

OBITUARY

BRIAN HARTLEY, 1939–94



Brian Hartley began his algebraic career as one of Philip Hall's research students in Cambridge. He obtained his Ph.D. in 1964, spent two post-doctoral years in the USA and, on his return to the United Kingdom, accepted a lectureship in the newly established Mathematics Department at Warwick University; there he was promoted to a readership in 1973. He was appointed to a chair of pure mathematics at the University of Manchester in 1977 and was Head of the Mathematics Department there from 1982–4. He was elected to the London Mathematical Society in 1968 and served on Council from 1987–9. He won an EPSRC Senior Research Fellowship, but died on 8 October 1994, a few days after taking it up. He travelled widely and took a lively interest in other cultures and languages. His intellectual energy, enthusiasm for algebra, direct manner and dry sense of humour endeared him to the many mathematical friends he made around the world. He was devoted to mathematics and gave generously of his time and energy in support of younger colleagues.

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1. *Early life, 1939–56*

Brian Hartley was born in Accrington, Lancashire, on 15 May 1939, the only son of Thomas Hartley (1907–69) and his wife Annie (née Kay) (1912–87). His sister, Mavis, was born three years later. Thomas and Annie had married in 1934 and, as was common then, waited to have children until they could afford them. Until 1946, they lived with Amelia Kay, Annie's mother. Since both Tom and Annie went out to work, Brian was largely brought up by Mrs Kay. Annie worked in a cotton mill. Tom Hartley was a joiner and later a foreman for a firm of building contractors; he wanted his son to become an architect, so that he could work indoors and tell other people what to do.

Soon after his fourth birthday, Brian started at Spring Hill Infants School; he transferred to Peel Park Infants in June 1945, then to Spring Hill County School in August 1946, just after the family had moved to a small terraced house in Fife Street. His success in the 11+ examination in 1950 won him a place at Accrington Grammar School. He was good at everything and eventually became head boy. The headmaster, Mr Bernard Johnson, described him as 'a boy of most unusual ability and power of concentration' and said that he did not remember 'any pupil in over 30 years' teaching experience who was superior to him. His results in the General Certificate were phenomenal, easily a record for the school.' Like many boys good at mathematics, Brian also excelled at chess. Usually top of the school chess table, he became captain of the school chess team, won local championships and played for the Lancashire team. He was one of the school's most enthusiastic musicians; this early love showed itself principally in trumpet playing and jazz. At Cambridge he played the trumpet in the Concert Orchestra, but, after taking lessons during his year in Chicago, mostly played the guitar. In 1956 Brian took GCE Advanced Level in Mathematics, Physics and Chemistry, and passed each with Distinction. Curiously, in the same year, he passed O-Level *Geometrical and Engineering Drawing*, a subject he did not keep up in later life: he inherited his father's tools, but they gathered rust in the garage. On the basis of his A-Level results he was awarded a State Scholarship and, in December 1956, won a Major Scholarship in Natural Sciences at King's College, Cambridge.

Accrington was an industrial town, predominantly engineering works and cotton mills, with acres of back-to-back houses and everywhere large numbers of smoky chimney stacks. Brian and his friends would escape into the surrounding countryside through running and cycling. In his last two years at school he was in the cross-country team, captained by Ron Hill, who three times represented Great Britain in Olympic Games. In his life story (10) Hill describes what marvellous runs there were 'along cobbled streets, up on to gritty, muddy farm tracks, across fields, over damp, mossy wooden stiles, down through woodland paths with the smell of cold rotting leaves rising as feet slithered'. Brian's life-long love of cycling also began in Accrington, with trips to the Yorkshire dales with school friends and events with the North Lancashire Clarion Cycling Club. Although in later life, and wherever he was, he often cycled 40 or 50 miles on a Sunday, his glory days were at Cambridge, where he was Secretary of the University Cycling Club and could ride 100 miles in 7 hours. In time trials, which he won in 1960, his speed was over 22 m.p.h. and in 1960, 1961 and 1962 he represented Cambridge against Oxford in the 25-mile cycle trials. His love of the outdoors stayed with him throughout his life.

2. Cambridge and America, 1956–66

At the end of 1956, Hartley was told in a letter from D. Stockdale, his prospective Director of Studies at King's, that in the scholarship examinations he had 'put up a very good show' and that 'They gave you high (almost very high) marks in Mathematics and Chemistry.' Stockdale discussed some of the pros and cons of reading Natural Sciences as opposed to Mathematics and invited Hartley, if he was considering 'pursuing the mathematical path' to 'come down and have a talk with Mr Ingham (Director of Studies in Mathematics) and myself.' Hartley accepted this invitation, with the result that he began in October 1957 to read for Part I of the Mathematical Tripos.

The Mathematics Faculty in the late 1950s was remarkably strong in pure mathematics: P. Hall, W. V. D. (later Sir William) Hodge and A. S. Besicovitch, succeeded by H. Davenport in 1958, held Chairs, whilst J. F. Adams, M. F. (later Sir Michael) Atiyah, J. W. S. Cassels, H. P. F. (later Sir Peter) Swinnerton-Dyer, J. A. Todd and E. C. (later Sir Christopher) Zeeman were some of those in junior positions. Besides Ingham, the mathematics teaching fellows at King's were N. C. Routledge, J. H. Williamson and F. D. Ursell, one of Hartley's future colleagues at Manchester. Hall, who was also at King's, no longer supervised undergraduates. In 1958 Hartley passed Part I of the Mathematical Tripos with first class honours and in 1960 was a Wrangler in Part II. During the summer of 1960, with a travel award from King's, he and a group of friends cycled through Scandinavia up to the Arctic Circle and back. In October of that year, he moved out of College to live in a top-floor room at 5 St. Mary's Chambers, just off King's Parade. At a Christmas dance at Hockerill Training College, he met Mary E. Mawer, whom he married eighteen months later in Clare College Chapel. In June 1961 he passed Part III with Distinction; others to do so that year included his future colleague at Warwick, T. O. Hawkes, his King's friend J. K. M. Moody, and N. T. Varopoulos who everyone believed came top.

Hartley began research under Philip Hall in October 1961, and for the next three years mainly worked on stability groups. Most of his initial results on this subject are contained in his first paper [1], which he wrote with Hall; his doctoral dissertation [109] contained more extensive results. During 1962–3 he took his first steps in Russian, and at the end of the university course *Russian for Scientists* passed the examination in 'translation into English (with the aid of a dictionary)'. Other research students doing group theory in Cambridge in the early 1960s included D. J. S. Robinson, J. S. Rose, A. H. Rhemtulla and S. E. Stonehewer; J. H. Conway was doing number theory with Davenport. In December 1961, Hartley was elected to a Research Fellowship at Clare College. This unusually early election, before he had had time to do any research, seems to have been made on the basis of his Part III results because Clare was short of mathematics teaching. He lived in Memorial Court until his marriage to Mary in the summer of 1962. Lord Ashby, then Master of Clare, kindly let them have their reception in the Master's Lodge. After their wedding they lived in Blinco Grove. It was not until 1972 that they had children: Catherine Frances, born in January 1972, and Christine Elaine, born in April 1974. Both grew up to obtain first class degrees and work in the City.

Hartley spent 1964–5 at the University of Chicago amongst distinguished algebraists: A. A. Albert, I. N. Herstein, I. Kaplansky, S. MacLane, R. G. Swan and J. G. Thompson. It was the time of the 'brain drain' when the British Government was worried about the number of able academics who were deserting for higher

salaries and better conditions in the USA; at Heathrow on the way out, Hartley was interviewed as the 100,000th brain. He hoped 'to take advantage of the presence of numerous distinguished ring theorists in Chicago to learn some ring theory'. J. L. Alperin, then an associate professor in Chicago, thought 'Chicago was the exciting place to be if one was a group theorist simply because Thompson was there and at his peak'. There was much to excite besides algebra: organ recitals in the Rockefeller chapel; concerts by Julian Bream and Segovia; Miles Davis and Art Blakey at their zenith; Dizzie Gillespie; drinking at Jimmy's; driving away at Christmas in a big red Chevrolet; learning to ride, to ski, and to skate; camping in the Rockies and in Florida. It was all a far cry from Cambridge.

In January 1965, Hartley received an offer of a position at M.I.T. and in August of that year moved to Boston to become a C. L. E. Moore Instructor. This was a plum job, with only six hours of teaching and good pay. Alperin, who held a C. L. E. Moore three years earlier, estimates the present-day equivalent at about \$70,000. Just as in the previous year, Hartley made the most of it. On 29 July 1966, he and Mary sailed for home on the *SS United States*. A fortnight later they were in Moscow for the International Congress of Mathematicians. By the end of August they had moved into a flat in Coventry. On 1st September they bought a *Triumph Herald*. By early October Brian had enrolled at the Coventry Technical College for a further course in Russian and a week later was again taking guitar lessons. One of the precepts that Hartley passed on to his daughters was 'there is always another five minutes in the day for something you really want to do'.

3. Warwick, 1966–77

Whilst Hartley was in Chicago, E. C. Zeeman, helped by D. B. A. Epstein, was setting up the Mathematics Institute at the new University of Warwick. By October 1965, when undergraduates were first admitted, R. W. Carter, J. A. Green, L. H. Hodgkin, B. J. Sanderson and R. L. E. Schwarzenberger had joined them. One of Green's principal tasks was to recruit more algebraists. On the advice of Philip Hall, he wrote to Hartley at M.I.T. and offered him a lectureship; he also invited Hartley's Cambridge contemporaries S. E. Stonehewer and T. O. Hawkes. All three accepted; Hartley and Stonehewer began in October 1966 in time for the first Warwick Symposium, which was on finite groups; Hawkes joined them a year later. In 1970 Hartley and Hawkes published a successful undergraduate text [107]: *Rings, modules and linear algebra*. Hartley was promoted to Reader in 1973 and remained at Warwick until 1977. Until January 1971 the nascent Mathematics Institute was housed in 135 Kenilworth Road, on the corner of Stoneleigh Road. Life at 135 was exhilarating: everyone was young and felt the future was theirs to create. From the earliest days algebra was an important component of Warwick mathematics, and Hartley played his full part in both teaching and research. Hartley's first research student was I. N. Stewart, who wrote a dissertation on *Subideals of Lie algebras* and later became a leading writer of popular books on mathematics.

During his time at Warwick, Hartley spent two more productive years abroad. In July 1969 he flew with Mary to Canberra to take up a Visiting Fellowship for ten months at the Australian National University, where B. H. Neumann had attracted numerous group theorists. They arrived at University House to find everyone glued to the television set watching Neil Armstrong take his 'giant leap for mankind'. In October Hartley had briefly to return to England when his father died. In Canberra

he got to know R. M. Bryant (there as a post-doctoral visitor), who was to be close to him in Manchester, P. J. Cossey, R. A. Bryce and D. McDougall. While students back in Warwick, clamouring for change, were occupying the Registry, Hartley worked on Sylow subgroups of locally finite groups [15], but full-time research was not wholly to his liking because he missed the contact with students and staff.

He left Canberra in May 1970 to take up an invitation from G. E. Wall to lecture at the University of Sydney and in September returned home, via Bali, Singapore, Delhi, Agra and Vienna, to what he saw as the unappealing prospect of living, for the first time since 1964, under a Conservative government. Although over the years Hartley moved some way to the right from his political starting point of ‘red in tooth and claw’, he remained left-of-centre.

Throughout his life Hartley loved to travel. In August 1973 he went for seven weeks to the University of Alberta, to work with his old friend A. H. Rhemtulla. In September the following year he made the first of a number of visits to Novosibirsk. Later on he was a frequent visitor to Russia. He rarely missed a group theory meeting at Oberwolfach and seldom turned down an invitation to speak at other conferences. He was a particularly able speaker; his lectures, lively but not at all showy, and delivered in a crisp north-country accent, were informed by immense knowledge of group theory and, more often than not, contained significant contributions of his own. His attractive, rather dry, often self-deprecating, humour, signalled a fraction of a section beforehand by a twinkling in his eye, made them all the more enjoyable.

Accompanied by his family, Hartley spent 1974–5 in Madison at the University of Wisconsin, where I. M. Isaacs, D. S. Passman and L. Solomon all held permanent positions. Passman was his principal contact and became a good friend. He remembered they used to go cross-country skiing together, with Hartley the surer footed. Passman has said how generous and helpful Hartley was, ‘always willing to make helpful suggestions and never asked for credit. He appreciated mathematical talent in others and helped to nurture it.’ In [41], the last of six papers he wrote that year and where he proves theorems on groups whose irreducible modules are all injective, Hartley mentions the influence that Passman had on his work and the stimulating conversations they enjoyed. After spending the summer with H. N. Ward at the University of Virginia, Hartley returned home, and with S. E. Stonehewer began preparations for the 1977–8 Warwick Symposium *Infinite Groups and Group Rings*. Early in 1976 he was approached about a position to become vacant when I. G. Macdonald left Manchester for London and in the summer of that year accepted a chair at the University of Manchester.

4. Manchester, 1977–94

In October 1977, Hartley took up his new post in Manchester, but spent much of that academic year in Warwick taking part in the Symposium. In the Spring of 1978 Hartley moved with his family to Didsbury, an easy cycling distance of 3 miles from the University, and lived there for the rest of his life. Manchester’s proximity to Derbyshire and the Lake District allowed them all to indulge a passion for hill-walking.

Hartley brought a single-minded drive and much energy to Manchester mathematics. During the seventeen years he was there, he published some 60 papers, including significantly difficult and illuminating ones on simple and on locally-finite groups. He supervised many research students from overseas and kept in touch with

them after they went home. He was the driving force behind the Tuesday algebra afternoons that brought together group theorists from the University and from its Institute of Science and Technology (UMIST). He brought many visitors to Manchester, particularly during 1988 when, with P. J. Rowley at UMIST, he organized the *International Symposium on Representation Theory and Related Topics*. During that year he began important collaborations with I. M. Isaacs (see [82]) and with A. Turull (see [98]). Hartley was also the prime mover behind the highly successful international conference on *Finite and Locally Finite Groups*, which took place in Istanbul in 1994 (see (9)).

Hartley continued to travel widely. In 1981–2 he spent nine months at the University of Utah with W. R. Scott and F. Gross, then four months at the University of Singapore. He made another long visit to Singapore in 1985. With E. M. Kamal, one of his research students, he tried to establish a scheme (eventually unsuccessful) under which promising Egyptian students would pursue higher degrees in their home universities; he lent his support with visits in 1984 and 1985 to the University of Minia, taking cases of books and reprints for their library. In 1989 he spent the Easter vacation at MSRI in Berkeley and the summer visiting Padua and Moscow. In the autumn he returned for six weeks to Padua, then spent a month with R. Stöhr in East Berlin (leaving the day the Wall was first breached) and a month in Barcelona, where he collaborated with W. Dicks (see [88]). In 1990 he spent eight months at the University of Oregon, where C. W. Curtis and G. M. Seitz were his hosts. Hartley had developed a strong interest in groups of Lie type, and he and Seitz had much to talk about. He interested Seitz in locally-finite groups and formed with him a mathematical and personal bond that led to their collaboration as principal organisers of the Istanbul conference.

From 1982–4 Hartley was Head of the Mathematics Department and from 1985–7 sat on the Standing Committee of the University Senate. With colleagues from UMIST he consolidated their joint M.Sc. programme and was instrumental in securing its recognition by the Research Council and a consequent quota of advanced studentships. He also played a major part in the successful planning, development and launch in 1993 of a four-year undergraduate M.Math. programme, one of the first in the country. From 1986–9 Hartley served on the Mathematics Committee of the SERC and for many years was a member of various of the London Mathematical Society's editorial boards.

Hartley's position as a leading British algebraist, still at the height of his considerable powers, was acknowledged in 1994 by an award by the EPSRC of a five-year Research Fellowship. This began on 1 October of that year. He died of a heart attack in the Lake District, coming down from Helvellyn, on 8 October.

5. *Mathematical work*

Stability groups and augmentation ideals. In [1] Hall and Hartley prove that a stability group of an ascending series has a descending hypercentral series and that a finitely generated group can faithfully stabilize an ascending series if and only if it is residually nilpotent. The question of what groups can faithfully stabilize a descending invariant series turned out to be difficult and is left unresolved. This is connected with when a group G is descendant in the standard wreath product $W = W(A, G)$ of an abelian group A by G . Hall and Hartley prove that if G is a group with a descending series of infinite cyclic factors, then G is descendant in

W for every abelian group A . In [10] Hartley shows that for a nilpotent group the same is true if and only if for all primes p it is residually a nilpotent p -group of finite exponent. In [3] he determines the locally-finite groups that can be embedded in the stability group of some invariant series, whilst one of the main results of [4] shows that a locally-finite group can faithfully stabilize a descending invariant series only if it can faithfully stabilize a descending series of an abelian group. In [9] Hartley takes up the allied question of when W is residually nilpotent; the results are discussed (without detail) in the essay *Topics in the theory of nilpotent groups* that he wrote for the volume (5) of memorial essays for Philip Hall.

There is a close connection between the descendancy of G in the various groups $W(A, G)$ and the question of whether some (transfinite) power of the augmentation ideal of related group rings of G is zero. In [9] Hartley gives for nilpotent groups G necessary and sufficient conditions for the finite powers to intersect trivially. In [30] he completes some of the work begun in [3] and [4] by describing for locally-finite groups G the limit of the powers of the integral augmentation ideal in terms of special subgroups of G ; this allows him to say (in terms of subgroups of G) precisely when G can faithfully stabilize an invariant series. In [58], influenced by conversations with I. B. S. Passi, Hartley relates the augmentation powers of G to those of certain normal subgroups K with G/K torsion-free. His methods, which involve choosing bases of $\mathbb{Z}G$ closely related to the powers of the augmentation ideal, are similar to those he used in [9] and improved in [29]; these generalize methods of Hall (7). He proves similar results in [59] for nilpotent p -groups. Surveys of these and related topics are given in [49] and [61].

Free presentations. A free presentation $1 \rightarrow R \rightarrow F \rightarrow G \rightarrow 1$ (with F a non-cyclic free group) gives rise to a relation module R/R' and to higher relation modules $\gamma_c(R)/\gamma_{c+1}(R)$ ($c \geq 2$). In [55] Lichtman and Hartley define, for a characteristic subgroup N of R , relative higher relation modules $N/\gamma_{c+1}(R)$, about which are proved theorems known to be true for standard relation and higher relation modules: they show that relative higher relation modules can be embedded in free $\mathbb{Z}G$ -modules and under suitable circumstances contain non-zero free $\mathbb{Z}G$ -modules. The methods involve Lie theory and the relation between the actions of general linear groups and symmetric powers. They suggest a property of ‘persistence of free submodules for finite subgroups G of $GL_n(K)$ ’: in [71] Hartley shows that, with certain exceptions, if the natural module for $GL_n(K)$ contains a non-zero free $\mathbb{Z}G$ -module, then so does any finite-dimensional irreducible $GL_n(K)$ -module V over K , unless $SL_n(K)$ acts trivially on V . In [69] Hartley considers residual nilpotence of factors F/S , where S is a fully invariant subgroup of R such that R/S is torsion-free nilpotent, and with Gilchrist in [96] makes a similar investigation for factors R/S that are relatively-free nilpotent p -groups.

Hartley published two major papers with R. Stöhr. In this context C. K. Gupta (6) had discovered the surprising fact that free groups of varieties defined by outer commutator laws need not be torsion-free, and Kuz'min (11, 12) had brilliantly brought homological algebra to the study of higher relation modules. In [86] Hartley and Stöhr give a description of the torsion subgroup of $H_0(G, \gamma_p R/\gamma_{p+1} R)$ when p is prime and also deal with $H_j(G, \gamma_p R/\gamma_{p+1} R)$ for $j \geq 1$; neither description is at all easy. In [90] they prove a special case of a conjecture of Kuz'min (13); the main idea is to transform questions about the homology of G with coefficients in

exterior powers of R/R' (Kuz'min's approach) to ones where the coefficients are in symmetric powers of R/R' .

Group rings. In [16] (with D. McDougall) and in [42] Hartley proves results about both artinian and injective modules and applies them to soluble groups with the minimum condition for normal subgroups. In [41] he extends work of D. R. Farkas and R. L. Snider (3), and completes, for countable groups, the classification of group algebras whose irreducible modules are all injective; his approach is 'essentially the same as theirs, depending ultimately on work of Passman (16) on polynomial identities in group rings'. In 1983 he published in [63] necessary and sufficient conditions on a locally-finite group G and a field k of prime characteristic for every irreducible kG -module to have finite endomorphism dimension. 'This theorem was proved in 1975 as the result of a stimulating discussion with R. L. Snider' and also extends work of Farkas and Snider (loc. cit., but see also Wehrfritz (22)). In [35], with others, he exhibits startling group algebras whose only non-trivial ideal is the augmentation ideal, and in [45] he and J. S. Richardson give necessary and sufficient conditions for a group algebra to have a minimal right ideal. In his elementary talk [50] Hartley surveys results on irreducible modules.

Small centralizers. In a series of papers [54, 57, 60, 74, 76, 78, 80, 92], some joint, Hartley studies the influence of the centralizers of its elements on the structure of a group. The classical theorem due to Brauer and Fowler (1) states that the order of a finite non-abelian simple group G is bounded in terms of the order of the centralizer of any involution in G . In [54], Hartley and T. Meixner use and generalize both P. Fong's refinement of this theorem (4) and the famous theorem of V. P. Šunkov (20) that if a periodic group has an involution with finite centralizer, it must have a soluble normal subgroup of finite index. They show that a finite group G admitting an involutory automorphism α whose centralizer has order bounded by some number m must contain a nilpotent subgroup of class at most two and of index bounded by some function of m . In [57] they extend some of their argument to show that the Fitting subgroup of a finite soluble group admitting such an automorphism, but of prime order q , has index bounded by a function of q and m . As a corollary they prove that a periodic soluble group containing an element of prime order with finite centralizer must contain a locally-nilpotent subgroup of finite index. In [60] Hartley proves analogous theorems for periodic locally-soluble groups with an automorphism α such that $C_G(\alpha)$ is Černikov, whilst, with V. Turau in [76], he discusses the situation when α is of prime power order.

In [78], using the classification of finite simple groups, Hartley proves that if a locally-finite group G admits an automorphism of prime power order whose fixed-point group is a Černikov group, then G has a locally-soluble subgroup of finite index. Hartley and Kuzucuoğlu show in [80] that in an infinite locally-finite simple group the centralizer of every element is infinite. In [85] Hartley discusses the question of whether an arbitrary finite subgroup F in an arbitrary non-linear simple locally-finite group G must have an infinite centralizer and proves a theorem that 'points to an affirmative answer': that F must centralize a section C/D of G which it avoids and which is a direct product of finite alternating groups of unbounded orders. The main result of [92] is a general Brauer–Fowler theorem that the order of a finite non-abelian simple group G admitting an automorphism of order n with

at most k fixed points is (n, k) -bounded. ‘In sharp contrast to the real Brauer–Fowler theorem, its proof depends on the classification of finite simple groups.’ An easy consequence of this result is that if a locally-finite group contains an element with finite centralizer, then it has a locally-soluble subgroup of finite index.

Finite and locally-finite groups. Much of Hartley’s work on infinite groups was informed by an extensive knowledge of finite groups and their representation theory. His early work on soluble groups included an important contribution to the short but seminal paper [6], written jointly with Gaschütz and Fischer during the Warwick Algebra Symposium in 1977; this paper laid the foundations for a satisfactory dual of Gaschütz’s well-developed theory of formations. It immediately opened up the study of Fitting classes and associated canonical subgroups called ‘injectors’ which Hartley took a stage further in [7]. Although he remained interested in the developments of Sylow theory and its generalisations to formations and Fitting classes in the area of finite (and often soluble) groups (see [12] and [14] for instance), Hartley directed his energies more to the challenge of extending this theory to various classes of infinite groups, an enterprise in which there was a lively interest at the time. He achieved particular success for a certain class \mathfrak{U} of groups with ‘a civilized Sylow structure’; it comprises all locally-finite groups G with a finite locally nilpotent series, and such that for all sets π of primes, and for all subgroups H of G , all Sylow π -subgroups of H are conjugate in H .

The evolution of his work on \mathfrak{U} can be traced through the long sequence of papers [11, 13, 15, 17, 19, 20, 22, 23, 24, 25, 28, 39, 44, 77]. In [24, 25, 39, 44], for instance, Hartley investigated the class of \mathfrak{M}_c -modules M over $\mathbb{Z}_p G$; these are defined by saying that for all p' -subgroups H of G there is a finite subgroup F of H with $C_M(F) = C_M(H)$. Elementary-abelian normal p -sections of \mathfrak{U} -groups are of this type. His investigations allowed him to prove, in [25] for example, that for a \mathfrak{U} -group G , the factor group of G modulo its Hirsch–Plotkin radical has a normal subgroup of finite index which is a special sort of metabelian group; this brought \mathfrak{U} -groups into line with soluble linear groups that Mal’cev [15] had proved to have a nilpotent-by-abelian subgroup of finite index (see the paper [21] by Wehrfritz, where it is shown that periodic soluble linear groups are in \mathfrak{U}). They also allowed him in [39] to discuss cover and avoidance properties in \mathfrak{U} -groups. The upshot of all this is that most of the standard conjugacy classes from the theory of finite soluble groups have well-behaved analogues in \mathfrak{U} . Hartley gave a survey of some of these results in [28].

In the late 1970s there was heightened interest in the prospect of classifying the finite simple groups (the classification was announced in 1981), and Hartley turned his attention to the structure of locally-finite simple groups. Although a classification of such groups in the conventional sense is out of the question, the general theory of locally-finite groups had reached a sufficient level of maturity (see [14]) to make a structural investigation a realistic endeavour. In 1984, on a visit to Michigan State University, Hartley studied the PhD thesis of G. Shute, which stimulated his own ideas. In their joint paper [68] Hartley and Shute determine the structure of all infinite locally-finite simple groups satisfying $\text{min-}p$ for some prime p . These turn out to be the simple groups of Lie type over locally-finite fields. They also give a classification of all infinite linear locally-finite simple groups.

Over the next decade Hartley remained closely involved with progress on locally

finite simple groups. His survey article [102] is an account of his own contributions and thoughts on the subject. In [95] Hartley and Zalesskii determine the Zariski-dense subgroup of Chevalley groups and their twisted analogues over infinite algebraic extensions of finite fields. It follows from this work that Seitz's determination in (17) and (18) of the closed maximal subgroups of a simple algebraic group over the algebraic closure of a finite field gives all the maximal subgroups. Seitz has said how influential [95] was on his own work (19) on abstract homomorphisms of algebraic groups.

Representation theory and applications. In their celebrated work (8), P. Hall and Graham Higman describe the exceptional representations of elements of p -power order in a p -soluble group that acts on a module over a field of prime characteristic p and exploit this information to give bounds for the p -length of p -soluble groups in terms of certain invariants (such as the exponent) of their Sylow p -subgroups. Hartley knew these methods intimately (and contributed to them in [47]). He also knew well the subsequent developments by authors such as T. R. Berger, E. C. Dade, F. Gross, E. Shult, J. G. Thompson and A. Turull; in particular, he uses and extends the deep results in Dade's paper (2), where the notion of a Fitting chain first appears. By generalising Fitting chains, Hartley develops a method of p -chains in the difficult and subtle paper [19] on Sylow p -subgroups and local p -solubility. His principal result concerns, for an odd prime p , a locally-finite locally- p -soluble group G . He supposes that each countable set of elements of G is contained in a subgroup H with less than 2^{\aleph_0} Sylow p -subgroups and proves the remarkable fact that $G/O_{p,p',p}(G)$ is finite and $G/O_p(G)$ has the minimum condition for p -subgroups. The most obvious corollary is that if the Sylow p -subgroups of G are all conjugate, then G is p -separable. Using these p -chains, Hartley and A. Rae [27] give bounds on the p -length of finite p -soluble groups admitting a finite p -group of automorphisms acting fixed-point-freely on the p' -sections.

The ideas and methods described in the previous paragraph arise time and again in Hartley's work, in particular in two remarkable papers, with Isaacs [82] and with Turull [98]. They deal with an irreducible complex character χ of the semi-direct product $\Gamma = SG$ of a finite group S acting on a finite group G , where the orders of S and G are relatively prime and where, without prejudice to applications, it is assumed that the restriction of χ to G is irreducible, or at least homogeneous. The primary objective is to find the restriction χ_S of χ to the subgroup S of Γ . In both papers, an 'extraspecial replacement' theorem plays a central role: this guarantees the existence of a direct product E of extraspecial groups admitting S as a group of operators, together with an irreducible character ψ of SE such that $\psi_S = \chi_S$.

In [82] Hartley and Isaacs prove the replacement theorem when G is soluble. This gives information about the representations that enables them to derive bounds for the index in G of a certain member of its upper Fitting series in terms of the orders of S and its centralizer $C_G(S)$. An interesting corollary is that a periodic locally-soluble group G admitting a finite soluble group of operators S acting with finitely-many fixed points has a finite normal series with locally-nilpotent factors, provided that the elements of G have orders relatively prime to $|S|$.

In [98] Hartley and Turull call on their deep knowledge of groups of Lie type to deal with an insoluble group G . Here the Glauberman correspondence is available and plays an important part in their proof of the 'extraspecial replacement' theorem

in this complementary case. The Glauberman correspondence gives a canonical bijection $\theta \rightarrow \theta'$ between the S -stable irreducible characters of G and the irreducible characters of the centralizer $C_G(S)$. Under certain assumptions Hartley and Turull prove that the degree of θ' divides the degree of θ . The proof of this seemingly innocuous statement is a *tour de force* which exploits even the most sophisticated aspects of the Deligne–Lusztig character theory of groups of Lie type.

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