

ON ASTRONOMICAL REFRACTION FOR A THREE-DIMENSIONAL MODEL ATMOSPHERE.

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The existing refraction theories for a one-dimensional model atmosphere do not account for anomalous refraction. The latter can be calculated using a three-dimensional model atmosphere.

The author has deduced formulae for calculation of refraction from the zenith distance  $R_Z$  and azimuth  $R_A$  for

a three-dimensional model atmosphere. For the considered model atmosphere the variation of the refractive index  $n$  in the vertical plane is described by the exponential function (cases of some other simple functions are also considered). In the horizontal plane the variation of  $n$  against  $x$  and  $y$  (rectangular coordinate system) is accounted by horizontal gradients  $\epsilon$  and  $\omega$  respectively. The  $\epsilon$  and  $\omega$  can be calculated from the data of aerological probing. While gradients of meteorological element variation in the vertical plane is by three-four orders larger than that in the horizontal plane (Leichtman, D.L., 1970).,  $\epsilon$  and  $\omega$  are small compared with the vertical gradient of  $n$ . Then the path of a beam of light in the three-dimensional model atmosphere will be expressed by

$$\left. \begin{aligned} \frac{d\alpha}{ds} &= \omega(1-\alpha^2) - \epsilon\alpha\beta - \bar{B}\alpha\sqrt{1-\alpha^2-\beta^2} \\ \frac{d\beta}{ds} &= \epsilon(1-\beta^2) - \omega\alpha\beta - \bar{B}\beta\sqrt{1-\alpha^2-\beta^2} \end{aligned} \right\}$$

where  $\alpha$  and  $\beta$  are cosines of the tangent to the trajectory<sup>(1)</sup> of the beam,

$$\left. \begin{aligned} \omega &= \frac{1}{n} \frac{\partial n}{\partial y} \\ \epsilon &= \frac{1}{n} \frac{\partial n}{\partial x} \end{aligned} \right\} \quad \text{small parameters,}$$

$\bar{B} = \ln B$ ,  $B$  is taken from  $n=AB^H$ ;  $H$  is the height above sea level.

System of differential equations (1) is a non-linear dynamic system of the second order with two small parameters. The equation system is solved by the analytical method with application of Bernoulli's method (Danko P.E., Popov A.T., 1974). or methods of perturbation theory.

For the solution of the equations by the former method an assumption of was made on the invariability of  $n$  within each atmospheric layer. The practical application of this method is expedient if there are sufficient meteorological data describing in detail the structure of the atmosphere. The utilization of perturbation theory methods permitted to solve system (1) in general form. Series expansion of the solutions was done following the powers of the two small parameters and separately up to the terms of the first and second order. Boundary conditions for the adopted model atmosphere are values of  $n$  on the Earth's surface and at a 75 km height. Initial conditions are values of direction cosines of a light beam in the medium with  $n=1+10^{-6}$

Refraction was calculated using the following definitions;

Definition I. Refraction  $R_Z$  for the zenith distance  $Z$  is assumed to be an angle between two tangents of the trajectory of a light beam at the end points of the beam path in the atmosphere.

Definition II. Refraction  $R_A$  for azimuth  $A$  is an angle between two binormals to the spatial curve of a beam of light at the end points of the beam path in the atmosphere.

$R_Z$  calculated from the proposed formulae for a case when there are no horizontal gradients of the refractive index  $n$  coincides to  $\pm 0.02$  with the refraction calculated using Gylden's theory which was basic for the Pulkovo refraction tables.

For the three-dimensional model atmosphere the length of the trajectory to the beam has to be calculated to an accuracy of  $\pm 0.10m \operatorname{cosec} Z$  at zenith distances  $Z \geq 80^\circ$  in order to determine refraction with an accuracy of  $\pm 0.01$ .

The practical application of the deduced formulae is possible only the case when there are data of free atmos-

pheric probing. In the ground and boundary atmospheric layers these may be obtained using the collected meteorological information for the moments of observations of each star (Kudeeva V.S., Pavlov B.A., Sergienko V.I.).

The proposed formulae of the refraction calculation give a chance to account for the asymmetry of the atmosphere, calculate refractions for different azimuths and reach a higher accuracy of the results in dependence on the atmosphere.

### References

1. Danko P.E., Popov A.T. Vysshaya Matematika v Uprazhneniyakh i Zadachakh, part II, M., Izd-vo "Vysshaya Shkola, 1974, pp.160-161.
2. Leichtman D.L. Fizika Pogranichnogo Sloya Atmosfery. L., Hidrometeoizdat, 1970, pp.41-43.
3. Sergienko V.I., Pavlov B.A., Kudeeva V.S., Trudy VNIIFTRI, vypusk 31 (61) pp.38-43.

### DISCUSSION

G. Teleki: noticed the importance of such investigations but he expects the application of these theories in practice. The basic problem is how to get a real information on the variations (with height) of inclinations of the equal density layers.