lattices are said to be *equivalent* if they are stabilized by the same elements of $GL(V)$. The author proves that $GL(V)$ acts on a tree X in which each vertex is a lattice class and adjacent vertices are represented by nested lattices whose quotient is the residue field of $\mathcal{O}$. He shows that, provided a subgroup G of $GL(V)$ satisfies certain hypotheses (some of which are topological), the quotient graph $G\backslash X$ can be determined. Applying the fundamental theorem he reproves in this way a theorem of Ihara on torsion-free subgroups of $SL_2(\mathbb{Q}_p)$ and a theorem of Nagao on $GL_2(k[t])$, where k is a field.

In the last part of Chapter II the author considers the case where K is the function field of a smooth projective curve C over a field k_0 and he studies the subgroup $\Gamma = GL_2(A)$ of $GL(V)$, where A is the affine algebra of the curve $C - \{P\}$ with a single point P at infinity. (Nagao's theorem is concerned with the case $A = k_0[t]$.) By showing that there is a correspondence between the quotient $\Gamma\backslash X$ and vector bundles or rank 2 over C, he uses known results on such bundles to determine $\Gamma\backslash X$. The fundamental theorem is then applied. One important consequence of this, for example, is that, when k_0 is finite, $GL_2(A)$ has countably many congruence subgroups and uncountably many subgroups of finite index. (This is one of the main results in Serre's famous paper "Le problème des groupes de congruence pour SL_2", Ann. of Math., **92** (1970), 489–527.) Chapter II ends with sections on the homology of Γ and its Euler-Poincaré characteristic.

Serre's notes on groups acting on trees have appeared in various forms (all in French) over the past ten years and they have had a profound influence on the development of many areas, for example, the theory of ends of discrete groups. This fine translation is very welcome and I strongly recommend it as an introduction to an important subject. In Chapter I, which is self-contained, the pace is fairly gentle. The author proves the fundamental theorem for the special cases of free groups and tree products before dealing with the (rather difficult) proof of the general case. One word of warning however—although Chapter II is well presented, considerable background reading is required for a full understanding of its contents.

A. W. MASON

BELLMAN, RICHARD, *Analytic Number Theory: An Introduction* (The Benjamin/Cummings Publishing Company Inc., 1980), 195 pp., U.S. $19.50.

Dr Bellman is well known for his distinguished work in number theory and in a very wide variety of branches of pure and applied mathematics. His objective in the volume under review has been to provide an introduction to certain parts of analytic number theory. The reader's interest is stimulated by studying the list of contents. In addition to covering a number of useful analytic techniques (methods of estimation, transforms, Poisson summation formula, etc.) this list includes the gamma and zeta function and all the most important arithmetical functions. There is particular emphasis on mean-value and Tauberian theorems.

It is when he progresses further that the reader begins almost immediately to realize the difficulties in store for him. There can be very few authors of books who do not suffer disappointment at the number of undetected errors and misprints in their published work, but the number to be found scattered through this book is excessive by any standard. The expert may have little trouble in making the necessary corrections, but the novice's difficulties with a new subject will be aggravated and his confidence shaken. Thus, on p. 3, formula (3) is false, in (4) n should be replaced by N, and the formula at the bottom of the page is meaningless unless expressed more precisely. If the reader has courage to proceed he will find an average of one error per page in each of the first eleven pages. Mistakes such as *principle* for principal (p. 46), *formerly* for *formally* (p. 60) and k^2 for *k squares* (p. 68) strongly suggest that the text was dictated and not checked thereafter. There are few grammatical solecisms, but the great bulk of the errors are typographical and mathematical. The latter range from dangerous half-truths, such as that 'Congruences may be manipulated like ordinary equations' (p. 3), to false statements such as (p. 65)

$$|\zeta(\sigma + it)| = 0(\log t^{3/2})$$

for $\sigma > \frac{1}{2}$. (Is it too pedantic to object to the universal use of the symbol for a capital italic O?)

The preceding remarks give some indication of the traps that beset the reader. If he perserveres and is prepared to treat every statement with due caution he will find much to reward his efforts. To the reviewer the section (§9.8) on algebraic independence of arithmetic functions is one of the most interesting and owes much to the author's own work. Moreover, there is a wealth of bibliographical information at the end of each chapter to encourage the reader to further study.

R. A. RANKIN

MINC, H., *Permanents* (Encyclopedia of Mathematics and its Applications, vol. 6, Addison-Wesley Advanced Book Programme, 1978), xviii+205 pp., $21.50.

In his preface, the author states: "Permanents made their first appearance in 1812 in the famous memoirs of Binet and Cauchy. Since then 155 other mathematicians contributed 301 publications to the subject, more than three quarters of which appeared in the last 19 years. The present monograph is the outcome of this remarkable re-awakening of interest in the permanent function." (In fact, 303 publications are quoted in the Bibliography.)

In an attempt to give a complete account of the theory of permanents, the author has traced their development from their inception until the present day (1978). Chapter 1 contains a historical survey in which only the classical results are discussed in detail, while Chapter 2 covers the basic elementary properties of permanents and Chapter 3 is devoted to the permanent of (0, 1) matrices, including the classical theorem of Frobenius and König (Let A be an $n\times n$ matrix with non-negative entries. Then per $(A)=0$ if and only if A contains an $s\times t$ zero submatrix such that $s+t=n+1$.)

The next three chapters are given over entirely to inequalities involving permanents, either upper and lower bounds for permanents of (mainly) non-negative matrices, or, in the case of Chapter 5, the Van der Waerden conjecture. This states that if S is a doubly stochastic $n\times n$ matrix then per $(S)\geq n!/n^n$. The whole chapter is devoted to a discussion of this, giving the then most recent developments in the pursuit of a solution to this conjecture. In 1980 it was proved to be true by a Russian author, Egoritsjev, in Russian. However, J. H. Van Lint has produced an account of Egoritsjev's proof in Linear Algebra and its Applications, **39** (1981), pp. 1–8, entitled "Note on Egoritsjev's Proof of the Van der Waerden Conjecture."

Chapter 7 discusses methods of evaluating permanents and compares their efficiency, while the final chapter deals with a variety of applications of permanents, e.g. to the estimation of the number of latin squares of a given order, the number of non-isomorphic Steiner triple systems of a given order and to the n-dimensional dimer problem. A list of conjectures and unsolved problems completes this final chapter.

It has to be said that this book is well-written and beautifully produced, with few misprints. To anyone working with, or requiring knowledge of, permanents, it should be regarded as essential. It is likely to become the standard reference on permanents.

E. SPENCE

ASIMOW, L. and ELLIS, A. J., *Convexity Theory and its Applications in Functional Analysis* (London Mathematical Society Monograph 16, Academic Press, 1980), x+226 pp. £23.20.

Convexity theory is a beautiful subject, combining geometry with algebra and analysis, and providing a unified approach to classical and modern results in areas such as potential theory, ergodic theory and operator algebras. It tells us that it is always possible to decompose points in a compact convex set into suitable combinations of extreme points (vertices), and when it is possible to do so uniquely. In finite dimensions this is a classical theorem of Carathéodory; an infinite-dimensional example is Bochner's Theorem on positive-definite functions.

After the discovery of Choquet's Theorem in 1956 and the Bishop-de Leeuw Theorem in 1960, there was much activity in general convexity theory for about 10 years, before a change of emphasis occurred in the 1970s. Alfsen's book "Compact convex sets and boundary integrals" (Springer, 1971) therefore appeared at a propitious time, and it has become a standard reference