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Numerical continuation methods for nonlinear equations and bifurcation problems

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This thesis investigates some aspects of the continuation method for the solution of a system of nonlinear equations, $f(x) = 0$, $f : D \subset R^n \to R^n$. This approach is useful for generating methods which do not rely on a good initial estimate of a solution and the problem is converted to one of following the solution trajectory $x(t)$ of a problem of the form $H\{x(t), t\} = 0$, $H : D \subset R^n \times R \to R^n$, from the starting guess $x_0 = x(0)$, hopefully to the solution x^*.

In Chapter 1 we give a brief introduction and note that $x(t)$ also satisfies

$$\dot{x}(t) = -\partial_x H(x, t)^{-1} \partial_t H(x, t) \ , \quad x(0) = x_0 \ ,$$

and so we can follow $x(t)$ by applying methods traditionally used for the solution of ordinary differential equations. In Chapter 2 we consider general single-step methods and, in particular, Runge-Kutta methods, for following $x(t)$. We also give conditions on the methods to attain rapid convergence to x^* and, as a result, for a particular choice of $H(x, t)$ we are able to derive methods which have improved rates of convergence to x^*. We apply similar arguments in Chapter 3 to the class of linear multistep methods and again generate methods which follow $x(t)$ accurately and then give rapid final convergence to x^*.

In Chapter 4 we consider Newton-like methods for finding $x(t_i)$ for a

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sequence of values $\{t_i\}$, and discuss the accuracy and computational efficiency of the methods. We use the results of Chapter 2 to derive a method which changes in a continuous way from one which follows $x(t)$ accurately to one which converges rapidly to x^* .

Chapter 5 is concerned with problems where the need to follow the solution of $H(x(t), t) = 0$ arises naturally. We consider, in particular, the difficulties associated with certain critical points, that is, points on the solution branch $(x(t), t)$ at which $\partial_x H(x, t)$ is singular. We describe an efficient method for following a branch through a simple turning point and present an efficient method for determining such turning points accurately. This method is also useful for finding certain simple bifurcation points.

Finally, in Chapter 6, we consider the problem of finding several solutions of the equation $f(x) = 0$. We consider two recent approaches and show that the two methods are essentially the same. A reformulation of one of the methods indicates a technique which is, in some sense, more efficient than the other methods.

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