CORRESPONDENCE.

THE FIRST ENCOUNTER WITH A LIMIT.

DEAR MR. EDITOR,—When I read the above title for one of the topics at the Annual Meeting*, I imagined that the speakers would deal with an encounter: a meeting undesigned, unforeseen, perhaps even unwelcome, such as when a boy asks, " Please, Sir, is $\cdot\dot{9}$ equal to 1 ? "

The earliest encounter mentioned by the openers was the sum to infinity of a geometric progression. Mr. Robson claimed that the earliest encounter was two years earlier than this and instanced the case of the tangent to such a curve as a cubic. I should have put it much earlier. Last term, a class of beginners in geometry were calculating the angles of a regular n-gon for increasing values of n. A boy of 11 asked, " Can it be 180° ? "

Occasionally it is interesting to give a class an outline of the history of the evaluation of π. The mention of Archimedes' work brings in the idea of a limit. It is a simple and interesting example for IVth Form boys doing numerical trigonometry to calculate areas and perimeters of regular n-gons for different values of n. Here again π appears as a limit. In the case of the angle of the regular polygon, the class graphed their results and saw the asymptotic relation of $y=180°$ Graphing the results of the trigonometrical work brings in the same idea of the asymptote for π.

Some teachers of authority and experience use the proof by limits of the properties of a tangent to a circle including that of the equality of the angle between tangent and chord with the angle in the alternate segment. Even those who prefer the Euclidean treatment point out to their classes that the Euclidean definition of a tangent to a circle must be abandoned in dealing with tangents to curves in general.

In graphical work, the solution of a set of equations,

$$x^2-2x=0,$$
$$x^2-2x=3,$$
$$x^2-2x+7=0,$$
$$x^2-2x+1=0,$$

by using the graph of x^2-2x, suggests the view that the tangent is the limiting position of a secant.

In compound interest work, questions in which the amount in two years at 4% is compared with the amount in four years at 2% are more interesting than questions with no such connections, and still more interesting are a set of questions in which the amounts of £1 approach the limit e.

I may also add that as an occasional " side-track " with bright IVth Form boys, I obtain

* *See* the Report on pp. 109-123 of this *Gazette*.

$$x=\frac{1}{2+}\,\frac{1}{2+}\,\frac{1}{2+}\,\dots$$

as the solution of $2x+x^2=1$ and obtain by arithmetical calculation the successive convergents

$$\tfrac{1}{1},\ \tfrac{3}{2},\ \tfrac{7}{5},\ \tfrac{17}{12},\ \tfrac{41}{29},\ \tfrac{99}{70},\ \dots$$

for $\surd 2$ giving $\surd(2-1)$, $\surd(2-\frac{1}{4})$, $\surd(2-\frac{1}{25})$, $\surd(2-\frac{1}{144})$, ... as convergents with $\surd 2$ as a limit.

All these examples antedate the study of progressions, that is, they are within the range of certificate elementary mathematics. And indeed, for many schools the progressions are not included in the certificate syllabus. In other words, (i) if the first encounter is to be deliberately avoided till the progressions are studied or even later, many pupils will not encounter the idea of a limit at all ; (ii) there are opportunities of introducing the idea, and indeed the idea will obtrude itself, despite the teacher's intention to introduce it only at the moment when the pupil is deemed mature enough to study the calculus.

For my part, I welcome those recurrent intrusions which give the teacher an opportunity to clarify and reinforce ideas born in boys' minds and so to give them a background of experience which renders them more receptive when the time comes for rigorous treatment. The small boy's mental vision is often singularly clear and penetrating, and if he does say such things as "the sum of $1+\frac{1}{2}+\frac{1}{4}+\dots$ is 2", it must not be taken to mean that he thinks the series can be summed, but that he is at a stage of phraseology when he has no better way of saying what he does think.

Those present at the meeting will realise that although the President extended the time for the discussion of the topic, there was not time to bring up these issues.

There is another point to consider. The topic was introduced with the statement that its genesis was due to dissatisfaction felt with the treatment of limits in the *Algebra Report*. Mr. Daltry put in a plea for a statement to be made by competent mathematicians of the precise details in which the *Report* was unsound and of the mode of sound instruction suitable for the certificate stage. I should like to back his plea and ask you to open the pages of the *Gazette* to a discussion of the topic. I do so the more pressingly, as the *Gazette* will reach most of the readers of the *Reports*, whereas only about one-tenth of that number were present at the meeting.

I am, Yours truly,

F. C. Boon.

Sir,—I have to thank you for giving me this opportunity of expanding the remarks * I made at the Annual Meeting of the Association last January.

I take it that the "school certificate" idea of a limit is completely illustrated by the result

* *See* the Report on pp. 109-123 of this *Gazette*.