The Factorisation of $1-2x^n\cos\alpha+x^{2n}$.

By Professor JACK.

Let $S=\sin\theta+x\sin2\theta+x^2\sin3\theta+x^3\sin4\theta+$ *ad infinitum*

multiply by $2x^n\cos n\theta$

$$\therefore\ S.2x^n\cos n\theta=x^n(\sin\overline{n+1}\theta-\sin\overline{n-1}\theta)+x^{n+1}(\sin\overline{n+2}\theta-\sin\overline{n-2}\theta)$$
$$+x^{n+2}(\sin\overline{n+3}\theta-\sin\overline{n-3}\theta)$$

$$\therefore\ S.2x^n\cos n\theta=S-(\sin\theta+x\sin2\theta+\ \dots\ +x^{n-1}\sin n\theta)$$
$$+S.x^{2n}-(x^n\sin\overline{n-1}\theta+x^{n+1}\sin\overline{n-2}\theta+\ .\ +x^{2n-2}\sin\theta)$$

Transpose, etc.

$$\therefore\ (1-2x^n\cos n\theta+x^{2n})S=$$
$$\left\{\begin{matrix}\sin\theta+x\sin2\theta+\ \dots\dots\dots\dots\ +x^{n-1}\sin n\theta\\ +x^n\sin\overline{n-1}\theta+x^{n+1}\sin\overline{n-2}\theta+\dots\ +x^{2n-2}\sin\ \theta\end{matrix}\right\}$$

Let $n=1$

$\therefore\ (1-2x\cos\ \theta+x^2\ )S=\sin\theta$ (Bracket reduces to one term here.)

DIVIDE.

$$\therefore\ \frac{1-2x^n\cos n\theta+x^{2n}}{1-2x\cos\ \theta+x^2}=\frac{\{\sin\theta+x\sin2\theta+\ ..\ +x^{2n-1}\sin2\theta+x^{2n-2}\sin\theta\}}{\sin\theta.}$$

and when $\cos n\theta$ is given there are n values only of $\cos\theta$

$\therefore$ there are n quadratic factors similar to the above.

The C-Discriminant as an Envelope.

By Mr JAS. A. MACDONALD.

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