Journal of Glaciology, Vol. 32, No. 110, 1986

ASSESSMENT OF MASS-BALANCE VARIATIONS WITHIN A SPARSE STAKE NETWORK, QAMANÂRSSÛP SERMIA, WEST GREENLAND

By ROGER J. BRAITHWAITE

(Grønlands Geologiske Undersøgelse, Øster Voldgade 10, DK-1350 Copenhagen K, Denmark)

ABSTRACT. Variations of area-averaged annual balance can be calculated from sparse stake networks, if the specific balances satisfy a simple version of the linear model proposed by Lliboutry. The linear model is tested with 4 years of data at 13 stakes on Qamanârssûp sermia, West Greenland, and is satisfied with a model error of ±0.34 m water equivalent. From analyses of data from three stakes a few metres apart, the random error in the measurement of annual balance is estimated to be greater than ±0.23 m water equivalent. The variation in balance for the 4 years is ±0.52 m water equivalent which reflects the year-to-year variation in climate. The shortness of the present data series is admitted and its continuation is recommended.

RÉSUMÉ. *Evaluation des fluctuations du bilan de masse avec un réseau limité de balises sur le Qamanârssûp sermia, Groëndland occidental.* Les fluctuations du bilan moyen peuvent être calculées à partir d'un réseau limité de balises si l'on peut appliquer le modèle linéaire proposé par Lliboutry. Le modèle linéaire est testé sur 4 années d'observations de 13 balises sur le Qamanârssûp sermia (Groëndland occidental). L'écart standard du résidu est de ±0,34 m d'eau. L'étude de trois balises situées à quelques mètres les unes des autres montre que l'incertitude sur les bilans annuels est supérieure à ±0,23 m d'eau. La fluctuation pendant les 4 années d'observation est de ±0,52 m d'eau, correspondant aux fluctuations interannuelles du climat. On reconnait que la série étudiée est courte et on recommande la poursuite des observations.

ZUSAMMENFASSUNG. *Schätzung der Massenbilanz-Schwankungen innerhalb eines dünn besetzten Pegelnetzes, Qamanârssûp-Gletscher, West Grönland.* Schwankungen einer über die Fläche gemittelten, jährlichen Massenbilanz können auch aus dünn besetzten Pegelnetzen berechnet werden, wenn die spezifischen Bilanzen eine einfache Version des von Lliboutry vorgeschlagenen linearen Modell erfüllen. Das lineare Modell wird mit Daten aus 4 Jahren an 13 Pegeln auf dem Qamanârssûp-Gletscher in West-Grönland getestet und erweist sich als zutreffend mit einem Modellfehler von ±0,34 m Wasseräquivalent. Aus Analysen der Daten von 3 Pegeln, die nur wenige Meter auseinander liegen, lässt sich der Fehler bei der Messung der Jahresbilanz als grösser als ±0,23 m Wasseräquivalent abschätzen. Die Schwankung in der über die Bilanz für die 4 Jahre beträgt ±0,52 m Wasser-äquivalent; sie spiegelt die Klimaschwankungen von Jahr zu Jahr wider. Es wird nicht bestritten, dass die vorliegende Datenreihe kurz ist, weshalb sich ihre Fortsetzung empfiehlt.

INTRODUCTION

An important tool for understanding the effects of glaciers on stream flow is the measurement of mass balance which has been started on several glaciers in West Greenland, using methods freely adapted from Østrem and Stanley (1969). A basic reality of mass-balance measurements in Greenland, however, is the fact that the glaciers involved are often relatively large, e.g. a few hundred square kilometres in area, as well as being poorly delineated and located in inaccessible places. It is therefore impossible to measure specific mass balances within a stake network which would be judged adequate by international standards, e.g. with at least a few stakes per square kilometre rather than a few square kilometres per stake, which is the best that can be achieved on large glaciers in Greenland.

The purpose of the present paper is to discuss whether useful information about mass-balance variations can be obtained from such sparse stake networks. Data from Qamanârssûp sermia in West Greenland are used as an example.

THEORY

Suppose the specific annual balance y_{jt} is observed at J stakes distributed over an area S for T years. The area-averaged mean balance of the area S covered by the J stakes will be given by

$$Y_t = \sum_{j=0}^{j=J} (s_j/S) y_{jt} \qquad (1)$$

where s_j is the area represented by the jth stake such that

$$S = \sum_{j=0}^{j=J} s_j . \qquad (2)$$

It is generally accepted that a fairly large number of stakes is required to calculate accurate area-averaged balances because of the need to sample all parts of the area S, i.e. that s_j should be small.

It is assumed after the simplest case in Lliboutry (1974) that the annual balance at the stake j and for the year t is a random variable given by the following linear model

$$Y_{jt} = a_j + b_t + e_{jt} \qquad (3)$$

where a_j and b_t express the stake- and time-variations of the balance, while e_{jt} is a centred-random error whose stake- and time-averages are both zero. The term b_t fluctuates randomly from year to year with a time average of zero. The model error e_{jt} will reflect the effects of (i) errors in the data whereby observed balances are not true balances, and (ii) violations of the model whereby true balances do not behave exactly according to the postulated model.

Substitution of Equation (3) into Equation (1) gives

$$Y_t = Y_0 + b_t + E_t \qquad (4)$$

where Y_0 is the area average of a_j given by

$$Y_0 = \sum_{j=0}^{j=J} (s_j/S)a_j \tag{5}$$

and E_t is the area-averaged error e_{jt} given by

$$E_t = \sum_{j=0}^{j=J} (s_j/S)e_{jt}. \tag{6}$$

This term need not be zero, even if the simple average of the error e_{jt} is zero by assumption. Y_0 is also the time average of Y_t, and by re-arrangement of Equation (4), the deviation of Y_t from this time-average Y_0 is given by

$$Y_t - Y_0 = b_t + E_t. \tag{7}$$

According to Equation (7), the deviation of the area-averaged balance from its time average is given approximately by the index b_t as long as the area-averaged error E_t is not too large. This will be satisfied if e_{jt} is small compared with other terms. The error E_t is not, of course, the absolute error in the area-averaged balance but gives a measure of how well the stake network allows the errors at single stakes to cancel each other out.

The linear model is useful for isolating time variations in mass-balance data, so that they can be compared with climatological variables. For example, Martin (1975) correlated mass-balance deviations from Lliboutry (1974) with summer temperatures and annual precipitation. Alternatively, such deviations can be compared with time variations in run-off from glacierized basins to study the effects of glaciers on stream flow, e.g. as done by Braithwaite (1985[b]). Lliboutry (1974) has suggested that the linear model should be introduced into routine work but there is little sign of this yet. No doubt many potential users have been frightened off by the rigour and detail of Lliboutry's treatment.

RESULTS

Qamanârssûp sermia is an outlet glacier from the inland ice located to the east of Nuuk (Godthåb), the capital of Greenland. The part of the glacier lying between 80 and 1500 m a.s.l., which coincides approximately with the ablation area, is estimated to have an area of 146.3 km^2 (Olesen, 1981), whilst the delineation of the accumulation area is still unknown. The field programme on Qamanârssûp sermia was started by O.B. Olesen in 1979 and has been led by the author since the late summer of 1981. A summary of climatological and glaciological statistics for the five measurement years 1979/80 to 1983/84 has been given by Braithwaite (1985[a]), while the stake locations are shown in Figure 1.

The field data

The data analysed in the present study refer to specific annual balances measured at the 13 "centre-line" stakes for

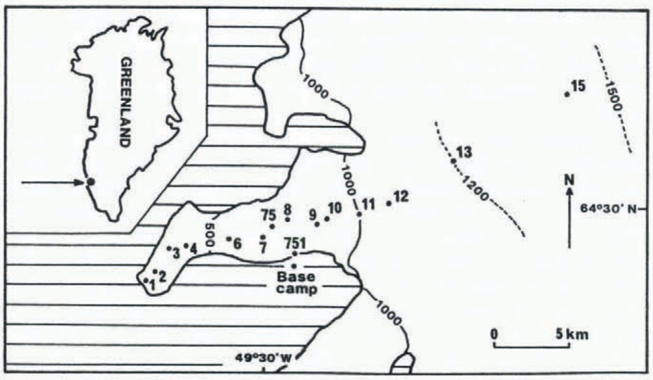

Fig. 1. Sketch map of the lower part of Qamanârssûp sermia, West Greenland, showing the locations of stakes.

TABLE I. ANNUAL SPECIFIC BALANCES AT 13 "CENTRE-LINE" STAKES FOR 4 YEARS, QAMANÂRSSÛP SERMIA, WEST GREENLAND. UNITS ARE m WATER EQUIVALENT AND THE MEASUREMENT YEAR IS 1 SEPTEMBER TO 31 AUGUST

Stake	*Elevation* m a.s.l.	*Area* km^2	*1980/81*	*1981/82*	*1982/83*	*1983/84*	*Mean*
001	110	3.5	−5.28	−4.90	−4.69	−4.64	−4.88
002	190	2.0	−4.50	−4.89	−4.28	−4.31	−4.50
003	320	4.5	−5.92	−5.44	−4.87	−4.45	−5.17
004	370	4.7	−6.33	−5.26	−4.68	−4.63	−5.23
006	580	8.2	−3.92	−3.74	−3.36	−3.65	−3.67
007	680	5.4	−5.18	−3.56	−2.71	−3.23	−3.67
075	760	3.8	−4.11	−4.57	−3.32	−3.09	−3.77
008	790	4.7	−4.23	−5.14	−2.52	−3.18	−3.77
009	910	4.7	−2.49	−2.19	−2.22	−1.97	−2.22
010	930	22.3	−1.89	−2.11	−0.67	−0.77	−1.36
011	1000	3.3	−1.38	−1.57	−0.61	−0.93	−1.12
012	1090	17.2	−1.96	−1.62	−0.87	−1.29	−1.44
013	1200	21.6	−0.41	−0.31	+0.80	+0.40	+0.12
Mean			−3.66	−3.48	−2.62	−2.75	−3.13
Total		105.9					

TABLE II. ANNUAL SPECIFIC BALANCES AT THE THREE "751" STAKES FOR 5 YEARS, QAMANÂRSSÛP SERMIA, WEST GREENLAND. UNITS ARE m WATER EQUIVALENT AND THE MEASUREMENT YEAR IS 1 SEPTEMBER TO 31 AUGUST

Stake	*1979/80*	*1980/81*	*1981/82*	*1982/83*	*1983/84*	*Mean*
751 A	−4.04	−4.62	−4.34	−3.64	−4.49	−4.23
B	−4.07	−4.51	−5.04	−3.47	−3.96	−4.21
C	−4.17	−4.93	−4.59	−4.11	−4.36	−4.43
Mean	−4.09	−4.69	−4.66	−3.74	−4.27	−4.29

the 4 years 1980/81 to 1983/84, together with 5 years of data from the three "751" stakes which are located within a few metres of each other on the margin of the glacier (at 790 m a.s.l.) near the base camp.

Data for the 13 "centre-line" stakes and the three "751" stakes are given in Tables I and II, respectively. The stake network is sparse as the stake density in the area of 106 km^2 between 60 and 1300 m a.s.l. is only 0.12 stakes/km^2.

Stakes up to and including stake 008 (790 m a.s.l) usually have little or no winter snow cover and their specific balances mainly reflect ice ablation, of which about 80–90% occurs in the period June–August. From stake 009 (910 m a.s.l.) there is a fairly substantial winter snow cover which starts to melt in mid May or early June. The "751" stakes were read almost every day during the summer whilst readings of the other stakes were more sporadic, for example, the more inaccessible stakes from 009 to 013 may have been read only two or three times during the summer. The last stake readings are made as close as possible to 31 August and then the data are adjusted with the help of data from the "751" stakes so that annual balances can be calculated for a measurement year 1 September–31 August.

Although the data in Table I refer to repeated measurements on the same stakes, it must not be assumed that conditions at each stake are constant. The "centre-line" stakes move with speeds of up to 250 m a^{-1}, whilst the main features on the glacier, e.g. ice falls, crevasse zone, and smoother areas, appear to remain fixed. This means that each stake passes through a variety of conditions with respect to local topography, so that the 4 year series at each stake are not homogeneous, but are sampled over trajectories of up to 1 km. By contrast, the total displacement of the "751" stakes is only of the order of 100 m so that the data in Table II are more homogeneous.

Testing the linear model

The model in Equation (3) is actually the simplest one of several discussed by Lliboutry (1974) and is chosen for that reason. The present sample sizes, e.g. a 13 × 4 matrix for the "centre-line" stakes, are probably too small to examine the more subtle effects of autocorrelation which Lliboutry considered. A simplification is gained by only considering complete data matrices rather than using more complicated procedures to take account of missing data. For this reason, incomplete data for 1979/80 at the "centre-line" stakes are excluded, as are incomplete data for some other stakes above stake 013. The reader is referred to Braithwaite (1985[a]) for the most complete summary for the 5 years of measurement.

The value of a_i for each stake is the mean value of each row (stake) in Tables I and II, respectively, whilst the value of b_t for each year is the mean value for each column (year) minus the mean of all stakes (Lliboutry, 1974, p. 382). The sources of variation within the data matrices in Tables I and II were computed by the "two-way analysis of variance" programme on a Hewlett-Packard HP-97 desk calculator. The results are summarized in Table III for the two data sets. A statistical test (Kreyszig, 1970, p. 277) shows that both stake and year effects are significant at the 5% level for the "centre-line" stakes. On the other hand, at the "751" stakes the year effect is statistically significant at the 5% level whilst the stake effect is not.

For the "centre-line" stakes, the stake effect is the largest source of variance, comprising 89% of the total variance. This illustrates the fact that differences between stakes may be very large compared with differences between years, i.e. the same stakes must be used for different years. The error in the linear model for the "centre-line" stakes is only 4% of the total variance, corresponding to a standard deviation of ±0.34 m equivalent.

The effect of errors in the measurements

In the case of the "751" stakes, the stake effect must be regarded as an additional error to the error effect, as both would be zero if readings on the three stakes were identical. The two effects have a combined standard deviation of ±0.23 m water equivalent. This gives a measure of the accuracy of single stake readings. However, the readings of the "751" stakes should be more accurate than those at the more remote "centre-line" stakes because they are read almost every day which allows the detection of gross errors or mistakes. By contrast, the "centre-line" stakes are only read a few times during the season and the data often suffer from the fact that the observers are tired, and a long way from home, when they make the readings. There is little doubt that there are some gross errors in the data for the "centre-line" stakes shown in Table I, e.g. the value for stake 008 in the year 1981/82 appears especially suspicious and is probably an error. The measurement errors at the "centre-line" stakes could therefore be greater than those at the "751" stakes, i.e. larger than ±0.23 m water equivalent. Measurement errors would then account for more than 45% of the total error variance, e.g. about one-half, whilst the rest would be caused by real violations of the linear model.

Variations of annual balance

The annual deviations of mass balance for the 13 "centre-line" stakes, given at the bottom of Table III, have a standard deviation of ±0.52 m water equivalent. The standard deviation of the corresponding error effect is ±0.34 m water equivalent, which is reduced to only ±0.05 m water equivalent by stake averaging according to Equation (6). This latter figure does not, of course, represent the absolute error, i.e. the difference between the data collected in the present sparse network and in a hypothetical dense network. However, it does indicate a reasonable degree of averaging out of errors in the present network.

The differences between the mass-balance deviations for the "centre-line" and "751" stakes have a standard deviation of only ±0.24 m water equivalent, which is of the same order as the error in the data for the "751" stakes. This represents reasonable agreement between the two sets of stakes. However, they cannot be expected to give identical results as their responses to climate control will not be the same, especially because winter snow cover tends to be absent from the "751" stakes. The discrepancy between the 1983/84 deviations, i.e. +0.41 and +0.07 m water equivalent, respectively, may reflect the fact that there was generally more winter snow at the "centre-line" stakes but not at the "751" stakes. On the other hand, the extreme positive deviations for both data sets for 1982/83 reflect the fact that the 1983 summer was exceptionally cool with reduced ablation at all stakes.

TABLE III. SOURCES OF VARIATION AMONG ANNUAL SPECIFIC BALANCES AT THE 13 "CENTRE-LINE" STAKES AND AT THE THREE "751" STAKES, QAMANÂRSSÛP SERMIA, WEST GREENLAND

	"Centre-line stakes"	*"751" stakes*
Number of stakes	13	3
Number of years	4	5
Sources of variation:		
Stake effect	145.06 (89%)	0.15 (6%)
Year effect	10.62 (7%)	1.90 (72%)
Error effect	5.99 (4%)	0.57 (22%)
Total variation	161.67 (100%)	2.62 (100%)
Standard deviation	±1.78 m water	±0.43 m water
Annual deviations in m water equivalent		
1980/81	−0.53	−0.35
1981/82	−0.35	−0.32
1982/83	+0.51	+0.60
1983/84	+0.38	+0.07
Mean	0.00	0.00
Standard deviation	±0.52	±0.44

DISCUSSION AND OUTLOOK

Statisticians can object that the samples analysed here are too small to draw general conclusions. This is correct and further analyses will be made as more data become available. However, it is a hard fact of life that the applied glaciologist cannot afford to collect data over a statistically adequate number of years before attempting to draw any conclusions at all. The present conclusion is that useful information can be obtained from sparse stake networks such that they should be continued in the future, rather than being discontinued; the alternative of making stake networks in Greenland more dense cannot be considered at present.

The present preoccupation is with the assessment of mass-balance variations on a scale of tens to hundreds of square kilometres. There is, however, the broader perspective of the whole Greenland ice sheet and its possible changes in response to climate. It would be quite impossible to measure the mass balance of the whole ice sheet within an adequate stake network. On the other hand, it might be possible to monitor time variations of the mass balance by a relatively small number of strategically spaced stakes, i.e. a sparse stake network on a scale of hundreds of thousands of square kilometres. Considering the global consequences of any major change in the mass balance of the ice sheet, the development of such a large-scale, albeit sparse, stake network should be considered seriously within the next few years.

ACKNOWLEDGEMENTS

This paper is published by permission of the Director, The Geological Survey of Greenland. The project at Qamanârssûp sermia was set up by O.B. Olesen. Many student assistants helped to collect data, especially J.-O. Andreasen and O. Bendixen from Aarhus University, and J. Andsbjerg and C.E. Bøggild from Copenhagen University.

REFERENCES

Braithwaite, R.J. 1985[a]. Glacier-climate investigations in 1984 at Qamanârssûp sermia, West Greenland. *Grønlands Geologiske Undersøgelse. Rapport*, Nr. 125, p. 108-12.

Braithwaite, R.J. 1985[b]. Relations between annual runoff and climate, Johan Dahl Land, south Greenland. *Grønlands Geologiske Undersøgelse. Gletscher-Hydrologiske Meddelelser* 85/2.

Kreyzig, E. 1970. *Introductory mathematical statistics.* New York, John Wiley and Sons.

Lliboutry, L. 1974. Multivariate statistical analysis of glacier annual balances. *Journal of Glaciology*, Vol. 13, No. 69, p. 371-92.

Martin, S. 1975. Corrélation bilans de masse annuels – facteurs météorologiques dans les Grandes Rousses. *Zeitschrift für Gletscherkunde und Glazialgeologie*, Bd. 10, 1974, p. 89-100.

Olesen, O.B. 1981. Glaciological investigations at Qamanârssûp sermia, West Greenland. *Grønlands Geologiske Undersøgelse. Rapport*, Nr. 105, p. 60-61.

Østrem, G., *and* Stanley, A.D. 1969. *Glacier mass balance measurements; a manual for field and office work.* Ottawa, Canadian Department of Energy, Mines and Resources/Oslo, Norwegian Water Resources and Electricity Board.

MS. received 14 March 1985 and in revised form 14 December 1985