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ON HOMOMORPHISMS OF AN ORTHOGONALLY DECOMPOSABLE HILBERT SPACE, III

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Abstract

A hyperfinite von Neumann algebra satisfies the condition that every o.d. homomorphism is a normal operator if and only if it is a factor of type I_n.

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Let M be a von Neumann algebra on a Hilbert space H and we assume that there is a cyclic and separating vector $\xi_0 \in H$ for M. We denote by J the conjugation operator associated with (M, H, ξ_0), and the natural cone H^+ is defined by

$$H^+ = \overline{\{xj(x)\xi_0 \colon x \in M\}},$$

where $j(x) = JxJ$.

A continuous linear operator ϕ on H is called an o.d. (orthogonal decomposition) *homomorphism* if the following condition is satisfied: if $\xi = \xi^+ - \xi^-$, where $\xi^+ \in H^+$, $\xi^- \in H^+$ and $(\xi^+, \xi^-) = 0$, is the orthogonal decomposition of $\xi \in H^J$, where $H^J = \{\xi \in H \colon J\xi = \xi\}$, then $\phi\xi \in H^J$ and $\phi\xi = \phi\xi^+ - \phi\xi^-$ is also the orthogonal decomposition of $\phi\xi$. It has been proved in [5] that a continuous linear operator ϕ on H is an o.d. homomorphism if and only if $\phi(H^+) \subset H^+$ and $\phi^*\phi \in M \cap M'$.

The aim of this note is to consider the following property:

($*$) *all o.d. homomorphisms are normal operators.*

We shall prove that ($*$) implies that M is a finite algebra, and, when H is

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separable and M is hyperfinite, (M, H, ξ_0) satisfies $(*)$ if and only if M is a factor of type I_n. However, the complete characterization of algebras with property $(*)$ will remain as an open problem.

1.

Let ϕ be a continuous linear operator on H. ϕ is normal if $\phi^*\phi = \phi\phi^*$. It is *quasinormal* if ϕ and $\phi^*\phi$ commute, and it is *paranormal* if $\phi^{*2}\phi^2 - 2\lambda\phi^*\phi + \lambda^2 \geq 0$ for all positive number λ [1]. Each of the three normalities implies the one that follows.

When M is a factor, an o.d. homomorphism ϕ satisfies $\phi^*\phi = \lambda 1$ for some number λ. Therefore, the following statement is obvious.

(1.1) *When M is a factor, all o.d. homomorphisms are quasinormal.*

When M is not a factor, there are o.d. homomorphisms which are not paranormal. In fact, we have the following.

(1.2) *If all o.d. homomorphisms are paranormal, all o.d. isomorphisms are normal.*

PROOF. A bijective o.d. homomorphism is an o.d. isomorphism, that is, the inverse operator, then, is also an o.d. homomorphism (see [5]). It has been proved in [5] that all o.d. isomorphisms are normal if and only if all unitary operators u such that $u(H^+) = H^+$ belong to the algebra $R(M, M')$ generated by M and M'. Now, suppose that u is a unitary operator such that $u(H^+) = H^+$. Let p be a central projection. Then, up is an o.d. homomorphisms. Hence, by the assumption, up is paranormal, that is,

$$(up)^*(up)^*upup - 2\lambda(up)^*up + \lambda^2 \geq 0$$

for all positive number λ, or,

$$pu^*pup - 2\lambda p + \lambda^2 \geq 0 \quad \text{for all } \lambda \geq 0.$$

Since u^*pu is a central projection (see [9], Theorem 2), $q = u^*pup$ is also a central projection and $q \leq p$. Hence, the above inequality with $\lambda = 1$ implies $q - 2p + 1 \geq 0$ and, therefore, $q \geq p$. Thus, $q = p$ and this implies $up = pup$. The same argument applied to u^*, instead of u, supplies $u^*p = pu^*p$. Thus, we arrive at $up = pu$. This proves $u \in (M \cap M')' = R(M, M')$.

Concerning the conclusion of (1.2), we have the following characterizations.

(1.3) *The following conditions are equivalent.*

(1) *All o.d. isomorphisms are normal.*

(2) *All unital Jordan isomorphisms are identical on the center.*

(3) *For any central projections p and q, if $u^*pu = q$ for some unitary operator u such that $u(H^+) = H^+$, then $p = q$.*

(4) *For any mutually orthogonal central nonzero projections p and q, there is no Jordan isomorphism β of Mp onto Mq such that $\beta(p) = q$.*

PROOF. The equivalence of (1) and (2) has been proved in [5]. (2) $\Rightarrow$ (3). Suppose that $u^*pu = q$ for some unitary operator u such that $u(H^+) = H^+$ and let α be the unital Jordan isomorphism determined by u. Then, by [2], Theorem 3.2.15, there is a central projection e such that $uxu^* = \alpha(x)e + J\alpha(x)^*J(1-e)$ for all $x \in M$. For $x = q$, we have $p = \alpha(q)e + J\alpha(q)^*J(1-e) = q$, because $\alpha(q) = q$ by (2).

(3) $\Rightarrow$ (4). Suppose that there is a Jordan isomorphism β of Mp onto Mq such that $\beta(p) = q$. Then, set $\alpha(x) = (1-(p+q))x + \alpha(px) + \beta^{-1}(qx)$ for all $x \in M$. α is a unital Jordan isomorphism on M and $\alpha(q) = p$. Thus, $u^*pu = q$ for the unitary operator u determined by α which satisfies $u(H^+) = H^+$.

(4) $\Rightarrow$ (2). Suppose that there is a unital Jordan isomorphism α of M and a central projection e such that $\alpha(e) \neq e$. Then, $f = \alpha(e)$ is also a central projection. When $fe \neq e$, set $p = e - ef$ and $q = f - f\alpha(f)$, and, when $fe = e$, set $p = f - e$ and $q = \alpha(f) - f$. Then, p and q are mutually orthogonal central projections such that $\alpha(p) = q$. Then, the restriction β of α on Mp is the Jordan isomorphism onto Mq and $\beta(p) = q$.

2.

In this section, we consider o.d. homomorphisms of the form $aj(a)$ for $a \in M$. We note that $aj(a)$ is an o.d. homomorphism if and only if $a^*a \in M \cap M'$ ([8], (3.4)).

(2.1) *All o.d. isomorphisms of the form $aj(a)$, $a \in M$, are normal.*

PROOF. If $aj(a)$ is an o.d. isomorphism, a^{-1} exists in M. Since $a^*a \in M \cap M'$, a^*a and a^{-1} commute. Hence, $a^*a = aa^*$ and $aj(a)$ is normal.

The corresponding statement for o.d. homomorphisms is as follows.

(2.2) *All o.d. homomorphisms of the form $aj(a)$, $a \in M$, are quasinormal.*

PROOF. When $\phi = aj(a)$ is an o.d. homomorphism, we have $a^*a \in M \cap M'$ and $\phi^*\phi = a^*aj(a^*a)$. Hence, ϕ and $\phi^*\phi$ commute.

The following statement shows that the quasinormality in (2.2) can not be replaced by normality. For the sake of convenience, we shall call two projections

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p and q on H *(j)-equivalent* if there exists a partial isometry v in M such that $p = \nu^*\nu j(\nu^*\nu)$ and $q = \nu\nu^* j(\nu\nu^*)$.

(2.3) *The following conditions are equivalent.*

(1) *If projections p and q on H are (j)-equivalent and $p \in M \cap M'$, then $p = q$.*

(2) *M is finite.*

(3) *All o.d. homomorphisms of the form $\nu j(\nu)$, for partial isometries ν in M, are normal.*

(4) *All o.d. homomorphisms of the form $aj(a)$, $a \in M$, are normal.*

(5) *If ν is a partial isometry in M and $\nu^*\nu \in M \cap M'$, then ν is normal.*

(6) *$x^*x = 1$ and $x \in M$ imply $xx^* = 1$.*

PROOF. The equivalence of (2) and (6) is well known ([6]).

(1) $\Rightarrow$ (2). We prove that 1 is a finite projection. Suppose that there is a projection e such that e is equivalent to 1. Then, there is a partial isometry $\nu \in M$ such that $\nu^*\nu = 1$ and $\nu\nu^* = e$. Then, $ej(e)$ is (j)-equivalent to 1. Hence, by (1), we have $ej(e) = 1$ and $e = 1$.

(2) $\Rightarrow$ (3). Let $\nu \in M$ be a partial isometry and $\phi = \nu j(\nu)$ be an o.d. homomorphism. Set $p = \nu^*\nu$ and $q = \nu\nu^*$. Then, $\phi^*\phi \in M \cap M'$ implies $pj(p) \in M \cap M'$ and, hence, $p \in M \cap M'$. Then, for the canonical central trace $\natural$, $p = p^\natural = (\nu^*\nu)^\natural = (\nu\nu^*)^\natural = q^\natural$. Hence, $(p - pq)^\natural = p^\natural - (pq)^\natural = p - pq^\natural = 0$. Since, $p \geq pq$, this implies $p = pq$. Therefore, $p \leq q$. Then, $(q-p)^\natural = 0$ implies $q = p$. Therefore, ν is normal and, hence, ϕ is normal.

(3) $\Rightarrow$ (4). Let $\phi = aj(a)$, $a \in M$, be an o.d. homomorphism and $a = \nu|a|$ be the polar decomposition. Then, $\phi = \nu j(\nu)|a|j(|a|)$ is the polar decomposition of ϕ. Therefore, $\nu j(\nu)$ is an o.d. homomorphism (see the remark below) and by (3) it is normal. Since $|a|j(|a|) \in M \cap M'$, ϕ itself is normal.

(4) $\Rightarrow$ (5). Let ν be a partial isometry such that $\nu^*\nu \in M \cap M'$, and set $p = \nu^*\nu$ and $q = \nu\nu^*$. Then, since $\nu^*j(\nu^*)\nu j(\nu) = pj(p) = p \in M \cap M'$, $\nu j(\nu)$ is an o.d. homomorphism. Hence, it is normal. Then, $p = \nu j(\nu)\nu^* j(\nu^*) = qj(q)$. Then, $p = pj(q) = qj(p)$ and $(p-q)j(p-q) = 0$. This implies $p = q$.

(5) $\Rightarrow$ (6). If $x^*x = 1$ and $x \in M$, x is a partial isometry in M and $x^*x \in M \cap M'$.

(6) $\Rightarrow$ (1). Suppose that $p = \nu^*\nu j(\nu^*\nu) \in M \cap M'$ and $q = \nu\nu^* j(\nu\nu^*)$. Then, $p = v^*v$. Since $e = 1 - p$ is a central projection, $(\nu + e)^*(\nu + e) - 1$. Hence, by (6), we have $(\nu + e)^*(\nu + e) = (\nu + e)(\nu + e)^*$, which implies $v^*v = vv^*$. Hence, $p = q$.

REMARK. When ϕ is an o.d. homomorphism and $\phi = \nu|\phi|$ is the polar decomposition, ν is also an o.d. homomorphism. This follows from $\nu = s - \lim_{n\to\infty} \phi(n^{-1}1 + |\phi|)^{-1}$, because $(n^{-1}1 + |\phi|)^{-1}$ belongs to the positive part of $M \cap M'$ and, hence, is an o.d. homomorphism.

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3.

When (M, H, ξ_0) has the property $(*)$, (2.3) implies that M is finite. Our conjecture is that (M, H, ξ_0) satisfies $(*)$ if and only if M is a factor of type I_n, but this has to remain as an open problem. In this section, we shall give some general consequences of $(*)$ and give an affirmative answer to this conjecture when H is separable and M is hyperfinite.

(3.1) *If* (M, H, ξ_0) *satisfies* $(*)$, *then every o.d. homomorphism belongs to* $R(M, M')$.

PROOF. Let ϕ be an o.d. homomorphism and p be a central projection. Then, $p\phi$ is also an o.d. homomorphism. Hence, ϕ and $p\phi$ are normal. This implies $\phi^* p\phi = p\phi\phi^* = p\phi^*\phi p = \phi^* p\phi p$. Therefore, $(p\phi - \phi p)^*(p\phi - \phi p) = 0$ and, hence, $\phi \in (M \cap M')' = R(M, M')$.

For the sake of convenience, we shall call two projections p and q on H (o)-*equivalent* if there exists a partial isometry v on H such that $\nu(H^+) \subset H^+$, $\nu^*\nu = p$ and $\nu\nu^* = q$. Note that, in this definition, p, q and ν are not necessarily in M.

(3.2) *The following conditions are equivalent.*

(1) *If* p *and* q *are* (o)-*equivalent and* $p \in M \cap M'$, *then* $p = q$.

(2) *All partial isometric o.d. homomorphisms are normal.*

(3) (M, H, ξ_0) *satisfies* $(*)$.

PROOF. (1) $\Rightarrow$ (2) and (3) $\Rightarrow$ (1) are immediate.

(2) $\Rightarrow$ (3). Let ϕ be an o.d. homomorphism and $\phi = \nu|\phi|$ be the polar decomposition. Then, ν is a partial isometric o.d. homomorphism and, hence, normal by (2). Furthermore, since $|\phi| \in M \cap M'$, ν and $|\phi|$ commute by (3.1). Therefore, ϕ is normal.

When p is a central projection, Mp is a von Neumann algebra on pH, and $p\xi_0$ is a cyclic and separating vector for Mp. The natural cone associated with $(Mp, pH, p\xi_0)$ is equal to $p(H^+)$.

(3.3) *Suppose that* (M, H, ξ_0) *satisfies* $(*)$ *and* p *is a central projection. Then,* $(Mp, pH, p\xi_0)$ *satisfies* $(*)$.

PROOF. If ϕ is an o.d. homomorphism on pH, $\psi = \phi p$ is an o.d. homomorphism on H. Hence, ψ is normal by the assumption, and ψ commute with p by (3.1). Hence, ϕ is also normal.

The next lemma will be used in (3.5). $M^n(\mathbb{C})$ denotes the algebra of all $n \times n$ matrices, the unit of which is denoted by 1_n.

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(3.4) *Let $n \leq m$ and $M = M^n(\mathbb{C}) \oplus M^m(\mathbb{C})$. Then, there is an element a of $M^{n+m}(\mathbb{C})$ such that $a^*a = 1_n \oplus 0$, $aa^* \leq 0 \oplus 1_m$, and the map $\nu: M \to M$ defined by $\nu(x) = axa^*$ satisfies $\nu(M^+) \subset M^+$.*

PROOF. M consists of the matrices of the following form:

$$x = \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}$$

for all $A \in M^n(\mathbb{C})$ and $B \in M^m(\mathbb{C})$. Since $n = m$, we shall write this matrix in the following form

$$x = \begin{pmatrix} A & 0 & 0 \\ 0 & B_1 & B_2 \\ 0 & B_3 & B_4 \end{pmatrix}$$

where B_1 is an $n \times n$ matrix. Then, the matrix

$$a = \begin{pmatrix} 0 & 0 & 0 \\ 1_n & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

satisfies the required properties.

Let M be a finite-dimensional von Neumann algebra and $M = Mp_1 \oplus Mp_2 \oplus \cdots \oplus Mp_n$ be the direct sum decomposition of M into factors where p_i $(1 \leq i \leq n)$ are mutually orthogonal central projections. Each Mp_i is the algebra of $n_i \times n_i$ matrices. Let 1_i be the unit matrix in Mp_i and set $\xi_0 = 1_1 \oplus 1_2 \oplus \cdots \oplus 1_n$. Let H be the Hilbert space consisting of elements of M with the inner product defined by the trace. Then, ξ_0 is a cyclic and separating vector for M.

(3.5) *Let M be a finite-dimensional von Neumann algebra on a Hilbert space H and ξ_0 be the cyclic and separating vector defined above. Then, (M, H, ξ_0) has the property $(*)$ if and only if M is a factor.*

PROOF. Suppose that M is a factor and ϕ is an o.d. homomorphism. Since $\phi^*\phi = \alpha 1$ for some number α and H is finite-dimensional, we have $\phi^*\phi = \phi\phi^*$. Conversely, suppose that (M, H, ξ_0) satisfies $(*)$. If M is not a factor, then, by (3.3), we can assume that $M = Mp_1 \oplus Mp_2$, where Mp_i are the algebras of $n_i \times n_i$ matrices such that $p_1 = 1_{n_1} \oplus 0$ and $p_2 = 0 \oplus 1_{n_2}$. Assume that $n_1 \leq n_2$ and take the operator ν defined in (3.4). Since $\nu^*\nu = p_1$, the central projection p_1 is (o)-equivalent to $\nu\nu^*$. Hence, by (3.2) and (3.4), we have $p_1 = \nu\nu^* \leq p_2$. Therefore, $p_1 = 0$, a contradiction. Hence, M is a factor.

Later, we shall give a negative example, which shows that the hyperfinite II_1-factor does not have the property $(*)$. In this example, ξ_0 is a trace vector and the o.d. homomorphism constructed there is ξ_0-preserving, that is, $\phi\xi_0 = \xi_0$. A linear operator α on M will be said to be τ-*bounded* if it is bounded with respect to the norm $x \to \tau(x^*x)^{1/2}$, where $\tau(x) = (x\xi_0, \xi_0)$.

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(3.6) *Let M be a finite factor and ξ_0 be a trace vector. Then, the following conditions are equivalent.*

(1) *Every ξ_0-preserving o.d. homomorphism is normal.*

(2) *Every ξ_0-preserving o.d. homomorphism is an o.d. isomorphism.*

(3) *Every τ-bounded unital Jordan homomorphism of M is a Jordan isomorphism.*

PROOF. (1) $\Leftrightarrow$ (2). This equivalence follows from the following fact: when M is a factor and $\phi \neq 0$ is an o.d. homomorphism, ϕ is normal if and only if ϕ is an o.d. isomorphism. To prove this statement, let M be a factor and ϕ be an o.d. homomorphism. Then, since $\phi^*\phi \in M \cap M'$ and ϕ is normal, $\phi = \lambda u$ for a positive number λ and a unitary operator u such that $u(H^+) = H^+$. Hence, ϕ is an o.d. isomorphism. The converse follows immediately from (1.3).

(2) $\Leftrightarrow$ (3). Since $\Delta_{\xi_0} = 1$, the map $x \to x\xi_0$ is an order isomorphism from M^+ onto the set $\{\xi \in H^+ : \xi \leq \lambda\xi_0 \text{ for some } \lambda > 0\}$, which is dense in H^+. Hence, the equation

$$\phi(x\xi_0) = \alpha(x)\xi_0 \quad \text{for all } x \in M$$

establishes a one-to-one correspondence between ξ_0-preserving o.d. homomorphism ϕ on H and τ-bounded unital Jordan homomorphism α on M, because a unital linear operator $\alpha: M \to M$ is a Jordan homomorphism if and only if $|\alpha(x)| = \alpha(|x|)$ for all sefladjoint elements x of M ([7], Theorem 6) and also, since $\Delta_{\xi_0} = 1$, we have $|x\xi_0| = |x|\xi_0$ for all selfadjoint elements x of M.

Let R_0 be the tensor product of (M_n, θ_n), $n = 1, 2, \ldots$, where $M_n = M^2(\mathbb{C})$ and

$$\theta = \theta_n = \begin{pmatrix} 2^{-1/2} & 0 \\ 0 & 2^{-1/2} \end{pmatrix}$$

for all n. This is a II_1-factor. A von Neumann algebra M is said to be *strongly stable* if M is isomorphic to $M \overline{\otimes} R_0$.

(3.7) *If M is strongly stable factor of type* II_1, (M, H, ξ_0) *does not have the property* $(*)$.

PROOF. We first show that R_0 does not have the property $(*)$. Let $K = K_n$ be the Hilbert space M_n with the inner product defined by the trace; $\theta_n \in K_n$ is a cyclic and separating vector for M_n. Let H be the tensor product of $\{K_n, \theta_n : n = 1, 2, \ldots\}$ and set $\xi_0 = \theta \otimes \theta \otimes \cdots$. Then, $\xi_0 \in H$ is a cyclic and separating vector for R_0. We prove that (R_0, H, ξ_0) does not have the property $(*)$. We denote the correspondence

$$\xi_1 \otimes \cdots \otimes \xi_k \otimes \theta \otimes \theta \cdots \to \theta \otimes \xi_1 \otimes \cdots \otimes \xi_k \otimes \theta \otimes \cdots$$

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by ϕ. It is extended to an isometric linear operator on the set of all finite linear combinations of the elements of the above form, and H is the closure of this set. Hence, ϕ can be regarded as an isometric linear operator on H. It satisfies $\phi^*\phi = 1$ and $\phi\phi^* \neq 1$. Therefore, if $\phi(H^+) \subset H^+$, ϕ is an o.d. homomorphism which is not normal. To prove $\phi(H^+) \subset H^+$, we note that $H^+ = \overline{\{A\xi_0 \colon A \in R_0^+\}}$. Now, suppose that $A \in R_0^+$. Then, A is the limit of a strongly convergent net of the elements of R_0^+ which are in the form

$$\sum_{\nu=1}^{p}(x_{1,\nu} \otimes \cdots \otimes x_{k,\nu}) \otimes 1 \otimes 1 \otimes \cdots$$

where $x_{n,\nu} \in M_n$ for all n. Therefore, the net

$$\left\{\sum_{\nu=1}^{p}(x_{1,\nu}\theta \otimes \cdots \otimes x_{k,\nu}\theta) \otimes \theta \otimes \theta \cdots\right\},$$

which is contained in H^+, converges to $A\xi_0$. However,

$$\phi\left(\sum_{\nu=1}^{p}(x_{1,\nu}\theta \otimes \cdots \otimes x_{k,\nu}\theta) \otimes \theta \otimes \theta \cdots\right) = \left(1 \otimes \sum_{\nu=1}^{p}(x_{1,\nu} \otimes \cdots \otimes x_{k,\nu}) \otimes 1 \otimes \cdots\right)\xi_0 \in H^+.$$

Since ϕ is an isometry, we have $\phi(H^+) \subset H^+$. Thus, R_0 does not have the property $(*)$. In the case of $M\overline{\otimes}R_0$, we define ϕ by

$$\eta \otimes \xi_1 \otimes \cdots \otimes \xi_k \otimes \theta \otimes \theta \otimes \cdots \to \eta \otimes \theta \otimes \xi_1 \otimes \cdots \otimes \xi_k \otimes \theta \cdots,$$

where η is an element of the Hilbert space on which M is defined. Then, exactly the same argument as above is applicable.

As a consequence, we have an affirmative answer to our conjecture when H is separable and M is hyperfinite (equivalently, injective).

(3.8) *Suppose that H is separable and M is hyperfinite. Then, (M, H, ξ_0) satisfies $(*)$ if and only if M is a factor of type* I_n.

PROOF. Suppose that H is separable. We first show that, if $M = M_1\overline{\otimes}M_2$, where $M_1(\neq \mathbb{C})$ is abelian, then M does not have $(*)$. Note, in this case, that M_1 has a direct summand $\mathbb{C} \otimes \mathbb{C}$ when it is purely atomic, and that M_1 has a direct summand isomorphic to $L^\infty([0,1])$ when it has a nonatomic part. Hence, there is an automorphism α_1 of M_1 with $\alpha_1 \neq \mathrm{id}_{M_1}$. Then, the automorphism $\alpha_1 \otimes \mathrm{id}_{M_2}$ of M is not identical on the center, so that, by (1.3), M does not have the property $(*)$. Now, suppose that M is hyperfinite and satisfies $(*)$. Then, by (2.3), M is finite. Through the central decomposition of M and by

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the uniqueness of injective II_1-factor on a separable Hilbert space (see [4]), M becomes a direct sum of von Neumann algebras of the form $M_1 \overline{\otimes} M_2$, where M_1 is abelian and M_2 is a factor of type I_n or $M_2 = R_0$. Thus, the desired conclusion follows immediately from (3.3), (3.5) and (3.6).

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