in a plane. Note that the common approximation of using a two-dimensional transform for VLA data always results in aberrations for emission away from the phase-tracking center. If the field of view is small, these errors are

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commonly small enough to be ignored. Second: arrays which suffer this problem must also suffer its consequence – a greatly increased computing load.

So how does one tell whether the coplanar problem will affect your data and your array? Unless your data lies on a plane, as explained above, you can use one of the following simple expressions, which are derived assuming the w term is of comparable size to the u and v terms. The number of required planes in the 'third' dimension is given by:

$$N_3 = \frac{\lambda B}{D^2}$$

which is appropriate for imaging of the entire primary beam, or:

$$N_3 = \frac{B\theta^2}{\lambda}$$

which is appropriate for an object with an angular size of θ radians. In both expressions, B is the maximum baseline, D is the antenna diameter, and λ is the wavelength. All length scales must be in the same units.

One can use these expressions to estimate the magnitude of the computing problem for a given instrument. It is easy to show that the current millimeter arrays, and the proposed NRAO mmA, are not affected by this problem, as the short wavelengths and baselines more than offset the small antenna element size. Similarly, the VLBA is not much affected, despite the enormous baselines – the size of objects which will be imaged is generally very small. For those fields containing widely separated knots of emission, such as maser spots, an MX-like approach will suffice. The instruments where the coplanar problem is serious are the VLA and the GMRT – especially the former. It is easily shown that for the VLA's A-configuration, the number of planes in the third dimension can be as high as 11 for 20cm, 60 for 90cm, and 225 at 400cm wavelengths. (However, there are a number of reasons why transforms such as the last two are not the correct way to solve the problem). The designers of the GMRT were fully cognizant of the problem, and took advantage of the strong dependence of the scale of computing on the primary antenna size. The GMRT's scale of the problem is about 5 times less than the VLA's, due to the 40 meter diameter of the primary antenna.

6. A Modest Summary

There can be no arguing with the success of interferometric techniques in radio astronomy. At this conference, we are seeing the emergence of infra-red and optical techniques, and I have no doubt that the impact of interferometry in these disciplines will ultimately equal that in radio astronomy. Radio interferometers are now nearly all of the aperture synthesis type, since that techniques provides maximum flexibility in post-observing image manipulation and error correction, while maintaining high sensitivity and resolution. The cost of these techniques is in the computing, and there has been enormous advances in both the algorithms and in reducing the cost of computation in the last ten years. A rather different technique of interferometry, suitable for surveys at high frequencies and for detecting short time-scale phenomena is not widely known, and might become

affordable as the cost of computing further declines. The 'bottom line' in all interferometers of the aperture synthesis type is the quantity of data they produce, and I argue that the main challenge for the future is not in the image formation from these data, but rather in the human interpretation of these images.

7. References

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Discussion:

Radhakrishnan:
You didn't mention the Arecibo correlator when you were discussing correlators.
Perley:
I wasn't aware of this correlator. In my oral presentation I have omitted mention of a number of arrays for which I could not obtain information.

Ekers:
A correlation synthesis array also makes $\sim n^2$ independent measurements of the sky so if it is configured appropriately for a survey it should have the same speed as the Dashida lens.
Perley:
You are correct, and I am guilty of changing the meaning of the variable 'N' in my talk. To avoid misunderstanding, I have modified the talk so all variables have a unique meaning.