A SIMPLE DERIVATION OF THE RADIATION FORCES FELT BY SCATTERING PARTICLES

Joseph A. Burns and Steven Soter
Cornell University, Ithaca, NY 14853 U.S.A.

Abstract. The radiation pressure (RP) felt by a perfectly absorbing particle is due to the momentum withdrawn each second from the beam. The Poynting-Robertson (PR) drag is produced since the particle continually absorbs mass in the form of radiation, which, upon re-emission, has the same mean momentum density as the particle itself. We find that, relative to the force felt by a perfectly absorbing particle, the RP+PR forces felt by a scattering particle must be multiplied by Q_{pr}, the radiation pressure coefficient, which can be evaluated from Mie theory.

The Poynting-Robertson drag, often invoked to move small particles sunward in the solar system - and thereby to eliminate them - is a force whose physical basis is poorly understood. There was a heated controversy at the beginning of this century between Poynting, Larmor, Plummer and others as to the precise values for the coefficients of radiation pressure and of the drag term. This dispute was resolved by Robertson's (1937) definitive derivation which, however, because it uses the metric tensor of special relativity, is incomprehensible to many. Here we show that radiation forces can be understood in largely classical terms. Furthermore we remove Robertson's restriction to perfectly absorbing particles since, after all, small particles often scatter and diffract efficiently. Recent attempts were made to overcome this last limitation by Mukai et al. (1974) and Lamy (1976), who claimed incorrectly that only the absorbed radiation is important in producing a drag, while others have treated a specific scattering law (Lyttleton 1976) or merely stated the final answer (Dohnanyi 1978).

We give first a heuristic derivation of the radiation forces on a perfectly absorbing particle in order to find the causes of the RP and PR forces and to define terms that will be used later in the derivation for a particle with more general optical properties. This first section is similar to derivations given by van de Hulst (1957), Best and Patterson (1962), Harwit (1973), Soter et al. (1977) and Burns et al. (1979). We consider a spherical particle of mass m and cross-section A moving with velocity $\underset{\sim}{v}$ relative to the Sun through a radiation beam having an

I. Halliday and B. A. McIntosh (eds.), Solid Particles in the Solar System, 281–284.

https://doi.org/10.1017/S0074180900066882 Published online by Cambridge University Press

energy flux density E. The energy flux intercepted by the particle is $E'A \doteq E\,A(1-\dot{r}/c)$, including the Doppler shift of the received signal; $\dot{r} = \underset{\sim}{v}\cdot\hat{e}$ with $\hat{e}$ the unit vector in the direction of the beam. The associated momentum flux taken out of the beam is $(E'A/c)\hat{e}$. Since this is a change in momentum per unit time, Newton's second law says it is a force, the radiation pressure force; it is due to the interception of the beam's momentum.

The absorbed energy flux $E'A$ will affect the dynamics in other ways since it is equivalent to a mass flux, which is reradiated continuously (assuming thermal equilibrium) and isotropically (since small particles are effectively isothermal). In the particle's frame this isotropic loss of mass produces no effect; however, since the particle moves with $\underset{\sim}{v}$, the mass flux carries momentum away from the particle at the rate $(E'A/c^2)\underset{\sim}{v}$ as seen in the solar frame. Throughout the transfer process the mass of the particle remains constant and so the loss in momentum requires a deceleration, the Poynting-Robertson drag; this drag results from "mass-loading", by which we mean the particle is continually loaded by the absorbed radiation while allowing momentum to stream away.

Summing the momentum changes in these two processes, we have

$$m\dot{\underset{\sim}{v}} = (E'A/c)\hat{e} - (E'A/c^2)\underset{\sim}{v} \quad , \tag{1}$$

or

$$\underset{\sim}{F}_{rad} \approx (EA/c)[(1-\dot{r}/c)\hat{e} - \underset{\sim}{v}/c] \quad , \tag{2}$$

to first order in v/c. We note that the RP is in the direction of the beam while PR is a drag along the particle's velocity vector.

We now consider a particulate model (Fig. 1) for radiation impinging on a scattering body; we believe it contains the essence of the radiation forces experienced by such an object. A beam of "bullets" of mass flux μ moves relative to inertial space with velocity $\underset{\sim}{c}$, not necessarily the speed of light. This beam strikes a target mass m which has a velocity $\underset{\sim}{v}$; a fraction i of the beam is absorbed and instantaneously re-emitted isotropically (i may also include the fraction of particles isotropically scattered) while the remainder s is scattered. The scattered portion of the beam either leaves the target precisely with its approach velocity, mimicking forward-scattering, or minus the approach velocity, when back-scattered. On the right of Fig. 1 we view the same scene one second after the first mass of μ arrives. Due to the target's motion, only $\mu' = \mu(1-v\cos\xi/c)$ has struck m and, of this, $s\mu'$ has been scattered (here shown back-scattered) while $i\mu'$ has been absorbed (or isotropically scattered); in either of the latter cases the material is considered to still reside in the particle since the momentum density of $i\mu'$ is the same as that of m itself, as our first example showed. The dynamical consequences of the interation with the beam are represented by two velocity changes: $\Delta\underset{\sim}{v}$ along the particle's original velocity, presumed to be due to mass-loading, and $\Delta\underset{\sim}{V}$ along the beam's direction, caused by interception of the beam's momentum.

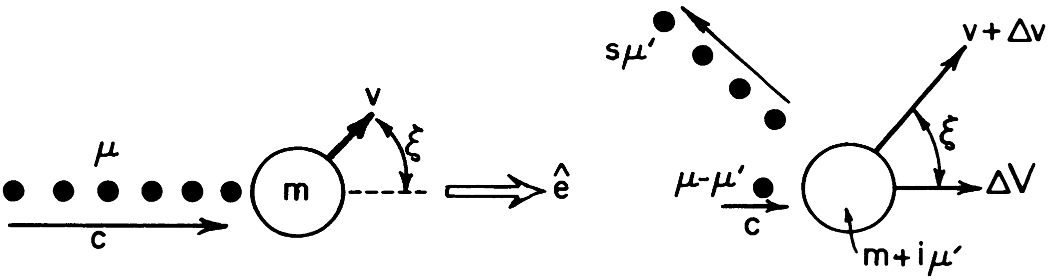

Figure 1. A model of the interaction between a radiation beam and a scattering particle. The left figure is just as the first part of the beam strikes m; on the right, one second later.

To solve for $\Delta \underset{\sim}{v}$ and $\Delta \underset{\sim}{V}$, we apply conservation of linear momentum between the two situations shown. Equating the vertical momenta,

$$m\,v\sin\xi = (m+i\mu')(v+\Delta v)\sin\xi + s\mu'[(\mp)v\sin\xi + v\sin\xi] \quad , \tag{3}$$

where here, as well as later, the upper sign in the last term corresponds to forward-scattering while the lower sign is for back-scattering. Solving for the deceleration along v,

$$\Delta v = -(1\mp s)\mu' v/m \quad . \tag{4}$$

Conservation of horizontal momentum lets ΔV be evaluated from

$$m\,v\cos\xi + \mu c = (m+i\mu')[(v+\Delta v)\cos\xi + \Delta V] + s\mu'[\pm(c - v\cos\xi) + v\cos\xi] + (\mu-\mu')c \tag{5}$$

or

$$\Delta V = (1\mp s)\mu' c/m \tag{6}$$

along the beam. Adding $\Delta \underset{\sim}{v}$ and $\Delta \underset{\sim}{V}$ vectorially, the mass times the change in velocity per second (i.e., the force felt by m) is

$$\underset{\sim}{F} = m(\Delta\underset{\sim}{V} + \Delta\underset{\sim}{v}) = (1\mp s)\mu'(\underset{\sim}{c} - \underset{\sim}{v}) \quad . \tag{7}$$

The term in the first parenthesis on the right is the total fraction of the radiation striking the particle minus (or plus) the fraction scattered in the direction of the beam as seen by the particle; in the terminology of Mie scattering it is Q_{pr}, the radiation pressure coefficient. We may transform from our particulate model to a continuous stream of radiation by equating μ' with $E'A/c^2$, the equivalent mass striking the particle. Finally the radiation force felt by a scattering, spherical particle is

$$\underset{\sim}{F}_{rad} \approx (EA/c)Q_{pr}[(1-\dot{r}/c)\hat{e} - \underset{\sim}{v}/c] \quad , \tag{8}$$

to order v/c. This same expression has been confirmed in a totally independent way by Burns et al. (1979) using the special relativity transformation laws for momentum and energy. This result agrees with the traditional view that the Poynting-Robertson drag is due to an aberration of the radiation pressure force through the angle v/c.

Equation (8) shows that both the radiation pressure and the Poynting-Robertson drag are proportional to Q_{pr} and thus, since Q_{pr} is small for particles less than ~0.03 μm in radius for most compositions (Burns et al., 1979), radiation forces are ineffective on such interplanetary particles. The drag (Whipple, 1967) produced by the streaming of solar wind particles past the target can be handled the same way as above. With a target size > 1 μm, Q_{pr} for both electromagnetic and corpuscular radiation is near 1 so that the solar wind drag is only about 10% that due to solar radiation; however, since Q_{pr} for the solar wind is always near 1, solar wind drag dominates at sizes < 0.03 μm. Our view of the Poynting-Robertson drag as due to "mass-loading" allows us to claim that the orbit will collapse in a characteristic time approximately equal to that in which the particle interacts with its own equivalent mass in radiation. This insight means that the orbital collapse time for particles on heliocentric orbits is approximately the same as for those on planetocentric orbits.

REFERENCES

Best, G.T., and Patterson, T.N.L.: 1962, *Planet. Space Sci.* 9, pp. 801-9.
Burns, J.A., Lamy, P.L., and Soter, S.: 1979, *Icarus* 40, in press.
Dohnanyi, J.S.: 1978, *Cosmic Dust* (J.A.M. McDonnell, Ed.), pp. 527-605. Wiley, Chichester.
Harwit, M.: 1973, *Astrophysical Concepts*. Wiley, New York.
Lamy, P.L.: 1976, *Interplanetary Dust and Zodiacal Light* (H. Elsässer and H. Fechtig, Eds.), pp. 437-442. Springer-Verlag, Berlin.
Lyttleton, R.A.: 1976, *Astrophys. Space Sci.* 44, pp. 119-140.
Mukai, T., Yamamoto, T., Hasegawa, H., Fujiwara, A., and Koike, C.: 1974, *Publ. Astron. Soc. Japan* 26, pp. 445-458.
Robertson, H.P.: 1937, *Mon. Not. Roy. Astro. Soc.* 97, pp. 423-438.
Soter, S., Burns, J.A., and Lamy, P.L.: 1977, *Comets, Asteroids and Meteorites* (A. Delsemme, Ed.), pp. 121-125. Univ. Toledo Press.
van de Hulst, H.C.: 1957, *Light Scattering by Small Particles*. Wiley, NY.
Whipple, F.: 1967, *The Zodiacal Light and the Interplanetary Medium* (J. Weinberg, Ed.), pp. 409-426. NASA SP-150.