basic theory and applications, and the second containing a useful list of transforms and their inverses. Throughout the text, it is assumed that the reader has had a good deal of experience with the operational calculus in one variable. The notations used in the book are those of Continental Europe, and the reader accustomed to the Anglo-American usage must continually consult the list of notations given in Part II.

Part I consists of two chapters. Chapter 1 deals entirely with the definitions and properties of the two-dimensional Laplace transform. In the second chapter, the authors state the basic definitions and prove some of the important theorems of the operational calculus in two variables. Also, some of these results are applied both to the evaluation of certain definite integrals and to the solution of some partial differential equations. The authors base their discussions on the Laplace-Carson transform,

$$F(p, q) = pq \int_0^\infty \int_0^\infty e^{-px-qy} f(x, y)dxdy.$$

They show that it is a simple matter to prove that the relations which hold for the Laplace transform of Chapter 1 also hold for this transform. The authors illustrate the basic theory in the last section of the second chapter by using this transform to find solutions of some first order partial differential equations, the equation for the vibrating string, the heat-conduction equation, and Laplace's equation. An excellent list of formulae is given in the second part of the text.

On the whole the book is well written. However, one should observe that it is written primarily for the reader who wishes to apply the operational calculus in two variables, rather than for those who are interested in a more rigorous treatment of the subject.

M. LOWENGRUB

SCHWERDTFEGER, H., *Geometry of Complex Numbers* (Oliver and Boyd, Edinburgh, 1962), xi+186 pp., 30s.

One way of introducing complex numbers into geometry is to admit them as individual coordinates of points, thus obtaining a complex geometry. Alternatively, the real and imaginary parts of a single complex number may be used as the real coordinates of a point in a plane, the familiar Argand diagram or Gauss plane of elementary mathematics. The second approach forms the topic of the present book so that, as we are warned in the Introduction, the geometry is essentially real, though " imaginary circles " have to be considered in so far as they define real inversions.

The first of the book's three long chapters deals with the analytic geometry of circles, which are represented by hermitian matrices, with stereographic projection and cross-ratios. Chapter II gives a very comprehensive account of the properties and classification of Moebius transformations. In the final chapter these results are applied to hyperbolic, spherical and elliptic geometry. The author's aim is to broaden the student's view of the relations between modern algebra, the geometrical properties of complex numbers and other geometrical theories. That students should be discouraged from assigning knowledge to water-tight compartments is most praiseworthy, but the reviewer wonders whether time could be found in the average course for so detailed a discussion of the book's specialised topic. Those who are interested in such an exhaustive treatment will, however, undoubtedly find it useful.

Some points of minor detail might perhaps have been treated more economically. a somewhat simpler construction for the fourth harmonic of z_3 with respect to z_1 and z_2 (page 38) is to join z_3 to the point of intersection of the tangents at z_1 and z_2 to the circle through the three points; the second intersection of the line and the circle is the point required. If the condition for a Moebius transformation to be an

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involution is obtained by interchanging the variables in a bilinear relation and subtracting, instead of by squaring a matrix as on page 49, the fact that it suffices for *one* pair of distinct points to have the involutory property is proved simultaneously; the author has to give a separate proof on the following page. A slight modification of the argument on page 54 shows that *three* perpsectivities suffice to construct a projectivity on a line.

The book is clearly printed and contains a large number of examples, the most important of which are accompanied by solutions, thus augmenting the theory developed in the text.

D. MONK

KEENE, G. B., *Abstract Sets and Finite Ordinals* (Pergamon Press, 1961), 106 pp., 21s.

This book shows how the theory of finite sets and ordinals may be based on set theory. To avoid Russell's paradox (about the set of all sets which do not belong to themselves) set theory must be rather complicated. Here Bernays' version of set theory is used. Bernays distinguishes between *classes* and *sets*. Every set determines a class having the same members, but not vice versa. The only objects that can be members either of classes or sets are sets. There is an axiom that if x is a set and y is a set, there is a set whose only members are y and the members of x. Taking $x = y$, we have x', the set whose only members are x itself and the members of x (it is called the self-augment of x). Starting with the empty set O, we obtain O' which we denote by *1*, *1'* which we denote by *2*, and so on. Every class C determines a predicate or property, that of belonging to C; but not every predicate P defines a class, so we cannot necessarily speak of the class of all sets having property P. Writing capitals for classes, small letters for sets, the predicates defining classes include those of the forms $x \in y$, $x = y$, $x \in C$, and any predicate obtainable from such expressions by means of *and*, *not*, $\exists$. We cannot obtain Russell's paradox, but we can do ordinary mathematics.

The book requires no previous knowledge of logic or mathematics. It is intended for the general reader and for students of logic or mathematics. Most undergraduate students of mathematics would find it rather hard reading. It can be highly recommended to graduate students of mathematics.

D. G. PALMER

ROTH, K. F., *Rational Approximations to Irrational Numbers* (University College London Inaugural Lecture) (H. K. Lewis & Co. Ltd., 1962), 13 pp., 3s. 6d.

In this inaugural lecture Professor Roth surveys the field of Diophantine approximations, a subject to which a new lease of life has been given by his own work. A series of theorems are stated and explained, culminating in the Thue-Siegel-Roth theorem, and the lecture concludes with a brief discussion of simultaneous approximation and other unsolved problems.

R. A. RANKIN

SHKLARSKY, D. O., CHENTZOV, N. N., AND YAGLOM, I. M., *The USSR Olympiad Problem Book*; Revised and edited by IRVING SUSSMAN; Translated by JOHN MAYKOVICH (W. H. Freeman and Company, 1962), 452 pp., £3, 3s.

This book contains 320 unconventional problems in algebra, arithmetic, elementary number theory and trigonometry. Most of them first appeared in the competitive examinations sponsored by the School Mathematical Society of the Moscow State University and in the Mathematical Olympiads held in Moscow. The book is designed for Russian students between thirteen and sixteen years of age, and very bright they must be. As the authors say, there are few problems whose solutions require mere