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RESEARCH ARTICLE

# Of pashas, popes, and indivisibles

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**Argument**
The studies of Bonaventura Cavalieri's indivisibles by Giusti, Andersen, Mancosu and others provide a comprehensive picture of Cavalieri's mathematics, as well as of the mathematical objections to it as formulated by Paul Guldin and other critics. Issues that have been studied in less detail concern the theological underpinnings of the contemporary debate over indivisibles, its historical roots, the geopolitical situation at the time, and its relation to the ultimate suppression of Cavalieri's religious order. We analyze sources from the seventeenth through twenty-first centuries to investigate this relationship.

**Keywords and phrases:** indivisibles; atomism; matter and form; Lateran; Trent; Bonaventura Cavalieri; Stefano Degli Angeli; René Descartes; Galileo Galilei; Paul Guldin; Antonio Rocca; Evangelista Torricelli

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As regards the opinion on quantity made up of indivisibles, I have already written to the Provinces many times that it is in no way approved by me and up to now I have allowed nobody to propose it or defend it.

Mutio Vitelleschi, S.J.

We consider that this proposition not only stands in opposition to the doctrine of Aristotle that is commonly assumed, but also cannot be proven in itself.

Jakob Bidermann, S.J.

However, the stain of this doctrine was subsequently revealed, disturbing us concerning the mysteries of the Eucharist, as we are taught by the Church.

Sforza Pallavicino, S.J.

## 1. Introduction

Studies of Bonaventura Cavalieri's indivisibles by Giusti (1980), Andersen (1985), Mancosu (1996), Delgado (2017) and others provide a comprehensive picture of Cavalieri's mathematics, as well as of the mathematical objections to it as formulated by Paul Guldin and other critics. Issues that have been studied less systematically concern the theological underpinnings of the contemporary debate over indivisibles, its historical roots, the geopolitical situation at the time, and its relation to the ultimate suppression of Cavalieri's religious order. We analyze sources from the seventeenth through twenty-first centuries to investigate this relationship.

An attentive reader of a recent article on Cavalieri will perhaps be puzzled by the following passage:

> In 1668, Pope Clement IX suppressed the Jesuats, probably at the request of the Republic of Venice to finance the fight against the Ottoman Turks for the rule of the island of Crete. [Amir] Alexander ... has recently ... argued that hostility of the powerful Jesuits to Cavalieri's mathematics and everything it represented played a role in this suppression. (González, Rabiti, and Cartwright 2018, 26)

The Jesuates were founded in 1360; Bonaventura Cavalieri (1598–1647) was a Jesuate, as was his student Stefano Degli Angeli (1623–97). The Jesuits were founded in 1540. In the decades between 1630 and 1668, these two orders held opposing views on indivisibles. Mathematically inclined readers of this journal may have learned about Cavalieri's principle in their undergraduate calculus classes.[1] What did Cavalieri's mathematics represent that would provoke the hostility of the Jesuits? Why would the Republic of Venice make an extraordinary request to suppress a religious order? Doesn't the practice of suppressing such orders so as to finance military campaigns seem unusual? For what reason would a pope accede to such a request?

We will report on what we have been able to discover concerning this case of science operating in, and influenced by, a historical and theological context that goes back to the thirteenth century. We will first seek to document Jesuit opposition to Cavalieri's indivisibles in Sections 2 and 3, and explore its animus in Section 4. Section 5 deals with the contemporary geopolitical situation, the 1668 suppression of the Jesuates and the 1669 Ottoman takeover of Candia. Sections 6 and 7 deal with the aftermath. Section 8 contains suggestions for further research.

## 2. Cavalieri's books

In this section, we review Cavalieri's books on the geometry of indivisibles and their influence on his contemporaries. Bonaventura Cavalieri was encouraged to study mathematics by his teacher Benedetto Castelli (a Benedictine). They met at the Jesuate convent San Girolamo in Pisa in 1616 (Giusti 1980, 2; Drake 1981a, 115; Dufner 1975, 351–52; Feingold 2003, 12). Galileo had composed a treatise on continuous quantity (now lost) as early as 1609 (Drake 1981b, 245). Cavalieri, who took his start from Galileo's analysis, importuned him to publish that work in order that Cavalieri might proceed with the publication of his own *Geometry of Indivisibles* (ibid.).

### 2.1. New approach

The earliest indication we have of Cavalieri's interest in a new approach to calculating areas and volumes occurs in one of his first letters to Galileo, dated December 15, 1621 (Cavalieri [1621]

[1]If two plane domains have the same height and the same cross-sectional length at every point along that height, then they have the same area. A similar relation holds between volumes of solids and their cross-sectional areas. Using a method resembling Cavalieri's principle, Archimedes was able to find the volume of a sphere given the volumes of a cone and cylinder in his work *The Method of Mechanical Theorems*.

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2009). A substantial part of the contents of Cavalieri's book can already be found in letters from 1622–23 (Jullien 2015a, 89). In 1623, in a letter now lost, Galileo expressed reservations concerning Cavalieri's approach (Delgado 2017, 239). (Galileo's reservations point in a different direction from that implied by Alexander's claim that "Letters from Cavalieri in the following months suggest that Galileo, at the very least, encouraged Cavalieri to continue his investigations.")[2] Manuscript copies of the book were circulating no later than 1627 (Festa 1992a, 310 n. 7; Arrighi 1973; Andersen 1985, 296). Cavalieri used the term "indivisible" in reference to his approach for the first time in 1627 (Delgado 2017, 31). The book *Geometria indivisibilibus continuorum nova quadam ratione promota* appeared in 1635 (Cavalieri 1635). De Gandt (1991, 160) observed that "[Cavalieri] tried, as very few did, to cast his new instruments in the mold of classical Euclidean exposition. It is striking to see how 'Euclidean' Cavalieri is, compared to other creators in mathematics at the same time." As noted by de Gandt, Cavalieri devoted a lot of attention to a mathematical justification of his approach.[3] The book provoked several responses by Jesuit scholars. High-placed jesuit scholars like Paul Guldin published tracts highlighting alleged inconsistencies of the indivisibles; see Section 3.

### 2.2. Exercitationes *(1647) and philosophy*

Cavalieri sought to demonstrate that alleged paradoxes attributed to his indivisibles stemmed from their unrestricted use (without respecting the conditions he specified).[4] Cavalieri published responses to some of the criticisms in his 1647 book *Exercitationes geometricae sex*. Here he wrote:

> You know the arguments I used to prove the first Method of the *Indivisibili*, and I think to defend it from criticisms. But if this is not enough, you have my other method, free of the notion of infinite indivisibles taken together. And if you are not satisfied, consider the arguments I used to validate the use of the indivisibles following the style of Archimedes, mainly in Exerc. II. In this way I think that you will be free from any doubt.[5]

Alluding to the distinction between *discovering* and *proving*, widely discussed at the time, Cavalieri continues:

> Even [Paul] Guldin admitted that they are very useful for discovering things (p. 331 lib. IV). You have to decide if they can be accepted also for proving, and in what form: as I used them or with reference to the Archimedean method, by which my discoveries have been validated. Finally, use it as you prefer: in this kind of discussion, more philosophical than geometrical, I do not want to lose any time. The Geometers will judge ...

The thrust of Cavalieri's remark about philosophy is that the discussion concerning which approach is preferable—indivisibles or Archimedean exhaustion—is a philosophical rather than a geometrical issue. The discussion not being a mathematical one, Cavalieri has no interest in entering it. Notably, he does not assert that rigor is the concern of philosophy rather than of geometry. The term *rigor* does not occur in the passage.

[2]Alexander 2015, 87. In fact, when Castelli's position at Pisa became available in 1626, Galileo recommended not Cavalieri but rather Niccolò Aggiunti to fill the position; see Giusti 1980, 13.

[3]Developed by a devout Jesuate, Cavalieri's indivisibles were neither "materialistic in tone" (Alexander 2018, 394) nor "riddled with paradoxes" (ibid.), a mathematical claim for which the historian Alexander provides no convincing evidence; see further in Sections 4.5 and 4.7.

[4]E.g., in his April 5, 1644 letter to Torricelli; see Delgado 2017, 149–51.

[5]Cavalieri, Exercitatio Tertia (Cavalieri 1647, 241); translation ours. A Russian translation can be found in Cavalieri [1635] 1940, 61–62.

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Cavalieri's point concerns the discovery of a new useful approach exploiting indivisibles. He employs it to obtain new results. If he is requested to "justify" his findings using traditional methods, namely Archimedean exhaustion, he is able to do so. However, he discovered them in a different way. Is his way acceptable? Cavalieri the geometer prefers to leave such questions to the philosophers.

### 2.3. The Boyer–Kline "affair"

Inexplicably, Cavalieri's comment on philosophy has been distorted out of recognition by numerous commentators starting with Carl Boyer. Boyer (1959, 123) claimed the following: "Rigor, he [Cavalieri] said, was the affair of philosophy rather than geometry" (emphasis added). Boyer's footnote 106, reading "*Exercitationes geometricae sex*, p. 241," attached to this claim, leaves no doubt that he is referring to the passage we quoted from page 241 of Cavalieri's *Exercitationes* in Section 2.2.[6] Boyer's claim was picked up by Morris Kline (1980, 133), who paraphrased Boyer by adding two pairs of quotation marks, and claimed the following concerning Cavalieri:

> "Rigor," he said, "is the concern of philosophy not of geometry."

Kline provided no source for his claim (which would be literally true if the third-person pronoun were taken to refer to Boyer). The Boyer–Kline distortion of Cavalieri has been since copied by numerous sources in multiple languages.[7] While lacking a source, the portrayal of Cavalieri as unrigorous dovetails with Boyer's (1959, 298) narrative of the conceptual development of mathematics toward the predetermined pinnacle reached in the 1870s by the "great triumvirate."

We should note that Cavalieri at no time seeks to sum "infinitesimal magnitudes" even though his language sometimes seems to point in that direction: "All the lines . . . ," "All the surfaces . . . ." In particular, the technique of discarding negligible (infinitesimal) terms—that would prove to be controversial in the history of the calculus—was never used by Cavalieri.[8] Starting in 1641, Evangelista Torricelli will reconceptualize indivisibles by assigning to them width (see, e.g., Delgado 2017, 15–16, 117–21). When Torricelli evaluates areas of plane figures, he uses only the lines corresponding to the partition points of an infinitesimal partition, rather than "all the lines."

### 2.4. Adequality in Cavalieri and Fermat

Interestingly, Cavalieri used the term *adequal* to describe the relation between the collection of lines on the one hand and the space they occupy on the other.[9] He did not mean it in the sense of *approximately equal*. Breger's claim that in Fermat, the term *adequal* does not mean "to be almost equal" is endorsed by Madeline Muntersbjorn (2000, 250, n. 5), who also admits that Fermat used the term when performing quadratures via a bunch of parallelograms (p. 243). Such a procedure (not used by Cavalieri) necessarily involves discarding negligible terms. Itard noted the use of the term by Cavalieri, observed that Fermat used it in various senses (Itard 1975, 117), and clarified such senses:

[6]Boyer's dismissive comment is typical of some historians' tendency to attribute to Cavalieri the role of an early purveyor of unrigorous mathematics, and to attribute more validity than is warranted to the criticisms by Cavalieri's Jesuit opponents.

[7]Thus, Brisson and Ofman have two versions of their erudite article: an archive version (Brisson and Ofman 2020) with 140 footnotes, and a shorter published version (Brisson and Ofman 2022). The archive version not only reproduces the Boyer–Kline distortion, but actually claims that Boyer is "quoting Cavalieri" (Brisson and Ofman 2020, n. 88) (the claim is not found in the published version). O'Connor and Robertson (2014) give the following purported quotation: "Rigour is the affair of philosophy, not of mathematics," but like Kline provide no source.

[8]It is therefore inexact to claim, as Brisson and Ofman do in the archive version of their text, that in Cavalieri, "Geometrical surfaces are infinite sums of parallel lines that can be treated, in some way, as finite sums" (Brisson and Ofman 2020, 15). This claim is not found in the published version (Brisson and Ofman 2022).

[9]"magnitudinem, quae adequatur spatio ab eisdem lineis occupato, cum illi congruat" (Cavalieri 1635, 17; cf. Muntersbjorn 2000, 235).

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> Lorsque l'on compare deux grandeurs de même espèce on peut le faire soit en étudiant leur différence, soit leur rapport. On pourra les considérer comme voisines si l'on trouve leur différence voisine de zéro, ou leur rapport voisin de 1 ... Fermat donne ainsi au concept le sens d'infiniment petits équivalents, pour utiliser un langage leibnizien. (Itard 1974, 339–40)

Accordingly, Fermat, unlike Cavalieri, did use the term in the sense of a relation of infinite proximity.[10]

In sum, Cavalieri's books introduced a novel approach to problems that had traditionally been approached via the method of exhaustion. Torricelli carried the method further. Subsequent geometers were familiar with Cavalieri's method mainly through Torricelli's work.

## 3. Four Jesuits against the indivisibles

The first half of the seventeenth century witnessed various attempts to teach indivisibles in Jesuit colleges. Conflicting attitudes existed among the Jesuits of the period toward the new techniques. The rank-and-file were attempting to use them in their teaching, and sent repeated requests for authorization, from their various locations in the periphery, to the central authorities. Palmerino (2003, 188) writes:

> If one were to study systematically the many *Censurae opinionum* that preceded the *Ordinatio* of 1651 throughout the first half of the century, one could reconstruct in quite some detail the chronological and geographical spread of the new sciences all over Europe and their surreptitious entrance through the back door of the Jesuit educational institutions.

See further on the *Ordinatio* in Section 4.9. Grégoire de Saint Vincent was among the Jesuit mathematicians who worked with indivisibles. Among the Jesuits who sought to discredit indivisibles mathematically were Paul Guldin, André Tacquet, Antoine de Lalouvère, and Mario Bettini, dealt with respectively in Sections 3.1, 3.2, 3.3, and 3.4.

### 3.1. Paul Guldin

Paul Guldin's critique of Cavalieri's indivisibles appeared in the fourth (and last) book of his *De centro gravitatis* (also called *Centrobaryca*), published in 1641 (Guldin 1635–41). Mancosu writes:

> Guldin is taking Cavalieri to be composing the continuum out of indivisibles, a position rejected by the Aristotelian orthodoxy as having atomistic implications.[11] ... Against Cavalieri's proposition that "all the lines" and "all the planes" are magnitudes—they admit of ratios—Guldin argues that "all the lines ... of both figures are infinite; but an infinite has no proportion or ratio to another infinite."[12]

If, in computing areas, an infinite can have no proportion or ratio to another infinite, as Guldin claimed, then Cavalieri's principle (see note 1) certainly risks being unsound. According to Cavalieri, a key objection of Guldin's was that indivisibles only appear in the continuum *in potentia* (potentially) and not in *actu* (Delgado 2017, 163), harking back to Aristotelian strictures on philosophical discourse concerning infinity.

[10]Briefly, the original term is actually Diophantus' *parisotes*, which does refer to an approximate equality. Bachet, when translating Diophantus, performed a semantic calque, rendering it by a Latin term that corresponds to *adequality*. Following Bachet, Fermat used the term in the same sense to refer to his relation, which is arguably a relation of infinite proximity. See further in Katz, Schaps, and Shnider 2013; Bair, Katz, and Sherry 2018; Katz 2020, sections 3.2–3.5; and note 21.

[11]For an elaboration of Mancosu's comment on Guldin and atomism, see Section 4.5.

[12]Mancosu 1996, 54. For Guldin's original Latin see Festa 1992b, 200 n. 15 or Delgado 2017, 204, n. 456.

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It does not seem to have been noticed by commentators that some of Guldin's arguments are refutable even by seventeenth-century standards. Relying upon Euclid's Book 5, Definition 5 (known as Definition 4 in modern editions), Guldin argues as follows:

> For the parts both of this figure and of another can be multiplied, so that the small parts surpass the bigger ones, and vice-versa, and the same applies to the lines that designate them, and to what I said on the subject of lengths; and therefore, according to the fifth definition of Book 5 of Euclid's Elements, they are said to have a ratio to each other.[13]

Having invoked what he takes to be Euclid's authority, Guldin proceeds to attack Cavalieri's method of indivisibles in the following terms:

> And so, if we sum any accumulation either of lines, or of parts of planes of a surface, they are finite and have a ratio to each other, the lines obviously to the lines, and the parts to the parts, but not the lines to the parts. This is why, since there is not, and cannot be, any accumulation of all the lines or of all the parts, neither can there be any ratio between them, etc.

Euclid's *definition* is closely related to the Archimedean property (see, e.g., Giusti 1980, 33 and n. 17 there). In his argument against Cavalieri's method, Guldin did not relate to the fact that it is a definition rather than an *axiom*. Guldin's critique based on Book 5, Definition 5 is, strictly speaking, not applicable to Cavalieri's method, because Cavalieri was working with codimension-1 indivisibles (see further in note 50), rather than with infinitesimals à la Kepler and Leibniz (which violate the Archimedean property when compared to 1). Guldin's argument, which would in principle apply had Cavalieri dealt with infinitesimals, is refutable. Half a century later, Leibniz would make it clear that his incomparable infinitesimals violate the comparability notion put forward in this definition.[14]

Significantly, Guldin also accused Cavalieri of plagiarizing Kepler.[15] Given the dubiousness of some of Guldin's mathematical criticisms, one can wonder what his true motivations were and whether there may have been a hidden doctrinal agenda. The fact that he would pursue non-mathematical criticisms (such as allegations of plagiarism) lends credence to the hypothesis that his opposition to Cavalieri's indivisibles was not purely mathematical. As Festa, Mancosu, and others have pointed out, a well-known allusion by Guldin to the existence of extra-mathematical issues may have involved a doctrinal issue; see Section 4.7.4. See further in Section 4 on the connection to atomism.

Cavalieri observed that, in spite of Guldin striking the posture of a defender of mathematical certainty, the justification Guldin provided for his own rule (for bodies of revolution; see Delgado 2017, 174 n. 368) is in fact insufficient. Torricelli agreed with Cavalieri's assessment (Delgado 2017, 174–75). Cavalieri went on to point out that Antonio (Giannantonio) Rocca (1607–56) furnished a better-grounded justification for the rule in terms of indivisibles (Delgado 2017, 178–79; Giusti 1980, 50 n. 40), while Cavalieri himself provided a generalization for solids (Delgado 2017, 180). Of interest also is an error, discovered by Torricelli, on page 127 of the first book of Guldin's work (Guldin 1635–41; Giusti 1980, 75; Delgado 2017, 184–87).

[13]Guldin as translated by Radelet-de Grave 2015, 80.

[14]Leibniz 1695, 322. For further references see note 53.

[15]Mancosu 1996, 51. In short, Guldin accused Cavalieri of copying nonsense from Kepler. Radelet-de Grave (2015, 77) mentions "Guldins skill in successfully criticizing Cavalieri's method whilst simultaneously praising the one from which his inspiration was drawn, namely Kepler's." She also notes that "Giusti was surprised, quite rightly, that Cavalieri did not take the opportunity to counter that Guldin's rule was already to be found in Pappus" (p. 76). Cavalieri did point out the similarity of Guldin's rule to Kepler's; see Delgado 2017, 217.

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When Cavalieri first became aware of Guldin's criticism in 1642, he made the following visionary comment:

> I do not mind if this Father has taken upon himself to refute this method of mine on indivisibles, because, if I am in error I will see the truth, but if he is the one who is mistaken, at least he will do this favour to my Geometria, which is that some people, who otherwise might never have seen it, will make some reflection upon it.[16]

### 3.2. André Tacquet

André Tacquet was generally less extreme than some of the other Jesuit opponents of indivisibles. He tolerated their use in heuristic arguments leading to the discovery of results that must then be justified by traditional exhaustion arguments. As noted by Bosmans (1927, 71),

> [Tacquet] connaissait la *Géométrie des indivisibles* de Cavalieri et appréciait la fécondité des méthodes du géomètre italien. Elles étaient cependant, d'après lui, un simple moyen de recherche, sans force démonstrative, à moins qu'on ne les ramenât à la méthode d'exhaustion d'Archimède.

Nonetheless, he warned in his *Cylindricorum et annularium libri IV* (1651) that the idea of quantity composed of indivisibles makes war upon geometry to such an extent that "if it is not to destroy it, it must itself be destroyed"[17] (i.e., the method of indivisibles must itself be destroyed if it is not to destroy geometry). Tacquet's terminology of "quantity composed of indivisibles" echoes the language of the 1632 ban, suggesting that the ban applied to mathematical indivisibles, as well; see further in Section 4.5.

Giusti (1980, 21 n. 49) provides the following apt summary:

> Tacquet rejects as non-geometrical the theory of indivisibles, which he calls the method of proof per heterogenea. However, he proves many of his own theorems both by indivisibles and by classical Archimedean methods.

### 3.3. Antoine de Lalouvère

A declared enemy of indivisibles, jesuit Antoine de Lalouvère unconsciously used them in his own work, as noted by Descotes (2015, 273):

> What is quite striking in this procedure is that Lalouvère, who is a declared enemy of indivisibles, uses without even realizing it an equivalent, and even at heart an identical, process to Dettonville's,[18] given that all the surfaces are arranged on the same line AC, divided into small portions. This amounts to summing these magnitudes, taken all along the line AC, while disregarding the problem posed by the divisions of AC, and consequently, the presence of indivisibles. ... At the very moment when he attacks the indivisibles, Father Lalouvère unconsciously uses them.[19]

Lalouvère is one of the Jesuit mathematicians analyzed in a comprehensive study of the period 1540–1640 by Antonella Romano (1999). She writes:

[16]Cavalieri as translated by Giusti 1980, 55.

[17]Festa 1992b, 205; see n. 26 there for the original. Cf. Descotes 2015, 255.

[18]Dettonville is a pen name for Pascal.

[19]Giusti (1980, 45) similarly points out that Lalouvère "did not reject completely the method [of indivisibles] but tried to prove it by exhaustion, a proof much in the spirit of [Cavalieri's] *Geometria*, and of the *Exercitatio* II."

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> Un premier constat regarde les jésuites confrontés à la censure: sur tous les cas du Fondo Gesuitico qui concernent la France, deux professeurs seulement appartiennent à la liste établie dans le cadre de cet ouvrage, B. Labarthe et V. Léotaud. Si d'autres mathématiciens y apparaissent, c'est tout aussi exceptionnellement, et au titre de censeur, comme Antoine Lalouvére. (p. 512)

Romano's footnote 114 at the end of this passage reads: "C'est lui qui porte un jugement négatif sur l'ouvrage de B. Labarthe." Thus, Lalouvère in his capacity of censor sank at least one mathematical book, namely the *Hermetis Mathematici* praeludium (Romano 1999, 515) of his fellow Jesuit Bartholomé Labarthe, in 1662.[20] On the relation of Lalouvère and Fermat see the following note.[21]

### 3.4. Mario Bettini

To Jesuit Mario Bettini, Cavalieri's indivisibles were not merely "hallucinations" but glib philosophizing that makes geometrical theorems useless:

> being pressed, I respond to the counterfeit philosophizing about geometrical figures by indivisibles. Far, far be it from me to wish to make my geometrical theorems useless, lacking demonstrations of truth. Which would be to compare ... figures and philosophize about them by indivisibles.[22]

The criticism appeared in Bettini's 1648 *Aerarium philosophiae mathematicae*. A sharp-tongued rebuttal by Degli Angeli (1658) is analyzed by Alexander (2015, 168). Degli Angeli's (1659) response to Tacquet is analyzed by Festa (1992b, 205).

In sum, some leading Jesuit mathematicians attacked indivisibles ostensibly on mainly mathematical grounds (though there is an allusion in Guldin to broader issues; see Section 4.7.4). In Section 4, we examine the related actions by their doctrinal superiors in the order.

## 4. Bans on quantity made up of indivisibles

Parallel to the efforts sketched in Section 3 by Jesuit scholars such as Bettini, Guldin, Lalouvère, and Tacquet to discredit indivisibles mathematically, there was a series of bans against the teaching of indivisibles, issued by their doctrinal superiors in the order. The bans are catalogued by Festa (1992b), Palmerino (2003), and Alexander (2015).

### 4.1. Physical versus mathematical indivisibles

The Jesuit bans applied equally to physical atomism and mathematical indivisibles. On August 10, 1632, the Revisors General led by Jakob Bidermann formulated and then banned the following proposition:

[20]The fact that Jesuits Nicolas du Port Guichart and Jean-Paul Médaille considered Labarthe's book to be "correct" (Romano 1999, 515) suggests that Lalouvère's opinion may have been controversial.

[21]The presence of an opinionated Jesuit anti-indivisibilist at Toulouse may help explain Fermat's reticence when discussing the foundations of his method of adequality and the nature of the increment E; see Katz, Schaps, and Shnider 2013; Bair, Katz, and Sherry 2018. Lalouvère was at Toulouse during the periods 1632–35, 1643–54, and 1659–64 (Romano 1999, 584–85) (and taught logic in 1632–33 and physics in 1633–34 [Romano 1999, 548]). Fermat deposited his method of adequality with d'Espagnet already in the late 1620s. The fact that Fermat did not communicate it to Mersenne in Paris until 1636 (Mahoney 1994, 53)—after Lalouvère's departure from Toulouse the previous year, 1635—may have been a coincidence.

[22]Bettini as translated by Alexander 2015, 159.

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> A permanent continuum can be constituted by physical indivisibles alone, or atomic [i.e., non-divisible] particles, having *mathematical parts* that can be designated, even though the mentioned particles are distinct from each other. Time, too, [consists] of instants, and intense qualities consist of indivisible grades alone.
> We consider that this proposition not only stands in opposition to the doctrine of Aristotle that is commonly assumed, but also cannot be proven in itself.[23]

The explicit reference to Aristotelian doctrines is significant. Aristotle, unlike Plato, held that mathematical objects were all instantiated in physical objects, or that mathematical objects are abstracted from physical objects.[24] Accordingly, Aristotelian philosophy of mathematics would couple mathematical and physical indivisibles. What is meant exactly by "mathematical parts" of physical indivisibles? Festa (1992b, 199) speculates that Cavalieri's indivisibles may be alluded to in this ban.[25] As noted by Mancosu (1996, 56), "[Guldin's] preface defines mathematics according to the Aristotelian classification, as that part of philosophy lying between physics and metaphysics. Mathematics is the science that considers quantity abstracted from sensible matter." Guldin's mathematical critique of indivisibles was examined in Section 3.1. Guldin, too, would likely have interpreted the ban as applying equally to mathematical and physical indivisibles.[26]

Referring to the 1632 ban, Feingold (2003, 28–29) notes:

> Six months later, General [Mutio] Vitelleschi formulated his strong opposition to mathematical atomism in a letter he dispatched to Ignace Cappon in Dole: "As regards the opinion on *quantity made up of indivisibles*, I have already written to the Provinces many times that it is in no way approved by me and up to now I have allowed nobody to propose it or defend it." (emphasis added)

Significantly, Vitelleschi speaks of *quantity*, which Aristotle abstracts from matter, confirming the coupling of mathematical and physical indivisibles in the thinking of the General.

Given such an explicit linkage, it is difficult to agree with Alexander's view that specifically *physical* indivisibles were the target of the bans, which, he claims, lacked grounds to object to mathematical ones: "In as much as the technical details of the miracle of the Eucharist mattered, they provided no grounds for objecting to a mathematical (rather than physical) doctrine" (Alexander 2018, 393). See further in Section 4.6 for an analysis of Alexander's position. For the precise canon involved see Section 4.2.

### 4.2. Atomism versus canon

What was behind the Jesuit opposition to indivisibles? Sforza Pallavicino pinpointed the problem as a clash with (the catholic interpretation of) the Eucharist. Festa recounts Jesuit Pallavicino's 1647 recanting of his previous interest in physical atomism:

[23]Emphasis added. "Continuum permanens potest constare ex solis indivisibilibus physicis, seu corpusculis atomis, habentibus partes mathematicas, in ipsis designabiles, etiamsi realiter dicta corpuscula inter se distinguantur. Tempus quoque ex instantibus, & qualitates intensae, ex solis gradis indivisibilibus constant.

Hanc propositionem arbitramur, non modo repugnare communi Aristotelis doctrina, sed etiam secundum se esse improbabilem, etc." Festa 1992b, 207. For a French translation see Festa 1992b, 198.

[24]E.g., in Aristotle's view, a sphere is abstracted from the moon, by abstracting the sensible property (sphericity, say) from its matter. See further in Lear 1982.

[25]Manuscript copies of Cavalieri's book were already in circulation in 1627–29, thus several years prior to the ban; see Section 2.1.

[26]Guldin's book may contain an allusion to a doctrinal issue; see further in Section 4.7.4.

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> Cette doctrine, écrit Sforza Pallavicino, "flatteuse pour notre imagination, ... attira une foule de disciples. Mais son caractère destructeur a été détecté; elle trouble ce que l'Église nous enseigne sur les Mystères de l'Eucharistie et ne s'accorde pas suffisamment avec ce que le Concile du Latran affirme sur la nature de l'âme humaine."[27]

The endorsement of transubstantiation at the sixteenth-century Council of Trent, Session 13, Canon 2, was widely interpreted by Catholic theologians as an endorsement of the Peripatetic theory of matter and form sometimes referred to as hylomorphism (or hylemorphism),[28] which they saw as opposed to atoms, closely related to indivisibles.[29] Canon 2 sought to clarify rulings of the Fourth Lateran Council three centuries earlier; see Section 4.9.

Lüthy and Nicoli (2023a, 7) speak of atoms becoming a problem "in the Counter-Reformational context of tightening doctrinal rules, and more specifically ... the decision of the Council of Trent to insist on transubstantiation as the only correct way of interpreting the Eucharist, wherein transubstantiation was defined in the terminology of substance and accident, of matter and substantial forms."

### 4.3. Grassi and Inchofer versus Galileo

In the case of Cavalieri's mentor Galileo, there are multiple documents by Jesuits—specifically, Orazio Grassi and Melchior Inchofer—attacking his atomism with the weapon of eucharistic theology. Thus, Ferrone and Firpo (1986, 505) write: "in his *Ratio ponderum*, [Jesuit Orazio] Grassi was to include the relationship between atomism and the Eucharist among the many accusations he leveled at [Galileo's book] *Saggiatore*." The Galileo–Grassi debate and its theological underpinnings are analyzed by Festa (1999, items 7–8). A detailed denunciation, document labeled EE291, came to light in 1999 (Artigas, Martínez, and Shea 2005; Finocchiaro 2021, 291). The author of EE291 is thought to be the Jesuit Inchofer based on handwriting analysis (Festa 2007, 25). The criticisms voiced in EE291 echo the accusations against Galileo leveled by both Grassi and an earlier anonymous document labeled G3.[30] An alternative interpretation of the Jesuit opposition to indivisibles, proposed by Alexander, is examined in Section 4.6.

### 4.4. Galileo and Cavalieri

Significant differences existed between Galileo and Cavalieri with regard to indivisibles (Jullien 2015a, 95–97, based on Giusti 1980, 40–41). Galileo was initially reluctant to use them in geometry. Cavalieri, on the other hand, thought of indivisibles as a kind of "useful fiction" (of course a mathematical one), eminently applicable in geometry. His letters to Galileo suggest that Cavalieri was disappointed with Galileo's attitude. The long hiatus between the writing of Cavalieri's book (1627) and its publication in extended form (1635) was apparently due to his efforts to influence Galileo's opinion concerning geometric applications of indivisibles.

Delgado (2017, 45–46) argues that Cavalieri's use of indivisibles to provide a new solution to the problem of free fall in his 1632 *Lo Specchio Ustorio* succeeded in stimulating Galileo's interest

[27]Pallavicino as translated by Festa 1999, item 10; cf. Festa 1992b, 203.

[28]As a 0th approximation for the benefit of a reader not familiar with the circle of ideas related to hylomorphism, note that, for example, a clay pot can be viewed as having two basic components: (1) undifferentiated matter, or material (clay), and (2) its form or shape. Such a perspective creates tensions with atomistic approaches that go against the grain of thinking in terms of undifferentiated matter.

[29]On transubstantiation and the Council of Trent, see Armogathe 1977, 28–32; McCue 1968, 419; Festa 1991, 101; and Section 8 below. On Scotus concerning the incompatibility of atomism and the Eucharist, see Section 4.9.

[30]The centrality of the atomist issue to the background deliberations in the Galileo trial was the thesis of Redondi 1987. Our argument is independent of Redondi's thesis. Namely, the relevant issue for us is Grassi's critique of Galileo based on the relation between atomism and the Eucharist, a historical fact acknowledged by Redondi's critics Ferrone and Firpo.

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in Cavalieri's geometric method. Delgado (2017, 55) traces Galileo's evolution from a physical to a mathematical atomism. Palmerino (2000, 307; cf. Delgado 2017, 79) analyzes Cavalieri's influence on Galileo. It is interesting to note that in his first reaction to the 1638 *Discorsi*, where Galileo proves the law of fall, Cavalieri "criticizes Galileo for not having emphasized that the indivisibles have to be taken as equidistant" (Damerow et al. 2004, 251 n. 175).

### 4.5. The Festa–Mancosu thesis

Historians are divided with regard to the reasons behind the Jesuit opposition to indivisibles. Mancosu (1996, 219–20 n. 18) writes:

> To these open attacks [on Cavalieri's work] one should however add the general hostility of the Jesuits to Cavalieri's indivisibles. Festa (1990, 1992) provides archival evidence to show that the teaching of indivisibilist techniques in geometry, as well as the use of the atomistic theory in physics, was forbidden in the Jesuit schools by means of decrees, the first dated 1632, issued by the "revision" fathers of the Collegium Romanum. Festa argues that the hostility to atoms and indivisibles was motivated by theological concerns about the dogma of transubstantiation.

Accordingly, the Festa–Mancosu thesis is that the teaching of indivisibilist techniques in geometry was included in the prohibition due to doctrinal concerns over transubstantiation.

### 4.6. Euclid's geometry as a world-ordering force

Meanwhile, Alexander proposes an alternative interpretation of the Jesuit opposition to mathematical indivisibles. He argues that to the Jesuits, starting with Clavius, Euclid's mathematics served as a world-ordering force that (as they were convinced) could be used in educating people and attracting them to Catholicism:

1. "The War Against Disorder: The Jesuits against the infinitely small."[31]
2. "And like each proof alone, geometry as a whole is universally and eternally true, *ordering the world* and governing its structure everywhere and always" (Alexander 2015, 67; emphasis added).
3. "It was clear to Clavius that Euclid's method had succeeded in doing precisely what the Jesuits were struggling so hard to accomplish: imposing a true, eternal, and unchallengeable *order* upon a seemingly chaotic reality" (ibid.; emphasis added).
4. "The Jesuits . . . believed that thanks to the rational rigor of its method and the unshakeable certainty of its results, mathematics would play a key role in imposing *order* on the chaos brought about by the Reformation, and reestablishing the authority of the Church hierarchy" (Alexander 2018, 394; emphasis added).

Accordingly, Alexander claims that Jesuit opposition to mathematical indivisibles was prompted by the allegedly paradoxical nature of the latter, inconsistent with the Euclidean ideal they sought to safeguard. On occasion, Alexander suggests that indivisibles are not merely paradoxical but inherently contradictory, such as when he writes:

1. "Zeno's mind-benders prove extremely difficult to resolve, based as they are on the inherent contradictions posed by indivisibles" (Alexander 2015, 9–10).
2. "Most damaging of all, whereas Euclidean geometry was rigorous, pure, and unassailably true, the new methods were riddled with paradoxes and contradictions, and as likely to lead one to error as to truth" (ibid., 120).

[31] Alexander 2015, 15. By mentioning the "war against disorder" already in the heading of an entire section, Alexander seeks to emphasize his idea that the Jesuits were out to save the world from chaos and disorder, and impose upon it mathematical order based on Euclid.

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Alexander may have magnified such an aspect of indivisibles so as to buttress his interpretation, analyzed in Section 4.7.

### 4.7. Analysis of Alexander's interpretation

Alexander's interpretation of Jesuit opposition to indivisibles is clearly at odds with the Festa–Mancosu thesis (see Section 4.5). Indeed, Alexander's interpretation can be challenged. We note the following six points.

#### 4.7.1. Galbraith's study

Galbraith's (2021) study of the history of the Jesuits in relation to mathematics suggests that the Catholic hierarchy was ambivalent about the involvement of members in mathematics in general (including the geometry of Euclid), and that many among the higher-ups thought that such involvement detracted members from their primary responsibilities. Accordingly, the attitude toward mathematics as a world-ordering force may have been mainly the attitude of Clavius himself[32] (see further in Section 4.7.2). Galbraith's analysis specifically undermines Alexander's (2015, 120) claim that "by the late sixteenth century, mathematics had become one of the most prestigious fields of study at the Collegio Romano and other Jesuit schools."

#### 4.7.2. Clavius

Significantly, Clavius was a proponent of viewing hornangles as quantities (an example of a hornangle is the crevice between a circular arc and the tangent ray at one of its endpoints). A well-known controversy opposed Clavius and Peletier on the subject of hornangles (with Clavius in favor and Peletier opposed; see, e.g., Maierù 1990; Axworthy 2018).[33] Whereas Alexander argues that the reason for the Jesuits' opposition is their desire to safeguard the world-ordering power of Euclid against an alleged threat stemming from the non-Archimedean behavior of infinitesimals, hornangles represent a non-Archimedean phenomenon: a hornangle is smaller than a half, a third, a quarter, etc. of any ordinary rectilinear angle. Therefore Clavius' advocacy of hornangles undermines Alexander's claim that, following Clavius, the Jesuits were duty bound to oppose Cavalieri's mathematics.[34]

#### 4.7.3. Ordinatio *on incompatibility*

Alexander (2015, 147–49, 151, 321) does discuss the 1651 Jesuit *Ordinatio*, but not the fact that it explicitly mentions the incompatibility of atomism with the Eucharist; see further in Section 4.9. This aspect of the *Ordinatio* points in a different direction from the one argued by Alexander.

#### 4.7.4. Guldin's allusion

As noted by Alexander (2015, 154),

> Guldin comes close to admitting that there are greater issues at stake than the strictly mathematical ones, writing cryptically that "I do not think that the method [of indivisibles]

[32]As noted by Hellyer (1996, 323 n. 12), "Clavius' textbooks standardized mathematical instruction throughout the Order to a very high degree well into the seventeenth century." See also Clavius 2002.

[33]An additional controversy concerning the status of mathematics opposed Clavius and Perera (Pereira, Pereyra); see Mancosu 1996, 14; Rossini 2022.

[34]Knobloch (1888, 337) writes concerning the Clavius–Peletier controversy: "Si nous laissons de côté sa querelle inutile avec Peletier au sujet de l'angle de contingence, il se montre le plus souvent comme un critique subtil repérant les constructions fausses, les démonstrations erronées ou les propositions mal formulées." We feel that analyzing non-Archimedean phenomena is not "useless," for both Clavius and Leibniz; see Katz et al. 2021, section 3.

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should be rejected for reasons that must be suppressed by never inopportune silence," but he gives no explanation of what those "reasons that must be suppressed" could be.

Guldin's mathematical critique of indivisibles was examined in Section 3.1. What issue with indivisibles could Guldin be alluding to in the passage above? If, as Alexander claims, Guldin is alluding to the world-ordering power of the timeless rigor of Euclid's mathematics, why should such a comment be suppressed by a "never inopportune silence"? On the contrary, Guldin should have emphasized it! We don't know exactly what Guldin had in mind, but he was likely referring to extra-mathematical issues. It is more plausible to interpret Guldin's quip as alluding to doctrinal problems with indivisibles than to their posing a perceived threat to the world-ordering power of Euclidean rigor.

#### *4.7.5. Bans as evidence*

Alexander (2015) presents seventeenth-century Jesuit bans against indivisibles as evidence for his thesis that Jesuits were opposed to mathematical indivisibles because indivisibles were paradoxical and contradictory, and therefore undermined Euclidean geometry, which, according to Alexander, the Jesuits viewed as a world-organizing force. If one assumes that the bans applied only to physical indivisibles (as claimed by Alexander; see end of Section 4.1), there is a basic incoherence in Alexander's approach: if the Jesuit mathematicians were mainly bothered by the contradictory nature of *mathematical* indivisibles, then their doctrinal superiors' bans on *physical* indivisibles constitute an unrelated issue that provides no evidence for Alexander's thesis.

#### *4.7.6. An old problem*

Catholicism's problems with indivisibles were not a Counter-Reformation phenomenon (as implied by Alexander), and in fact date back to the thirteenth century; see Section 4.9.

### *4.8. Atoms, shibboleths, Descartes*

Lüthy (2023, 114) notes that "At the latest by 1650, the atom had become a fetish to one party and a scandal to the other, and hence a shibboleth that defined one's adherence to the so-called new philosophy." Rossi (1998) presents a detailed study of "the radical incompatibility between atomist theses and the conclusions reached by the Council of Trent on the sacrament of the Eucharist," and analyzes attempts by maverick Jesuit scholars (Suarez, Pereira, Arriaga, and Oviedo) "to develop a natural philosophy alternative to the Aristotelian one, capable of retaining its distance from impious atomism, but, nonetheless, adopting certain central aspects concerning belief in discontinuity and indivisibles."

René Descartes questioned the explanatory power of prime matter and substantial forms (the two components of hylomorphism) in the following terms:

> We do much better to understand what takes place in small bodies, whose minuteness prevents us from perceiving them, by what we see occurring in those that we do perceive <and thus explain everything in nature, as I have tried to do in this treatise>. This is preferable to explaining certain things by inventing all sorts of novelties with no relation to those that are perceived <such as prime matter, substantial forms, and all the whole range of qualities which many are in the habit of assuming, any one of which is more difficult to understand than all the things they are supposed to explain>. (Descartes 2000, 269, article 201)

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While Descartes was an anti-atomist,[35] corpuscularism was a common feature of the approaches of Kepler, Galileo, and Descartes (Berchman 2009, 199–200).

In 1663, some of Descartes's writings were placed on the Index of Prohibited Books (Armogathe and Carraud 2004, 67). Stephanus Spinula was one of the two censors: "By the time the Congregation of the Holy Office gives [Spinula] the task of judging Descartes's *Principles* and the *Passions of the Soul*, he was already the author of an important book, *Novissima philosophia*, in which he attacked the Jesuit theologians Oviedo, Arriaga, and Hurtado" (Armogathe and Carraud 2004, 73). Among the items censured by Spinula is the opinion that "there is no prime matter" (Armogathe and Carraud 2004, 75).

In the second half of the seventeenth century, several authors attempted to reconcile atomism and catholicism (Festa 1999, item 22), including Donato Rossetti (1633–86) (Gómez Lopez 1996). In 1671, the Holy See alerted the archbishop of Naples concerning the existence of attempts to reconcile the Cartesian philosophical system and the Eucharist, in the following terms:

> Il y avait dans cette ville [i.e., Naples] des gens qui, voulant prouver leur supériorité, se faisaient les promoteurs des opinions philosophiques dun certain Renato des Cartes [i.e., René Descartes] qui, il y quelques années, a fait imprimer un nouveau système philosophique, réveillant ainsi les anciennes opinions des Grecs concernant les *atomes* . . . ;[36] or, certains théologiens prétendent prouver, à partir de cette doctrine, la manière dont les accidents du pain et du vin se conservent après que ce pain et ce vin sont changés en corps et sang. (translation by Festa 1999, item 28; emphasis added)

A trial of atheists commenced in Naples in 1688 (Festa 1999, item 29; see further in Fiorelli 2021). A number of atomists came under scrutiny, including the jurist Francesco D'Andrea (1625–98), who was acquitted nine years later (Festa 1999, item 29).

### 4.9. Sourcing opposition to atomism in Scotus

In 1651, indivisibles were placed on the Jesuit list of permanently banned doctrines (Hellyer 1996, 329). As noted by Leijenhorst and Lüthy (2002, 396; cf. Vanzo 2019, 211), "the incompatibility of atomism with the accepted interpretation of the Eucharist was invoked, for example, . . . in the Jesuit *Ordination* [i.e., Ordinatio pro studiis superioribus] of 1651." The same authors trace Catholicism's opposition to atomism back to Duns Scotus (c. 1265/66–1308): "Ever since Duns Scotus had interpreted the decisions of the Fourth Lateran Council of 1215 as entailing that the Church had formally established transubstantiation as the canonical interpretation of the Eucharist, physical atomism was generally viewed as a heresy" (Leijenhorst and Lüthy 2002, 396). Cross (1998, 118) also notes that "Scotus is fiercely opposed to any sort of atomism."[37]

The final years of the pontificate of Innocent III (1160/61–1216) had witnessed a tightening of doctrinal control. The turn of the twelfth century was a troubled time for Catholicism, which had to fend off a Manichaean challenge (the so-called Albigensian heresy), against which Innocent III declared a crusade. Besides the endorsement of transubstantiation as the canonical account of the Eucharist at the Fourth Lateran Council in 1215 (see above), prohibitions against the teaching of Aristotle's *Metaphysics* and commentaries by Avicenna and others could also be mentioned. They were promulgated in the same year by the papal legate Robert of Courçon in Paris (De Vaux 1934, 45; Bertolacci 2012, 214). Lahey (2009, 102) notes that in the fourteenth century, "Wyclif seems to

[35]Thus, in article 202 of his *Principles of Philosophy*, Descartes distanced himself from the atomism of Democritus in the following terms: "Que ces principes ne s'accordent point mieux auec ceux de Democrite qu'auec ceux d'Aristote ou des autres" (Descartes 1824, 62, 516). See also Lüthy and Nicoli 2023a, 3.

[36]Such a claim concerning Descartes is debatable; see note 35.

[37]See Ariew (2012) on the disagreement between Aquinas and Scotus concerning the nature of prime matter (p. 187) and on Descartes's anti-atomism (p. 193).

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have employed his conception of spatio-temporal indivisibles in arguing that the traditional doctrine of transubstantiation was, as normally held, impossible." Clearly, the tensions between indivisibles and transubstantiation were by no means a new problem in the seventeenth century.

In sum, the objections to indivisibles formulated by Jesuit mathematicians (see Section 3) were only part of a greater picture that involved acute doctrinal problems that had vexing consequences for leading scholars such as Galileo and Descartes (and, as we argue, the Jesuate mathematicians) as well as a slew of less prominent scientists.

## 5. Clemency, Pasha, and cannons

In this section, we provide a non-mathematical explanation of the suppression of the Jesuates that obviates the need for Alexander's unconvincing explanation analyzed in Section 4.7.

The established pattern is that, while the Jesuits published a series of decrees and books against indivisibles (see Sections 3 and 4), the Jesuates published a series of books on the geometry of indivisibles: Cavalieri published two books (see Section 2) and his student Degli Angeli published nine (see Section 6). Both of these scholars were highly placed in the Jesuate order,[38] a fact that surely colored their adversaries' attitude toward the order itself. The parties interested in making sure that the Jesuate problem "penetrated the supreme ears" (see Section 5.1) were likely to include Jesuits. Cavalieri died in 1647, but Cavalieri's student Degli Angeli saw their order of the Jesuates suppressed with little clemency by Pope Clement IX, on December 6, 1668, by dint of a papal bull.[39] Alexander's hypothesis that the suppression of the Jesuate order may have been a reprisal to their members' work on indivisibles is less well established.[40]

### *5.1. Looming suppression*

In 1644 Pope Urban VIII died. As reported by the historian Uccelli, rumors of a possible suppression started circulating no later than 1646, when the General of the Jesuates pleaded with the current pope in the following terms: "From the beginning until these days there has not been a Pontiff who, as a loving father, has not loved and helped it [i.e., the Jesuate order]; for never have wrongdoings, that would discredit it, penetrated those supreme ears. Only today there are rumors of removing our habit."[41] Apparently the General was alarmed that damaging rumors concerning the Jesuate order were beginning to penetrate the supreme ears. Uccelli also noted that a similar letter had been sent by Cavalieri from Bologna on May 2, 1646 to the Grand Duke Ferdinando II de' Medici (1621–70) of Tuscany, in recommendation of the same congregation.

An official historian of Catholicism, Ludwig Pastor, glossed over the suppression[42] despite devoting over a hundred pages to the papacy of Clement IX (Pastor 1940, 314–430). By contrast, volume 1 of Pastor's monumental work does duly mention that in 1367, "Giovanni Colombini, the founder of the Gesuati, and his religious came as far as Corneto to meet the Pope [Urban V], singing hymns of praise."[43] Pastor mentioned the founding of the jesuate order, but failed to

[38]Thus, "about 1652 [degli Angeli] was appointed prior of the monastery of the Gesuati in Venice, and shortly afterward he was given the post of provincial definer, a position he held until Pope Clement IX suppressed the order in 1668" (Carruccio 1981, 164).

[39]Hélyot 1863, 623. Alexander (2015, 171, 323) erroneously refers to this as a *papal brief*.

[40]"[The Jesuits] turned ... to the papal Curia in Rome, where their influence was decisive. They could not punish [Degli] Angeli directly, so they let their fury rain on the order that sheltered him and his late teacher" (Alexander 2015, 173).

[41]Uccelli 1865, 90. Gagliardi (2004, 483) attributes a similar passage to Cavalieri.

[42]"On August 18th, 1668, Venice was authorized to alienate Church property in aid of the Turkish war, and, subsequently, to raise a million ducats from suppressed monasteries" (Pastor 1940, 422). The "monasteries" in question remain unspecified, and the suppression of the Jesuate order goes unreported by Pastor.

[43]Pastor 1899, 95. What matters, of course, is not that the Jesuates were chanting hymns, but rather that the *founding* of the Jesuates is covered in detail (11 lines) by Pastor, as this quotation illustrates. Significantly, the *suppression* of the Jesuates is not even mentioned by Pastor, suggesting that it was perhaps a nettlesome issue he preferred to avoid.

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mention its suppression. Indeed, the reader of the official *History of the Popes* will find nothing to disabuse him of the idea that Colombini's order still existed in 1940, the publication date of volume 31 of the English translation of Pastor's work.

### 5.2. *The* aquavitae *issue*

There is some debate concerning the appellation *Aquavitae Fathers* (or *Brothers*). The 1911 *Encyclopedia Britannica* (1911; henceforth "EB") claims the following: "[In] the 17th century the Jesuati became so secularized that the members were known as the Aquavitae Fathers, and the order was dissolved by Clement IX in 1668." Thus the EB posits a causal link between alleged "secularization" of members and the appellation *Aquavitae Fathers*, apparently insinuating a charge of uncatholic dealings in alcohol. Another account has it that in the hospitals, the Jesuates distilled liquors and gave them to the sick in order to help them bear the pain (Eamon 2011). Accordingly, the term *Aquavitae Fathers* as applied to the Jesuates is linked to their deeds of charity. As reported in the Catholic Encyclopedia (1913), "[I]n 1606 the Holy See allowed the reception of priests into the congregation." Such a boost in their status is unlikely to have occurred had the EB's linkage been correct. Furthermore, in 1640, Urban VIII approved new constitutions for the Jesuate order (Hélyot 1863, 623). The obscurity of the reasons sometimes given for the suppression of the order ("some abuses crept in")[44] suggests that they may have been introduced after 1668 so as to retroactively justify the suppression—coming as it did from an infallible source.[45]

### 5.3. *Crusading for Candia*

Hippolyte (Pierre) Hélyot (1660–1716) was eight years old when the Jesuates were suppressed. Some decades later, Hélyot (1863, 623) detailed the circumstances of the suppression as follows:

> [D]ans l'Etat de Venise ils étaient assez riches, ce qui fit que la république demanda leur suppression à Clément IX, afin de profiter de leurs biens, qui furent employés à soutenir la guerre que cette république avait contre les Turcs qui assiégeaient pour lors Candie; ce que le pape accorda l'an 1668.

The reference is to the Fifth Ottoman–Venetian War, or the Cretan War (1645–69), and to Candia, the capital of Crete at the time. Uccelli (1865, 91) provides a similar account. Thus, Clement IX appears to have determined that "some abuses crept in," suppressed the order, and transferred its assets to the Venetian Republic, at the precise moment the latter was the most in need of funds to fight Köprülü Fazıl Ahmed Pasha, the Ottoman Grand Vizier.[46]

[44]Thus, Eves (1969, 35), in his entry 220 starting with the words "Written histories often contain hidden perpetuated errors," claimed that "Certain abuses later made their way into the Jesuat order" and proceeds to speculate darkly concerning "the manufacture and sale of distilled liquors, apparently in a manner not acceptable to Canon Law, etc." (Eves 1969, 36), without, however, providing any evidence as to his Canon Law claim. Simmons (1992, 106) lodged a similar allegation concerning liquors contrary to Canon Law, again without evidence. A canon, namely Council of Trent, Session 13, Canon 2 (see Section 4.2), may have been at odds with the manufacture not of liquors but of theorems by leading Jesuate scholars.

[45]Gagliardi (2020, 36) claims that "the Order was subsequently suppressed by Clement IX in 1668 due to an insufficiency of vocations. The suppressive records indeed reveal a substantial number of friars, yet a meager count of novices and young aspirants." It is open to question what the cause was and what the effect. The Jesuates having been the target of a negative campaign for at least two decades prior to the ultimate suppression (see Section 5.1), one can wonder whether such a campaign may have had an effect on potential recruits.

[46]Uccelli (1865, 91) also notes the following: "The Grand Duke [Ferdinando II de' Medici of Tuscany] wrote on December 18 of that year to his ambassador in Rome, so that he could interpose himself with his Holiness: so that 'in the distribution of the effects and goods of similar monasteries existing in this dominion, S. B. may be to regard with the eyes of his supreme piety the needs of Tuscany, as he has regarded those, even if more urgent, of the Venetian dominion'." The tenor of Ferdinando II's plea confirms that the papal bull consigned the Jesuate assets to the Venetian Republic even outside its realm.

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It is instructive to compare, following Hélyot, two of the orders suppressed by the 1668 papal bull: the *Georges in Algha* and the Jesuates. With regard to the former, Hélyot (1863, 400) writes: "Ils se sont bien éloignés dans la suite de la pauvreté et de l'humilité dont leurs fondateurs avaient fait profession et dont ils leur avaient laissé l'exemple." He goes on to detail their arrogant ways, and concludes: "c'est avec raison que Clément IX les supprima en 1668 et donna tous leurs biens à la république de Venise pour s'en servir dans la guerre qu'elle avait contre les Turcs." By contrast, Hélyot (1863, 623) details the rigors of the daily routine of the Jesuates, including prayers, dietary restrictions, and fasts, and, unlike the case of the *Georges in Algha*, reports—*without endorsing it*—their 1668 suppression. It is possible that the remarks in the EB are due to a confusion of the two orders.

### 5.4. Danger of Venice making peace

In a crusading effort, Pope Clement IX was able to mobilize contingents from several European nations in 1668–69 to fight the infidels (the sentiment was surely mutual). Thus, "[Clement IX] met the wishes of [France and Spain with regard to the creation of Cardinals] on August 5th, 1669, in order to secure their help for the Turkish war" (Pastor 1940, 345). As a further illustration of papal savvy in contemporary *Realpolitik*, Pastor (1940, 347) notes the following:

> It was an act of sheer favour on the part of the Pope if he had taken into consideration the wishes of France and Spain; . . . it was hoped that this act of grace would spur on Louis XIV to more intense military action on behalf of Crete for *there was danger of Venice making peace with the Turks*, thereby exposing both Italy and the Emperor's own territories to Turkish aggression. (emphasis added)

Pastor (1940, 419) relates further that

> Towards the end of May 1668, the papal galleys, under Vincenzo Rospigliosi, put to sea in order to effect their junction with the Venetian fleet commanded by Francesco Morosini. Soon after Abbot Airoldi was sent as special agent to obtain help for Crete from the Italian and German princes.

Pastor (1940, 420–21) furnishes a detailed account of all the princes, knights, and dukes participating in the war effort, as well as a papal quote of Numbers 10:35 in support of the effort (p. 424). Concerning the Ottoman side of the conflict in the 1660s, Baer (2004, 160) writes: "Ottoman military policy, especially regarding the siege of Crete, showed little sign of success, and the Ottomans, who considered themselves to be the protectors of Islam, were demoralized." Such difficulties may have contributed to the Grand Vizier's eagerness to conclude a peace treaty with Captain General Francesco Morosini. The showdown between Köprülü Fazıl Ahmed and Clement IX ended in an Ottoman victory by sword and cannon at Candia on September 6, 1669 (Pastor 1940, 427), less than a year after the transfer of the Jesuate assets to the Venetians.

## 6. Gregory's books held in great esteeme, but . . .

Whatever the reasons for the summary suppression of the 300-year-old Jesuate order, founded by the Blessed Giovanni Colombini,[47] collateral damage of the suppression included the books of

[47]Pope Gregory XIII included Colombini's name in the 1578 edition of the *Martyrologium Romanum* (the official list of the saints, blessed, and martyrs). The 1578 inclusion (over two centuries after the recognition of the Jesuates by Urban V and less

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James Gregory (who visited Degli Angeli in the 1660s),[48] which were held "in great esteeme, but not to be procured in Italy."[49] Degli Angeli himself published nine books on the geometry of indivisibles, including his *De infinitis spiralibus inversis* (1677), which appeared merely a year before the suppression (Giusti 1980, 50 n. 39). He did not write a word on them following the suppression, though he remained active in mechanics, publishing a seventy-nine-page treatise in 1671, as well as in university teaching. Decades later, in the 1690s, we find Jacopo Riccati attending Degli Angeli's course in astronomy at Padua. In a touching episode of historical continuity, the seventy-two-year-old Degli Angeli encouraged the nineteen-year-old Riccati to study the *Principia Mathematica* (Bertoloni Meli 2006, 288).

## 7. Aftermath: *Le vide intégral* and a new dawn

Andersen (1985, 355) notes that Degli Angeli, who was a Jesuate like Cavalieri, remarked in his *De infinitis parabolis* (Degli Angeli 1659) that the circles opposed to the method of indivisibles mainly contained Jesuit mathematicians. In the aftermath of the hostility toward indivisibles, Italian mathematics suffered a period of stunted growth in the emerging infinitesimal analysis, and the centers of activity in what was becoming mathematical analysis shifted north of the Alps. Among the Jesuits, "le grand nombre des mathématiciens ... resta jusqu'à la fin du XVIIIe siècle profondément attaché aux méthodes euclidiennes" (Bosmans 1927, 77). By 1700, Italy was a mathematical desert with regard to the new mathematics: "en 1700, c'est le vide intégral en ce qui concerne la pratique des mathématiques nouvelles en Italie" (Robinet 1991, 183).

Less than four years after the suppression of the Jesuates, Friedrich Leibnütz's twenty-six-year-old son arrived in Paris at the end of March 1672 (see Antognazza 2009, 139) on a diplomatic mission aimed at diverting Louis XIV's appetite for conquest away from the Dutch Republic and toward Egypt. His mission was aborted before it could start, the preparations for the French military campaign being well underway by the time Gottfried Wilhelm reached Paris (see Lynn 1999, 112–13). But his arrival would have momentous significance in an entirely different realm. Perhaps sensing the doctrinal burden of the (overly) evocative term *indivisible*, Leibniz coined the term *infinitesimal* in 1673,[50] taking up a proposal by Nicolaus Mercator.[51] Within a few years, infinitesimal calculus would be born.

Leibnizian infinitesimals were controversial in their own right, both in Leibniz' time and in current Leibniz scholarship.[52] Did the Jesuits' sacramentally inspired opposition to atomism and indivisibles also apply to the new technique of infinitesimals? Sherry (2018, 367) argues that it did not necessarily: "indivisibles but not infinitesimals conflict with the doctrine of the Eucharist, the

than a century before the suppression by Clement IX) endorsed a long-standing practice in Siena (Colombini's birthtown), even though there was no official act of beatification.

[48]For details on Gregory's Italian trip see Crippa 2020. Gregory visited Degli Angeli during the period immediately preceding the suppression of the Jesuates. The fact that, following the suppression of the order, Gregory's mathematical books were similarly suppressed in Italy supports the hypothesis that the suppression of the Jesuates may not have been unrelated to the mathematical investigations of their leading scholars.

[49]Jean Bertet, as reported by John Collins, quoted in Turnbull 1939, 107; see Bascelli et al. 2018 for details. Jean Bertet (1622–92) was a Jesuit and studied under Honoré Fabri, professor of mathematics at Aix (Wallis 2014, 506).

[50]The term *indivisibles* can be used in a narrower sense and in a broader sense. Cavalieri's indivisibles possessed no thickness. Torricelli started using indivisibles of variable thickness, i.e., already infinitesimals; see, e.g., Andersen 1986, 20–21. Many seventeenth-century pioneers learned about Cavalieri's method through Torricelli; see further Bascelli 2015. Leibniz' sources were Blaise Pascal, John Wallis, and Honoré Fabri's *Synopsis Geometrica* (1669). It is not surprising that Leibniz sometimes referred to his own method as *indivisibles* long after he introduced the new term *infinitesimal*. Adhering strictly to the narrower sense of *indivisibles* (as codimension-1 entities) is an innovation of twentieth-century scholars starting with Koyré. It makes for a neat classification but it does not faithfully reflect the variety of seventeenth-century practice.

[51]See Probst 2018, 200, who also notes that Wallis used the term *pars infinitesima* already in 1670.

[52]The related bibliography is vast. One could mention Bos 1974 and the numerous articles mentioned in note 53.

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central dogma of the Church." A vast correspondence between the Jesuit des Bosses and Leibniz (Leibniz 2007) attests to the former's keen interest in the latter's work.[53] Indivisibles were also taught by Cavalieri's successor Pietro Mengoli (1659), as well as by the Jesuits Fabri (1669) and Dechales (1674).

## 8. Final remarks

The theological underpinnings of the seventeenth-century debate over indivisibles require further study. It may be of interest to analyze the philosophical/theological pedigree of the Jesuates in relation to their being more accepting of the emerging mathematics of indivisibles. Indivisibles were controversial within the Jesuit order. While some maverick Jesuits, including Gregoire de Saint Vincent, attempted to exploit them (Section 3), mathematicians like Paul Guldin, André Tacquet, and others opposed them (Section 3), and their doctrinal superiors in the order issued a series of bans aimed against teaching indivisibles in Jesuit colleges (Section 4.1). Meanwhile, scholars in the Jesuate order were among the leading practitioners of indivisibles (Sections 2.2 and 6): Cavalieri and his student Degli Angeli published a series of books on indivisibles. Cavalieri attempted to avert the looming suppression of their order (Section 5.1).

Such a difference in attitude towards indivisibles between the two orders may be related to differences in the theological pedigree of the rival orders of the Jesuates and the Jesuits, respectively via Augustine of Hippo (354–430) and Thomas Aquinas (1225–74), back to respectively Neoplatonists and Aristotle. The relevance of such pedigree is due to the connection of transubstantiation to what many Catholic theologians conceived of as the Aristotelian doctrine of matter and form, the latter being thought of, in Thomist theology, as clashing with atoms and indivisibles. The Counter-Reformation context of the tensions over transubstantiation was analyzed by Festa (1990), Hellyer (1996), McCue (1968), and others. The Jesuit Pallavicino is on record concerning the tension between atomism and the Catholic interpretation of the Eucharist (Section 4.2), while Jesuits Grassi and Inchofer attacked Galileo with the weapon of eucharistic theology (Section 4.3). Hélyot (1863, 620) traces the theological pedigree of the Jesuates back to "la règle de saint Augustin." Some of the Italian atomist Catholic scholars were influenced by Descartes, whose doctines were deemed to be at odds with Thomist theology by the Catholic hierarchy (Section 4.8). A comparison of the intellectual roots of the Jesuates and the Jesuits in the thought of respectively Augustine and Thomas merits further analysis.

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[53]Bair, Blaszczyk et al. 2018, section 4.10. Interpreting Leibniz' infinitesimals is an area of lively debate. Bair et al. (2021) published a comparative study of three interpretations; Katz et al. (2021) presented three case studies in Leibniz scholarship; Katz et al. (2022) presented and analyzed a pair of rival approaches; Archibald et al. (2022) formulated some criticisms; Bair et al. provided a brief response (Bair et al. 2023) and a detailed response (Bair et al. 2022). Recent work includes Katz et al. 2023, Katz et al. 2024, and Ugaglia and Katz 2024. Arthur and Rabouin (2024) formulated further objections. In their latest book (Arthur and Rabouin 2025), they double down on their interpretation, following Ishiguro (1990, chapter 5), and claim that Leibnizian infinitesimals were not quantities. A rebuttal appears in Katz and Kuhlemann 2023.

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