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ARTICLE

Are Mathematical Explanations Causal Explanations in Disguise?

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Abstract

There is a major debate as to whether there are non-causal mathematical explanations of physical facts that show how the facts under question arise from a degree of mathematical necessity considered stronger than that of contingent causal laws. We focus on Marc Lange's account of distinctively mathematical explanations to argue that purported mathematical explanations are essentially causal explanations in disguise and are no different from ordinary applications of mathematics. This is because these explanations work not by appealing to what the world must be like as a matter of mathematical necessity but by appealing to various contingent causal facts.

1 Introduction

There is a major debate as to whether some physical facts have a mathematical, as opposed to causal, explanation. Among the various accounts of mathematical explanations (Baker 2009; Baron 2016; Lange 2016), Marc Lange's account of distinctively mathematical explanations (DMEs), as in Lange (2013, 2016), stands out as the most persuasive defense of the purported existence of mathematical explanations. This is because DMEs are claimed not only to explain the existence of certain physical phenomena but also to show how there is a mathematical necessity associated with the existence of such phenomena, regardless of the contingent causal laws in operation.

DMEs can be framed as modal conditionals of the following form: if P, then necessarily Q for the reason R, where P is the antecedent, Q is the consequent, and R is the explanans.[1] In a causal explanation, the phenomenon, or the explanandum,

[1] Although such a characterization of the conditional is endorsed in some recent works on non-causal explanations, such as that of Reutlinger (2018) and Rice (2021), this is not common in the literature. We

Q (understood with the background conditions P) is explained via R, where R is a contingent causal law, which takes into account, for instance, particular forces that causally affect the phenomenon. In a DME, however, the explanans R is a largely mathematical fact, and thus the explanation of Q derives from facts that are modally stronger than contingent causal laws, such as mathematical facts or framework laws that are modally stronger than contingent causal laws.[2] The mathematical constraints (R) relevant to the explained phenomenon in a DME thus have an alleged degree of necessity surpassing that of any contingent causal law (in P and Q) that may potentially be used in a causal explanation of that phenomenon.[3] DMEs are considered forms of non-causal explanations because the explanatory power of a DME is said to derive not from its description of the causal nexus of a target system but from the "non-causal" facts relevant to the explanation.[4]

We first clarify Lange's position on causal explanations, DMEs, and the role of contingencies in DMEs in section 2. In section 3, we highlight how Lange's account is inconsistent in classifying many examples of causal explanations as DMEs, and vice versa, despite the examples exploiting mathematical and contingent facts in an analogous manner. This inconsistency, we argue, comes from a misunderstanding of the role of the contingencies in purported mathematical explanations and from the way Lange reformulates and *manipulates* questions pertaining to the ordinary application of mathematics to the physical world, mischaracterizing them as DMEs. In several of these cases, an explanandum pertaining to natural facts is narrowed down (by presupposing various contingent, and effectively causal, facts) to a point that the only relevant fact left to be explained is a largely mathematical fact, which, not surprisingly, is then explained with an alleged DME. We conclude that either one must accept that all applications of mathematics to the world, suitably reformulated in such a manner, are DMEs—in which case the philosophical point of Lange's account is lost—or accept that all DMEs are causal explanations in disguise, where the guise primarily concerns how contingent facts are moved around in the associated conditional (as presuppositions) to fabricate DMEs.

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mainly use conditionals to bring out the presuppositions of purported DMEs, which helps clarify Lange's muddied and inconsistent classification of causal explanations and DMEs, as we show later. Lange also employs conditionals to defend his account: "[I]f we have a scientific explanation in which the explanandum E follows from the explanans C by some mathematical proof, then (in an appropriate context) an answer to 'Why is it the case that if C, then E?' can be [—] Because this conditional fact is mathematically necessary" (Lange 2018b, 87). Thanks to a referee for urging us to clarify the use of conditionals in the article.

[2] For instance, Lange (2016, 29) considers Newton's classical force-acceleration law as a framework law because, say, even if the peculiar form of Newton's gravitational law were to be different (inverse-cube instead of inverse-square, say), the framework law would still hold.

[3] One may note that other accounts of mathematical explanations, such as the extra-mathematical account by Baker and Colyvan (2011) and the deductive-nomological account by Baron (2019), do not stress modal differences as the primary line of demarcation between mathematical and nonmathematical (such as causal) explanations. We do not discuss accounts other than DMEs in this article, but our arguments generalize to all such accounts where mathematical explanations can be formulated as conditionals.

[4] We will often use the terms "DMEs" and "non-causal explanations" interchangeably throughout this article, even though the notion of non-causal explanations is wider. We are unable to discuss other kinds of non-causal explanations, such as grounding explanations, for instance, in this article.

2 Causal explanation and the "why" question

Lange (2013) says that to explain an event causally is to provide some information about its causal history (Lewis 1986) or to show how the event was embedded in the "causal structure of the world" (Salmon 1977; Sober 1984). Lange (2013) notes that a DME, a non-causal explanation,

> works instead by (roughly) showing how the fact to be explained was inevitable to a stronger degree than could result from the causal powers bestowed by the possession of various properties. If a fact has a distinctively mathematical explanation, then the modal strength of the connection between causes and effects is insufficient to account for that fact's inevitability. (487)

That is, even if both causal explanations and DMEs may cite facts about the explanandum's causes, a DME does not work by virtue of describing the explanandum's causes (Lange 2013, 496).

By way of clarification, consider the following example of Lange's:

> [T]hat twenty-three cannot be divided evenly by three supplies information about the world's network of causal relations: it entails that there are no causal processes by which twenty-three things are distributed evenly (without being cut) into three groups. But in the distinctively mathematical explanation of Mother's failure [of dividing 23 strawberries evenly among her three children], the fact that twenty-three cannot be divided evenly by three does not possess its power to explain by virtue of supplying this information about causal processes in particular. The distinctively mathematical explanation does not exploit what the world's causal structure is like as a matter of mathematical necessity. Rather, it exploits what the world is like as a matter of mathematical necessity: the fact that twenty-three things cannot mathematically possibly be divided evenly (while remaining uncut) into three groups explains why no collection of twenty-three things is ever so divided. (Lange 2013, 496)

Lange thus claims that DMEs show how certain facts about the world are modally constrained as a matter of mathematical necessity. A causal explanation does not necessarily provide information on such modal constraints and instead derives its explanatory power from a case-to-case causal description of the facts using contingent causal laws, which are modally weaker than mathematical facts.

Even though DMEs concern modal facts, Lange freely admits that contingencies (involving such modally weaker facts) can form part of the "why" question pertaining to a DME. Lange, however, clarifies that DMEs do not "exploit" these contingent causal powers—a claim we dispute—because these contingencies are subsumed as initial conditions in the "why" question:

> That bridges [concerning the Königsberg bridge case] are not brought into existence or caused to disappear by people travelling over other bridges, and that strawberries are not caused to replicate by being distributed, reflect the causal powers of various things and are matters of contingent natural law, not

> mathematical necessity ... the fixity of the arrangement of bridges and islands, for example, is presupposed by the why question that the explanation answers ... The laws giving the conditions under which the bridges' arrangement would change thus do not figure in the explanans. (Likewise, it is understood in the why question's context that the relevant sort of 'crossing' involves a continuous path.) (Lange 2013, 497)

According to Lange (2016), the explanatory facts in a DME thus explain either by their modal necessity or by "being understood in the why question's context as constitutive of the physical task or arrangement at issue" (506). We will go on to show in this article that the account of DMEs is fallacious on both of these grounds for the following reasons: either the necessity claimed by purported DMEs actually comes from the contingent causal presuppositions of a DME (which are explanatorily prior to the associated mathematical formalism), or the necessity becomes manifest only in those cases where an explanandum pertaining to a natural fact is narrowed down to a point where it effectively becomes a mathematical explanandum (with some added physical details). We first discuss how some DMEs championed by Lange rely on false conditionals and thus cannot be DMEs, such as the purported explanation of the existence of at least two antipodal points on the surface of the earth, at any given moment, with equal temperature and pressure. We then demonstrate that any attempt to save these DMEs by narrowing down the associated conditional fails because such a narrowing down only highlights the role of contingent causal presuppositions in the explanation of an allegedly "natural" fact. We finally demonstrate that the cases in which the conditionals do hold true are the cases where the explanations offered are merely instances of ordinary applications of mathematics to the physical world, where such applications have been suitably reformulated to appear as if they were DMEs.

3 Contingency in DMEs: Inconsistent examples

What is to be understood as denoting the "why-question's context" or "constitutive of the physical task" is critical for our thesis. We show that the distinction between background facts and contingent facts rests on what is explanatorily relevant and thus context sensitive (but not necessarily in the way Lange thinks of them). We argue that Lange's account is inconsistent in how this "why" question is formulated: some "why" questions that include contingent causal facts are considered causal explanations, whereas some are not, despite them exploiting mathematical facts in an analogous manner. This inconsistency, we argue, comes from the way Lange reformulates and manipulates questions pertaining to the ordinary application of mathematics to the physical world by classifying them as DMEs. We first illustrate how Lange reformulates the "why" questions by moving contingencies from the explanans to the explanandum. Then we discuss some examples to highlight what is inconsistent about such a reformulation, and in the next section, we discuss what is wrong with such a strategy more generally.

Consider a conditional, C, of the following form: If P, then Q because R, where P includes the given or initial conditions, Q is the explanandum, and R is the explanans. Now consider an example involving a reformulation of this conditional:

CB1: If bees make honeycombs (P), then these honeycombs take the shape of a hexagon (Q) because it is selectively advantageous for honeybees to minimize the wax they use to build their combs—together with the mathematical fact that a hexagonal grid uses the least total perimeter in dividing a planar region into regions of equal area (R).

The explanans in CB1 contains both contingent and mathematical facts—the contingent fact pertains to the (causal) evolutionary history of bees, and the mathematical fact pertains to the properties of hexagons. Lange claims that CB1 is a causal explanation because the explanans consists of both contingent and mathematical facts—that is, the explanation of why bees make hexagonal honeycombs perhaps comes partially from the (causal) selection pressures felt by bees in the course of their evolutionary history. If, however, the contingent fact concerning the (causal) evolutionary history of bees is subsumed in the "why" question instead, then we get CB2, which Lange claims to be a DME:

CB2: If bees create the most efficient honeycombs (in terms of the amount of wax used), then these honeycombs take the shape of a hexagon because of the mathematical fact that a hexagonal grid uses the least total perimeter in dividing a planar region into regions of equal area.

Notice that in the narrowed-down conditional of CB2, the explanans is largely mathematical, and the contingent facts about the (causal) evolutionary history of bees form a part of the explanandum—the DME is allegedly saved. In this reformulation, it appears that Lange is seeking an explanation of a fact *related* to the original fact (about why bees make hexagonal honeycombs), not the original fact itself. And thus one might say that the original (natural) fact had no DME, whereas the "related" (largely mathematical) fact has a DME.

However, we now show how analogous reformulations reveal the inconsistency in Lange's account concerning which "why" questions (pertaining to certain physical facts) have a DME and which ones have a causal explanation. We discuss an example of a purported DME of there being two antipodal points with the same temperature and pressure on the earth (at any given instant) and an apparently causal explanation for the existence of Gaussian distribution curves for gas diffusion. We later show what is wrong with such reformulations more generally.

3.1 Antipodal temperature patterns and the case of symmetric gas diffusion

Several philosophers and mathematicians argue that the existence of at least two antipodal points on the earth with the same temperature and pressure at any given moment of time has a mathematical explanation (Lange 2016, 7; Colyvan 2001, 49–50; Kosniowski 1980, 157–59).[5] If there is a continuous mapping between a manifold (such as a sphere, assuming the earth to be one) and a physical variable (such as temperature or pressure), then the Borsuk–Ulam (BU) theorem entails the existence

[5] Although Lange only mentions the case of antipodal points with the same temperature around the equator, the idea and the supporting mathematical framework extending this to such antipodal points around the earth remain essentially the same.

of at least two such antipodal points at any given instant (Kosniowski 1980, 157–59). Consider two antipodal points p_1 and p_2 on a sphere S. There are two facts to be explained here, as Colyvan (2001, 49) notes: "(1) why are there any *such* antipodal points? and (2) why p_1 and p_2 in particular?" Colyvan further notes that an explanation of (2) needs to be provided in causal terms (based on a causal history of the event that traces the distribution of temperature and pressure over the earth) but the explanation of (1) can be given in non-causal or distinctively mathematical terms using a corollary of the BU theorem. We do not go into the details of this theorem; however, for our purposes, it is crucial to note that the theorem exploits the continuous mapping of an n-sphere (mapped to the distribution of temperature or pressure, for instance) onto the Euclidean n-space such that a pair of antipodal points on the sphere (pertaining to the value of the physical variable) are mapped to the same point in $\mathbb{R}^n$.

Lange believes that the application of the BU theorem to this case is a DME because once the underlying physical assumptions (such as the continuity of temperature) are packaged into the "why" question or the explanandum, the resulting explanation is distinctively mathematical because the explanandum obtains with a greater necessity than the contingent causal laws. In a charitable interpretation of Lange's position, irrespective of how different meteorological conditions varied or evolved around the earth or however the micro-causal properties/laws of temperature distribution were to be, at least two antipodal points are still guaranteed by the BU theorem in each of these instances even if they differ in their causal details. More so, Lange (2016, 21) argues that in the event that we have a pair of causal explanations pertaining to two different meteorological conditions (where such antipodal patterns were observed), it "inaccurately depicts this similarity between the two moments [of having the same antipodal temperature at each moment] as utterly coincidental—as having no important common explainers—since the earlier meteorological conditions relevant to one moment are largely disjoint from those relevant to the other moment" (Lange 2016, 21). This shows that the "necessities [revealed in this explanation] are stronger than the variety of necessity possessed by ordinary laws of nature, setting explanations like these apart from ordinary scientific explanations" Lange (2016, 9). Also, notably, the explanation does not take into account any physical fact or law governing or defining temperature except the fact that the temperature and the pressure need to be considered as continuous variables, which, as per Colyvan (2001, 49) is a "minor structural" assumption. (This "minor structural" assumption cannot always be justified, as we discuss later.) This makes the explanation, as per Lange, distinctively mathematical.

Now consider an example that Lange claims is not distinctively mathematical. Consider the diffusion of gas molecules, being infinitely concentrated initially and starting from the origin in a homogeneous, boundless, and two-dimensional medium. Lange (2016, 33) asks, Why is it that the concentration curve $\Phi(x, y, t)$ of the gas diffusion at each subsequent time t looks like a Gaussian bell-shaped curve around the origin on the x–y axis, namely, one in which the concentration of the gas is proportional to $e^{-(x^2+y^2)/b}$ for some constant b? We skip the details of explanation for the sake of brevity, but Lange (2016, 33) cites an explanation by John Herschel that appeals to the rotational and translational symmetry of gas diffusion, which gives rise to a Gaussian distribution of the diffusion of gas molecules.

Lange points out that similar symmetry-based reasoning can be used to explain why the resulting distribution pattern of an archer aiming at a target or the velocity distribution of the molecules of an ideal gas is Gaussian. Interestingly, Lange (2016) claims that such examples are genuine instances of DMEs or non-causal explanations because the rotational symmetry of gas molecules or the distribution of the shots of a skillful archer or of the velocities of the molecules of an ideal gas result from the micro-causal processes governing them:

> [I]n appealing to the fact that a parcel's likelihood of having a given *x*-coordinate is equal to its likelihood of having the negation of that *x*-coordinate, Herschel is appealing to features of the initial velocity distribution of the diffusing particles, the initial state of the diffusing medium, and the laws governing the micro-causal processes. In short, the various formal features to which Herschel's derivation appeals pertain directly to the micro-causal processes underlying diffusion and in this way derive their explanatorily priority over ϕ's exponential character: the explanandum is an effect of causes described by the explanans. Therefore, Herschel has given us a causal explanation. (35)

Thus, as per Lange, the Gaussian distribution, and its various formal features, achieved in gas diffusion, archery errors, or the velocities of ideal gas molecules is "neither modally more necessary than ordinary laws of nature nor understood in the why question's context as constitutive of the physical task or arrangement at issue" (Lange 2016, 35).

However, the categorization of one example as a DME and the other as a causal explanation is inconsistent because they both include contingencies in the explanans. Consider a formulation of these explanations as conditionals:

> CG1: If a gas parcel starts diffusing from a defined position, where it is infinitely concentrated, in two directions, then the resulting concentration curve of the gas must be a Gaussian bell-shaped curve, for the reason that the gas diffusion equation evolves in an exponential manner *owing to the rotational symmetry of gas diffusion*.

> CT2: If the temperature around the equator can be defined at each point and time, with the meteorological parameters allowed to vary in whichever manner possible, then the resulting distribution of temperature must be such that there are at least two antipodal points with the same temperature, for the reason that the formal properties of the BU theorem are satisfied over such distributions *owing to the continuity of temperature*.

The general form of CG1 and CT2 is of CEx:

> CEx: If P obtains, within a range of initial conditions, then the mathematical structure/distribution resulting from its time evolution has the formal properties denoted by Q, for the reason R.

Lange claims that CG1 is a causal explanation because R1 (the reason that CG1 holds) involves contingent causal phenomena, notably the symmetries pertaining to the case of gas diffusion, which is "explanatorily prior" to the formal structure pertaining to Q (exponential form of the Gaussian) that provides the rest of the explanation. But CT1 is an analogous explanation—there are contingent causal facts that are explanatorily prior to the consequent Q2 of CT1 as well. That is because the formal properties of the BU theorem are satisfied only for continuous functions, and thus, the continuity of temperature is explanatorily prior to the formal topological structure of the BU theorem that provides the rest of the explanation in CT1; the explanation collapses without the continuity. The continuity of temperature is, however, a contingent causal phenomenon that depends on various factors, such as the vapor pressure of the droplets evaporating in the atmosphere; their interfacial conditions, including surface contamination; and so on (Jha et al. 2023). Temperature discontinuities across interfaces has been widely confirmed both on theoretical and experimental grounds in various studies of heat transfer across solid–gas interfaces, such as those by Fang and Ward (1999) and Chen et al. (2022). The idea is that "abrupt changes in the microscopic material conditions at liquid-vapour interfaces (phase transitions) and at solid-solid interfaces (thermal boundary resistance) enters the thermal transport models as temperature discontinuities" (Jha et al. 2023, sec. 6). This is because there is a resistance to heat flow at such interfaces, and this may well prevent the establishment of an equilibrium and a uniform continuous distribution of temperature across such interfaces. Jha et al. (2023, sec. 3.1) further point out that extrapolating a continuous function, overlooking the underlying discontinuities, will produce an incorrect distribution of temperature variation across the interface. Therefore, both the continuity of temperature and the symmetry of gas diffusion are analogous contingent causal facts that play an important role in making sure that the relevant mathematical formalism applies to the case at hand and produces correct predictions. Thus, Lange cannot claim that CT1 is a non-causal explanation if CG1 is not one—they are both indeed causal explanations.

We anticipate two objections here. Primarily, one may object that continuity is not really a "causal thing," in that continuity does not *cause* the values of temperature to be what they are around the surface of the earth. We reply to this objection by stating that the continuity of temperature does enter the network of causal relations and causes the temperature distribution to be what it is, in the sense that if one point is wiggled in the continuous chain, the other point would be wiggled as well (by virtue of the chain of the values of temperature being continuously connected around the sphere). In fact, this is why continuity is crucial for the physical application of the BU theorem—and indeed for analogous topological explanations in general. Moreover, as pointed out before, the continuity enters the explanation as a modally weaker fact (being dependent on micro-causal processes), in that it does not necessarily obtain from the modally stronger fundamental conservation laws of physics or by virtue of some other mathematical fact. However, Lange's account of DMEs requires that "constraints [that appear in the 'why' question of a DME] must be explained via other constraints rather than by modally weaker facts" (2016, 129). His mistake is not to recognize that the continuity of temperature is a modally weaker fact than the mathematics that is doing the rest of the explanation in E2, even as this fact critically underwrites the purported mathematical explanation. Lange's account further requires that "[a]ny causes cited by a causal explanation explain by virtue of being causes and thereby supplying information about the network of causal relations ... [and] when causes figure in non-causal explanations, the source of their explanatory power is not their status as causes" (2016, 18). However, the symmetry of the gas distribution and the continuity of temperature figure in the alleged DMEs as causes in the sense that they supply information about the network of causal relations (namely, regarding what kind of concentration curves are permitted in CG1 and what temperature distributions are feasible in CT1).

A related objection could be: Can one not presuppose continuity to be a part of the background condition for the "why" question, as done for other cases, like that of the Königsberg bridge case, where the "fixity of the arrangement of the bridges" (including the crossing of a "continuous" path on the bridges) is presupposed to be a part of the background condition? We respond that contingencies like the "fixity of the arrangement of the bridge" and the continuity of its crossing are of a distinct kind (given the context) than the contingency of the continuity of temperature or the symmetry of the diffusion of gas molecules. That is because the facts related to the "fixity of the arrangement of the bridges" or the continuous crossing do not participate in the explanation as contingent causal facts but instead participate only as ordinary background facts in the relevant explanations (as also noted by Lange 2013, 491–97).[6] In contrast, contingent facts like the continuity of temperature or the rotational symmetry of the diffusion of gas molecules participate in the relevant explanations as contingent causal facts. Here is why. One could perhaps imagine a hypothetical flexible arrangement of the Königsberg bridges where each node of the bridge moves by a few centimeters every few minutes (preserving its overall graph-theoretic structure), and that would make no difference to the explanation of why one cannot cross all the bridges exactly once in a single continuous crossing. Even if there were minor discontinuities in the structure of the bridges (cracks, for example), as long as one can walk on the bridge, the purported topological explanation would still apply. (One can suitably approximate that they are still continuous.) That is because the exact shape of the bridges is irrelevant to the explanation; they can be distorted or bent in any way as long as the graph-theoretical structure of the bridges (containing information about the relative positioning of the bridge nodes) is maintained. As for the case of untying a trefoil knot: although the explanation of why one cannot untie a trefoil knot is a (purported) topological explanation and the continuity of the knot is critical for its success, the continuity of the knot participates in the explanation only as an ordinary background fact. This is because if the knot is not continuous, it can no longer be a (topological) trefoil knot, and thus the related "why" question as to why one cannot untie it becomes moot—that is, task constitution fails. However, in the case of the antipodal temperature patterns, if temperature is not continuous, it may still be true that under some arrangement of the meteorological conditions, there may be two antipodal points with the same temperature around the equator—that is, one may still be able to constitute the related task, so to say, of finding such antipodal points. But under such circumstances, one can no longer be certain that there *must* be two such points because the BU theorem, which underpins the purported DME, no longer applies to discontinuously

[6] We thank a referee for pressing us to clarify this point.

distributed functions. Thus, the conditional associated with the purported DME will be false because the continuity of temperature participates as a contingent *causal* fact in the purported DME. Moreover, continuity cannot be assumed away (via mathematical extrapolations or approximations) for the purported explanation to work (unlike in the case of the Königsberg bridges) because doing so would produce incorrect predictions about the distribution of temperature, given how abrupt and large temperature discontinuities can be across small regions of space, as Jha et al. (2023) argue. Similarly, if the gas molecules are not symmetrically diffused around a supposed origin, the mathematical distribution of their concentration may not be exponential at all. This tells us that facts like continuity and symmetry have a different status in these contexts: they participate as (contingent) causal facts that are directly relevant for explaining the purported existence of antipodal points and the exponential distribution of gas molecules, respectively. Presupposing such contingent facts as supposedly ordinary background facts, in what is claimed to be a DME, is misleading. This is because the physical facts (in need of explanation) that were claimed to be "necessary" by virtue of the associated mathematical formalism fail to be necessary—their alleged necessity (so to say) actually arises from the contingent causal facts presupposed in the explanation. Therefore, a purported mathematical explanation that presupposes such contingent causal facts, which participate as causal facts, is essentially a causal explanation in disguise and, indeed, an ordinary application of mathematics, as we discuss shortly. For this purpose, the distinction between what constitutes an ordinary background fact or a contingent causal fact is context sensitive and can be crucial to evaluate whether the explanation of a "physical" or "natural" fact qualifies as a DME.

Jha et al. (2022) discuss an analogous problem concerning the narrowing down of the conditional for the case of double pendulums discussed by Lange. They show that Lange's explanation of the minimum number of equilibria of a double pendulum (a DME that is championed throughout his book) is false because not all double pendulums have four or more equilibria. Only those double pendulums that have Morse potential energy functions (simply put, well-behaved functions with no degenerate critical points) have four or more equilibria, and thus a suitable topological explanation is forthcoming only when the system has such a potential energy function. Jha et al. (2022) further argue that a narrowing down of the conditional that attempts to save Lange's claim (by presupposing pendulums with Morse potential energy functions) will sneak in causal reasoning through the back door (concerning contingent causal factors) because this requires prior knowledge of the particular forces acting on the pendulum, which affects the form of its potential energy function. This makes the role of contingent causal presuppositions obvious in the explanation because the necessity–because it mistakenly appears to arise from the mathematical formalism—is not distinctively mathematical.

3.2 More on the contingent causal presuppositions of a purported DME

Lange (2018a), in his response to Craver and Povich (2017), says:

> Of course, what is understood as constituting the physical task or arrangement at issue can sometimes obviously be affected by including slightly more in the

> why question. Returning to the case of the flagpole and shadow, Craver and Povich (2017: 36-7) take the why question "Why is the shadow's length l?" (where the answer would cite the flagpole's height h, the sun's angle of elevation θ, that the flagpole and ground are straight and form right angles, that light travels in straight lines, that space is Euclidean, etc.) and compare it to the why question "Why is the shadow's length l when the flagpole's height is h, the sun's angle of elevation is θ, the flagpole and ground are straight and form right angles, light travels in straight lines, space is Euclidean (etc.)? Regarding that question, they say that "the only relevant fact left to do the explaining seems to be the trigonometric fact that $\tan\theta = h/l$" (p. 37). I roughly agree: "[I]f we have a scientific explanation in which the explanandum E follows from the explanans C by some mathematical proof, then (in an appropriate context) an answer to 'Why is it the case that if C, then E?' can be 'Because this conditional fact is mathematically necessary.' This would be a 'distinctively mathematical' explanation" (Lange 2017: 419). In this case, the explanandum is changed from E to the fact that if C then E. The reason why it is the case that if C then E, rather than its not being the case that if C then E, is that the latter is mathematically impossible. (87)

If one continues to restrict the explanandum by including more and more contingent facts (including both background facts and causal facts) in C—that is, in "if C, then E"—then the explanandum (which was earlier supposed to be about a natural fact) can be narrowed down to a largely mathematical fact. One wonders as to what constitutes a reasonable limit for tightening the conditional in such cases. Should one be allowed to exploit all the nonmathematical facts (including contingent causal facts and ordinary background facts) as presuppositions or, assuming them to constitute the task at hand, cherry-pick the left-out mathematical facts (wrapped in a physical guise) and then claim that we have a DME of a physical phenomenon? That would be absurd because if that were the case, then all that one would be left with is a mathematical explanation of barely mathematical facts, and that being so, Lange's pitch about DMEs being "scientific explanations" of physical phenomena would be a hopeless endeavor. Apparently, this is what is going on in these cases. We discuss some further examples illustrating this claim. Our claim will be that such explanations are either causal explanations in disguise (mixed explanations, where both mathematical and causal facts provide the explanation) or ordinary applications of mathematics where the associated conditional has been narrowed down to a point that it involves a largely mathematical explanandum (which then is explained mathematically).

Consider Peter Lipton's stick toss discussed by Lange. Lipton (2001, 49) argues that "a mid-air snapshot of a collection of randomly tossed sticks will tend to show more of the sticks in a near-horizontal orientation than in a near-vertical orientation." An explanation E of this (alleged) fact is that "there are more ways for a stick to be near the horizontal than the vertical" (Lipton 2001, 49). Lange (2013, 508–9), however, argues that whether Lipton's explanation qualifies as a causal explanation or as a DME depends on how the explanandum is framed.

> Perhaps what is doing the explaining there is a propensity of the stick-tossing mechanism (that it is equally likely to produce a tossed stick in any initial orientation) together with a propensity of the surrounding air molecules (that they are equally likely to push a tossed stick in any way). (After all, if the tossed sticks were instead all spinning uniformly about axes that lie in the horizontal plane, they would be as likely at any moment to be vertical as horizontal, contrary to what we observe.) If the explanans in Lipton's example includes these propensities, then the explanation seems more like the explanation of a fair coin's behavior in terms of the propensities of the chance set-up than like [Lange's] . . . examples . . . of distinctively mathematical explanations [DMEs] in science.

Lange (2013) thus argues that if the explanans E subsumes the evenness (symmetry) of the propensity of the stick-toss mechanism, then Lipton's stick-toss case qualifies as an ordinary causal explanation. However, if the evenness of the propensities (a contingent causal fact) is subsumed in the explanandum rather than in the explanans, Lange (2013) argues that Lipton's stick-toss case then qualifies as a DME. Consider CS1, a causal explanation, and a reformulated CS2, a purported DME, as follows:

> CS1: If one takes a snapshot of a collection of randomly tossed sticks, then the snapshot tends to show more of the sticks in a near-horizontal orientation than in a near-vertical orientation because all the orientations of the sticks are equally likely and there are more ways for a stick to be near the horizontal than the vertical.
>
> CS2: If one takes a snapshot of a collection of randomly tossed sticks, given that all orientations of the tossed sticks are equally likely (i.e., possible orientations are symmetrically distributed along each orientation), then the snapshot will asymmetrically tend to show more of the sticks in a near-horizontal orientation than in a near-vertical orientation because there are more ways for a stick to be near the horizontal than the vertical.

According to Lange (2013), CS2 highlights the fact that despite the symmetry in the orientation of tossed sticks, with each orientation of the sticks being equally likely, there is an asymmetry in the outcome—that is, more sticks are observed to be in a near-horizontal orientation than in a near-vertical orientation, and thus, CS2 qualifies as a DME; CS1 remains as a causal explanation. However, CS2 suffers from the same problem as discussed earlier: the symmetric distribution of the orientation of the stick, the symmetric propensities of the gas molecules to diffuse in all directions, and the continuity of temperature are all analogous contingent causal facts that participate as causal facts in the explanation. Whether a stick can be oriented along each possible direction depends on whether the stick's angular momentum is conserved in that direction and a host of other factors, including the kind of forces acting on the stick, the viscosity of the medium in which the stick is tossed, and so forth (Galdi 2002, 656–57). Discussing an analogous case of a fair coin's toss (which is essentially a stick much shorter than it is wide), Mahadevan and Yong (2011) show that pure geometric reasoning in the case of coin-tossing, which assumes that all coin orientations are equally likely (along a sphere, which is the coin's configuration space, i.e., a locus of all possible positions the coin could be positioned in, given its geometry), is flawed, and thus one needs to engage in dynamical reasoning to consider only those coin orientations that conserve the angular momentum of the tossed coin. CS2 thus cannot be a DME either.

If one objects to this claim, then one ought to respond to the question: What is the distinctively mathematical component in the purported DME of the stick-toss case? After all, CS1 and CS2 are true not only in a horizontal direction but also in any plane that constitutes two orthogonal directions. There is a greater number of lines in any plane versus a single line, and therefore if we reformulate CS2 as there being a greater number of sticks oriented in the x–y (horizontal) direction as against the z direction (vertical), or in the x–z direction as against the y direction, or in the y–z direction as against the x direction, then, admittedly, the same form of explanation will be forthcoming. One may ask: Is it a "mathematical fact" that a plane (representing a plane in physical space) has a greater number of lines than a line (representing a line in a physical space), or is it merely a physical fact that is indexed by a mathematical fact?[7] Why is CS2 even a DME when all that is left to explain (once the causal contingencies about the equal propensity of the sticks to orient themselves along each direction have been assumed away in the explanandum) is that there are more ways for this to happen along a direction with a greater number of lines (orientations) versus a direction with a lesser number of lines? This seems like an obvious mathematical fact that is in no need of a DME. If it was the case that the symmetric propensities of the sticks were modally stronger than the contingent causal laws, we may have had a DME of a "physical" phenomenon. Left without an explanation of the "physical" phenomena (concerning the likely orientation of sticks in cases that include all propensities of sticks, both symmetrical and asymmetrical), one only has a mathematical explanation of a bare mathematical fact put in a physical guise, namely, that a plane (representing the orientation of sticks) contains a greater number of lines than a single line (the vertical orientation of sticks). The inclusion of contingent causal phenomena in the antecedent thus only reveals the causal dependencies of what was claimed to be a DME. Also, if the conditional is narrowed down to an extent (every possible contingency being tucked away in the explanandum) that we lose the more interesting original explanandum, then the DME loses its appeal because the strong necessity with which the original phenomenon was thought to obtain no longer stands true. Therefore, such reformulations of purported DMEs, where all the contingent causal facts contributing to the explanation are moved away to the explanandum, are not only illicit but also of little explanatory value because they take away too much and leave too little to explain. This can be illustrated in another way, in some more detail, as we show next.

3.3 Ordinary application of mathematics are no different from DMEs

What if we can narrow down an explanandum in a way that the narrowing down only appeals to ordinary background facts that do not seem to participate in the

[7] See Saatsi (2011) for a discussion of related issues.

explanation as causal facts? For instance, presumably, the question as to why honeybees make hexagonal honeycombs has a causal explanation (CB1), but for the question as to why, given that honeybees make such honeycombs, their wax-use efficiency is maximized when they construct hexagonal honeycombs has an alleged DME (CB2). We deny this claim as well, for the reason that in such cases (to reiterate), the explanandum gets narrowed down to a point where all that is left to explain is a bare mathematical fact in a physical guise. Such reformulations lose their appeal as an attractive philosophical account illuminating the distinctive role that mathematics may play in such explanations because these reformulations are essentially run-of-the-mill applications of mathematics to the physical world. To make this point, we now demonstrate that such a narrowed-down explanandum is essentially a reformulation of a mathematical fact as a "constraint," dressed up with physical facts, that is then explained mathematically. We argue that if such a reformulation of a mathematical fact qualifies as a DME, then every such application of mathematics to the physical world (the vast majority of the application of mathematics to the world) qualifies as a DME. This trivializes Lange's account because it fails to distinguish between the applicability of mathematics and its role in a purported DME.

Consider the following questions in need of an explanation. These questions gradually move from seeking an explanation of purely mathematical facts to seeking explanations of their constraint-based reformulations that include some physical details:

Q1: Why does the number 23 divided by 3 give the answer 7.66...?

Q2: Why does the number 23 divided by 3 give a non-integer?

Q3: Why is the number 23 not divisible (evenly) by 3?

Q4: Why are 23 objects not evenly divisible into three collections of whole and unbroken objects?

Q5: Why are 23 strawberries not evenly divisible between three children?

Q1 is a straight-up mathematical fact, and Q2–Q5 are constraint-based reformulations of the same fact—that is, we are moving to higher-order, more abstract reformulations of the same mathematical fact as seeking the explanation of a constraint (being a non-integer, being indivisible, and so forth). Q4 and Q5 are higher-order reformulations analogous to Q2 and Q3 but also include some physical facts attached to explanation-seeking mathematical facts (constraints). Admittedly, the explanation for Q1–Q3 coming from number theory should be characterized as a mathematical explanation of mathematical facts. And if this is so, then one wonders whether it is legitimate to merely add physical details to these reformulated questions (Q2 and Q3) depicting mathematical constraints and then present them as DMEs of physical facts (Q4 and Q5). In moving from Q3 to Q4, Lange (2013, 496) claims that the reason why Q4 holds, as mentioned in section 2, is because "the fact that twenty-three things cannot mathematically possibly be divided evenly (while remaining uncut) into three groups explains why no collection of twenty-three things is ever so divided." Hence, Lange argues that the mathematical fact about the indivisibility of 23 by 3 provides a general explanation for why all such causal instances (of dividing a certain number of objects evenly between a certain number of groups) will also meet the same fate. But this general explanation is essentially a higher-order reformulation of a mathematical fact (presented as a constraint), in the guise of a physical constraint that is in need of explanation—if this explanation qualifies as a DME, then every such application of mathematics qualifies.

However, one may object that Q3 and Q4 are different "why" questions and have different answers: whereas the former involves physical objects, and an answer to it can be presented in allegedly distinctively mathematical terms (that 23 is not divisible by 3), the latter involves purely mathematical objects, and presumably an answer to that will be a mathematical proof.[8] But this only highlights that an answer to the former question (Q4), a physical question, cannot be purely or distinctively mathematical, unlike what Lange argues. This is because such an answer not only involves contingent background facts (e.g., the existence of strawberries, a mother dividing them, and so on) but also, crucially, disguised contingent causal presuppositions, namely, the principle of mass conservation and the individuality of the strawberries (i.e., their integrity as they are divided—or any individual object, as the case may be). These presuppositions do not participate as trivial, ordinary background facts merely constituting the task at hand; these are the very facts that make a purely mathematical result applicable, relevant, and meaningful in a physical context. Without these fundamental and contingent causal principles in place, there is absolutely no necessity enforced on the physical world by the mathematics. The purely mathematical result—that 23 is not evenly divided by 3—is thus not a complete answer to Q4.

Further, because these principles are presupposed as a part of the "why" question in Lange's account, thereby reducing a physical situation to a largely (if not purely) mathematical situation, there is nothing interesting (or even barely scientific) left to be explained, really! Consider this example on the use of integrals in calculating the electric field strength pertaining to a linear charge distribution, which Lange (2013) believes to be a causal explanation but when seen in the light of the points just discussed (and based on Lange's own position) should actually be classified as a DME:

> to explain why the electric field strength at a distance r from a long, linear charge distribution with uniform charge density is equal (in Gaussian centimeter–gram–second [CGS] units) to $2\lambda/r$, we can integrate the contributions to the field (given by Coulomb's law) from all segments of the line charge. When the integral is simplified, it becomes $\frac{\lambda}{r}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \cos\theta \, d\theta$. The argument then makes special appeal to the fact that $\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \cos\theta \, d\theta = 2$. But intuitively it is not a distinctively mathematical explanation in science. (491)

We could reformulate this case in the same format as previously, starting from a purely mathematical question Q1 (e.g., Why does cosθ, when integrated over infinitesimal intervals of θ between $-\pi/2$ to $+\pi/2$, yield 2 as the answer?) and moving to a physical question Q5 (i.e., Why does a cosine function, given that it correctly quantifies over the electric charge contributions from an infinitely long

[8] Thanks to a referee for pressing us on this objection.

charge [with uniform charge density], when integrated over appropriate intervals [to yield the electric field strength at a certain radial distance from this charge distribution], always yield a constant quantity as the answer?), where all relevant contingent causal presuppositions have been made. When reformulated like this, subsuming all the causal contingencies into the "why" question, this explanation, too, fits into Lange's version of DMEs, even as he claims otherwise.

Coming to the reduction of natural facts into mathematical facts by presupposing ordinary background facts, consider CB2 again and its reformulation, CB3.

> CB2: If bees create the most efficient honeycombs (in terms of the amount of wax used), then these honeycombs take the shape of a hexagon because of the mathematical fact that a hexagonal grid uses the least total perimeter in dividing a planar region into regions of equal area.

> CB3: If one were to create the most efficient polygon division (in terms of having the least total perimeter) into regions of equal area, then one would make hexagons because of the mathematical fact that a hexagonal grid uses the least total perimeter in dividing a planar region into regions of equal area.

CB3 is essentially an explanation of the mathematical fact that hexagons maximize planar area while minimizing perimeter and thus demonstrates a more general physical fact than CB2. Apparently, then, CB3 shows that CB2 has nothing to do with honeycombs or honeybees as such because it is merely a reformulation of the properties of hexagons by adding the physical details of the context in which these hexagons exist, namely, in the honeycombs created by honeybees. Thus, CB2 (generalized as CB3) applies to any hexagonal shape that exists physically. One is free to choose any relevant context for its existence and then formulate a question, based on that context, requiring an explanation. For example, while dividing his paper sheets into regions of equal area while minimizing the total perimeter of the region, why did Sam manage to achieve this by drawing hexagonal shapes? An analogous pattern of explanation can thus follow for any geometrical fact (a geometrical fact reformulated as a physical constraint), and thus every bit of geometry that applies to the world, when reformulated so, will then appear to be a DME. For instance, Why does the shortest distance between two people on the two corners of a square-shaped courtyard lie along the diagonal? Or, Why do two people, initially situated on the two corners of a square-shaped flat courtyard, collectively exert the least energy when they walk along the diagonal to meet each other? Both these questions are essentially reformulations of the mathematical fact (in a physical guise) that the shortest distance between two corners of a square runs along the diagonal connecting them. The cases of the Königsberg bridges and the trefoil knot mentioned earlier are merely more sophisticated examples of analogous formulations. Lange, in fact, grants that the geometrical explanations of these physical facts are DMEs; he argues that the word allusions one finds in mathematics textbooks illustrate instances of DMEs. For example, "if Farmer Brown, with fifty feet of negligibly thin and infinitely bendable fencing, uses the fencing to enclose the maximum area in a flat field, then Brown arranges it in a circle" is an instance of a DME as per Lange (2013, 500). However, we have shown that any geometrical fact, when reformulated as a constraint along with the relevant physical context of the instantiation of that geometrical fact, counts as a DME. And if that is the case, then all other applications of mathematics (suitably reformulated) must be given the same status: this trivializes DMEs as torch-bearing the "distinctive" role played by mathematics in such explanations. All these cases merely illustrate mathematical explanations of mathematics facts that find instantiation in the physical world. Lange seems to completely miss this point, and this results in his inconsistent classification of such examples.

One could presumably reformulate all mathematical facts applicable to the world as constraints, add some physical details (and context) to them, and then present their explanations as DMEs. If Lange agrees with this assessment, then his account loses its appeal as a philosophically illuminating account of the "distinctive" role mathematics plays in explaining such physical facts in the world because any application of mathematics, when suitably reformulated as a constraint, can be termed as a DME. (How does Lange's account illuminate our philosophical understanding of the role of mathematics, then?) The dilemma confronting Lange is this: if one grants that all kinds of quantification of physical facts by mathematical facts and their suitable reformulations (as higher-order, constraint-based, or abstract questions) can be classified as DMEs, then there is nothing to gain from Lange's account, and if one does not grant that, then there are no DMEs—all purported DMEs are essentially run-of-the-mill examples of the ordinary applications of mathematics in causal explanations.

4 Conclusion

We have shown that the facts allegedly explained by a DME do not obtain because of a mathematical necessity but by appeal to the world's network of causal relations. Such an explanation may best be conceived as a mixed explanation where mathematics participates in the explanation analogous to how it participates in its ordinary applications to the world. An upshot of this thesis is that mathematics operates *within* the contingent domain and not as a constraint on what the physical world *must* be, as mathematics by itself contains insufficient information to do so. This, however, leaves some important questions unanswered. One important question relates to the applicability of mathematics to the natural world. Even if causal contingencies are presupposed in a "why" question, mathematics still plays a substantial role in the explanation of some physical facts (e.g., equal values of temperature and pressure on antipodal points on a sphere in the case that these variables behave continuously over the sphere): How is mathematics able to play such a role if mathematics cannot non-causally explain the existence of a physical fact? This question presumes that indexation is not enough to fully account for the applicability of mathematics.[9] Does this mean that mathematical structures essentially represent physical structures in some deep way?[10] Levi (2012) mentions several cases where a mathematical fact can be understood and reasoned in physical terms. For instance, one can understand why the Pythagorean theorem is applicable to the world by understanding that the theorem can be "derived" (or understood, less controversially) from energy-

[9] Saatsi (2011) and Knowles and Saatsi (2019), however, argue that the representational nature of mathematics is sufficient to explain its applicability in such explanations.

[10] Aristotelian realists may be inclined toward this view. See Franklin (2014) for a discussion.

conservation principles.[11] Levi's (2012) book—*The Mathematical Mechanic: Using Physical Reasoning to Solve Problems*—demonstrates several other examples of a similar "derivation" of mathematical facts from physical facts or principles, including mathematical facts belonging to differential calculus, complex numbers, trigonometry, and so forth. We suspect that the book is an excellent place to start looking for an answer to how mathematics applies to the natural world in purported non-causal explanations and more generally about the applications of mathematics to the natural world. The reason behind our suspicion is that if mathematics contains insufficient information to provide a non-causal explanation of a physical fact but is still able to feature in a mixed explanation where causal facts support the explanation, where is the rest of the information (relevant to explain the physical fact) coming from? Put differently, how is mathematics able to provide a way to reason about physical facts in the world *after* causal facts have been accounted for? We hope future research sheds light on these questions.

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[11] See Skow (2015) for a discussion on the "derivation" of mathematical facts via physical facts.

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