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# JAAKKO HINTIKKA 1929–2015

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The logician and philosopher Jaakko Hintikka died on August 14, 2015, only a few days after giving a lecture entitled *Distributive normal forms 2015* in the European Summer Meeting of the Association for Symbolic logic (Logic Colloquium 2015), and another entitled *Entanglement and probability* in the co-located 15th Congress of Logic, Methodology and Philosophy of Science in Helsinki, Finland. He was perhaps best known for his distributive normal forms but in recent years his main focus was a complete rethinking of mathematics and quantum physics, the topic of his lecture *Entanglement and probability*. In his last lecture his radical conclusion was:

> "Why cannot the behavior of entangled particles be explained in terms of shared attributes? Allegedly because of Bell's inequality. But if that inequality is merely a mathematical truth, it cannot have any factual consequences. Hence we should look for a better mathematics, including new logic and new concept of probability ... This removes all obstacles to local realism and hence vindicates the Einstein vs. Copenhagen interpretation."

Hintikka was born in Vantaa, Finland, in 1929. He received his Ph.D. from the University of Helsinki in 1953. He held positions at Harvard University, the Academy of Finland, the University of Helsinki, Stanford University, Florida State University, and Boston University, from where he had recently retired to the position of Honorary Fellow at the Helsinki Collegium for Advanced Studies.

Hintikka's early work was in mathematical logic, and applications of logic in philosophy was his trademark all through his long career. His first contribution in logic was on *distributive normal forms* [8], a topic investigated and advocated by his teacher, George Henrik von Wright [26–28] in propositional logic, monadic predicate logic and fragments of full predicate logic. In propositional logic the distributive normal form is just the usual *disjunctive normal form*. Proofs of the decidability of monadic logic are usually based on an extension of the disjunctive normal form to monadic predicate logic. Hintikka extended distributive normal forms to

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*full* predicate logic as follows: Let $A_i^n(x_1,\dots,x_n)$, $i \in K_n$, list all atomic formulas in a finite relational vocabulary (without identity, for simplicity), and the variables $x_1,\dots,x_n$. If $F$ is a formula, let $[F]^0 = F$ and $[F]^1 = \neg F$. Let $C_i^{0,n}(x_1,\dots,x_n)$, $i \in I^{0,n}$, list all possible conjunctions $\bigwedge_j [A_j^n(x_1,\dots,x_n)]^{\varepsilon(j)}$ where $\varepsilon$ runs through all functions $K_n \to \{0,1\}$. Let $C_i^{m+1,n}(x_1,\dots,x_n)$, $i \in I^{m+1,n}$, list all possible formulas

$$\bigwedge_{j\in J} \exists x_{n+1} C_j^{m,n+1}(x_1,\dots,x_{n+1}) \wedge \forall x_{n+1} \bigvee_{j\in J} C_j^{m,n+1}(x_1,\dots,x_{n+1}),$$

where $J \subseteq I^{m,n+1}$.

The syntactic form of $C_i^{m+1,n}(x_1,\dots,x_n)$ suggests meaning in terms of a two-person "semantic game": Player $I$ picks $j \in J$ and then player $II$ picks a value for $x_{n+1}$. Alternatively, Player $I$ picks a value for $x_{n+1}$ and then player $II$ picks $j \in J$. After these moves the game continues similarly in the position $C_j^{m,n+1}(x_1,\dots,x_{n+1})$. In later work Hintikka indeed strongly advocated the resulting *Game-Theoretical Semantics*. A similar analysis of predicate logic was introduced by Fraïssé [6] in terms of back-and-forth systems, and by Ehrenfeucht [5] in game-theoretic terms. The connection is as follows: If $a_1,\dots,a_n$ satisfy $C_i^{m,n}(x_1,\dots,x_n)$ in a model $\mathcal{M}$ and $b_1,\dots,b_n$ satisfy $C_i^{m,n}(x_1,\dots,x_n)$ in a model $\mathcal{N}$, then $C_i^{m,n}(x_1,\dots,x_n)$ codes a winning strategy for player II in the $m$-move Ehrenfeucht-Fraïssé-game played on the models $\mathcal{M}$ and $\mathcal{N}$, starting from the position $\{(a_1,b_1),\dots,(a_n,b_n)\}$.

Every first order sentence $\phi$ of quantifier rank $m$ is logically equivalent to a unique disjunction of formulas of the form $C_i^{m,0}$. This disjunction is the *distributive normal form* of $\phi$. The process of finding the distributive normal form of a given sentence cannot be made effective. Intuitively, one pushes quantifiers as deep into the formula as possible.

Distributive normal forms were Hintikka's preferred tool in applications of logic to philosophy. In the logic of induction Hintikka used distributive normal forms to give, in contrast to Carnap [4], positive probabilities for universal generalizations [13]. He developed a theory of *surface information* [15] to support a thesis of the nontautological nature of logical inference, with applications to Kant's analytic-synthetic distinction. In model theory he used them to systematize definability theory [18], such as the Beth Definability Theorem, the Craig Interpolation Theorem and the Svenonius Theorem, and to systematize infinitary logic. Even if distributive normal forms have not become a standard tool as such, the more or less equivalent concept of Ehrenfeucht-Fraïssé-games, or back-and-forth systems, is nowadays widely used in model theory, especially in infinitary logic, where distributive normal forms are very close to so-called Scott Sentences [24]. Lindström's celebrated characterization of first order logic [21], as well as its infinitary extensions by Barwise [1], are based on a systematic use of back-and-forth systems.

A winning strategy of player $II$ in the semantic game associated with some $C_i^{m,0}$ true in a given model $\mathcal{M}$ gives rise to a certain limited set $S$ of information about the model $\mathcal{M}$. Now we can turn the tables, so to speak, and start from such limited sets $S$ and try to reconstruct the model $\mathcal{M}$, or at least *some* model $\mathcal{M}^*$, or consider $S$ as a "model", a "possible world". Hintikka combined this idea with his critical view of the Tarski truth definition and was led in [9] to the concept of a *model set* as a more constructive approach to semantics. A model set has enough information to build a canonical term model in which sentences belonging to the model set are true. However, one can argue with model sets directly and avoid building the models entirely. Hintikka argued in [9] vehemently in favor of this method as an alternative to the more set-theoretical approach, which was not very clearly articulated at the time, but was presented in all details subsequently in Tarski-Vaught [25].

A *model set* is a set $S$ of first order sentences without identity (for simplicity), with negation in front of atomic formulas only, in a countable vocabulary, and containing possibly new individual constants, such that (1) No atomic sentence $\varphi$ satisfies both $\varphi \in H$ and $\neg\varphi \in H$. (2) If $\varphi \wedge \psi \in H$, then $\varphi \in H$ and $\psi \in H$. (3) If $\varphi \vee \psi \in H$, then $\varphi \in H$ or $\psi \in H$. (4) If $\exists x\varphi(x) \in H$, then $\varphi(c) \in H$ for some constant $c$. (5) If $\forall x\varphi(x) \in H$, then $\varphi(c) \in H$ for all constants $c$ occurring in $H$.

A sentence has a model (in the set-theoretical sense) if and only if it is an element of a model set. Attempts to build a model set around the negation of a sentence $\phi$ form a tree, known as a semantic tree (or Beth tableau). Infinite branches of this tree are model sets for $\neg\phi$. If the tree has no infinite branches, it is finite, and can be considered a proof of $\phi$ in the style of Herbrand and Gentzen. Beth tableaux were introduced in the same year but independently by E. Beth [2]. Model sets came to play a central role in Hintikka's other work, such as possible-worlds semantics and game-theoretic semantics. One can see in the top-down analysis of the meaning of first order sentences by means of model sets, signs of what became Hintikka's life-long commitment to noncompositionality.

Another early but far-reaching contribution of Hintikka is the reduction of the theory of finite types (i.e., higher order logic) to a fragment of second order logic [10]. Hintikka associates with an arbitrary sentence $\phi$ of finite type theory an existential second order sentence $\phi^*$ such that $\phi$ is valid if and only if $\phi^*$ is. This shows that the existential fragment of second order logic is, in a sense, as complex as full second order logic, and even the theory of finite types. The proof is essentially an application of many-sorted logic and was later generalized by Montague [23]. With his Independence Friendly Logic [17], which is equivalent in expressive power to the existential fragment of second order logic, Hintikka came back to this reduction in the 90s.

Hintikka's formal definition of *possible-worlds semantics*, or model systems [11] for modal and epistemic logic, is based on his concept of model set, unlike Kripke's approach [20] which uses actual models as possible worlds. A *model system* $(\mathcal{S}, R)$ consists of a set $\mathcal{S}$ of model sets and a binary alternativeness-relation $R$ on $\mathcal{S}$ such that: (1) If $\Box\phi \in H \in \mathcal{S}$, then $\phi \in H$. (2) If $\Diamond\phi \in H \in \mathcal{S}$, then there exists an alternative $H' \in \mathcal{S}$ to $H$ such that $\phi \in H'$. (3) If $\Box\phi \in H \in \mathcal{S}$ and $H' \in \mathcal{S}$ is an alternative to $H$, then $\phi \in H'$. A set $S$ of formulas is defined to be *satisfiable* if there is a model system $(\mathcal{S}, R)$ such that $S \subseteq H$ for some $H \in \mathcal{S}$. A formula $\phi$ is *valid* if its negation is not satisfiable. Hintikka applied possible-worlds semantics to epistemic logic, deontic and modal logic, the logic of perception and to the study of Aristotle and Kant. See [14] for Hintikka's summary of his theory of possible-worlds semantics. His book *Knowledge and Belief* [12] became a classic, also outside philosophy in artificial intelligence and theoretical computer science. On the basis of his epistemic logic Hintikka formulated a *logic of questions* [16] and used it as an interrogative model for the philosophy of science. Hintikka wrote also extensively on the history of philosophy in a series of papers on philosophers from Aristotle through Kant and Leibniz to Frege and Carnap.

The idea of thinking of the meaning of a sentence in game-theoretic terms in implicit already in distributive normal forms and model sets. Eventually Hintikka made this idea completely explicit with his game-theoretic semantics. A *semantic game* of a sentence $\phi$ in a model $\mathcal{M}$ is a two-person game between $I$ and $II$ about a formula $\phi$ and an assignment $s$. For $\phi = \phi_1 \wedge \phi_2$, player $II$ chooses $\phi_i$. For $\phi = \phi_1 \vee \phi_2$, player $I$ chooses $\phi_i$. Then the game continues with $\phi_i$ and $s$. For $\phi = \forall x \psi(x)$, player $II$ chooses $s'$ which agrees with $s$ outside $x$. For $\phi = \exists x \psi(x)$, player $I$ chooses such an $s'$. Then we continue with $\psi(x)$ and $s'$. For negation, the players exchange roles. For $\phi$ atomic, the game ends. Player $II$ wins if $s$ satisfies $\phi$ in $\mathcal{M}$, otherwise player $I$ wins. Hintikka lists Wittgenstein's language-games [29], Lorenzen's dialogue games [22], Ehrenfeucht's back-and-forth games [5], and Henkin's game-theoretic interpretation of quantifiers [7], as the predecessors of his game-theoretic semantics.

Game-theoretic semantics became Hintikka's tool for analyzing natural language, particularly pronouns, conditionals, prepositions, definite descriptions, and the de dicto vs. de re distinction, and for challenging the approach of generative grammar. Sentences like *Every writer likes a book of his almost as much as every critic dislikes some book he has reviewed* led Hintikka to consider partially-ordered quantifiers and eventually IF (Independence friendly logic) logic [17], with existential quantifiers $\exists x/y$, meaning that a value for $x$ is chosen *independently* of what has been chosen for $y$. Thus the semantic game of IF-logic is a game of partial information. IF-logic is equal in expressive power to the existential fragment of second-order logic. Hence its validity problem is by Hintikka's earlier work [10] as difficult as that of full second order logic. On the positive side, the truth of almost any mathematical statement can be reduced to the validity of a suitable sentence in IF-logic. The satisfiability of a sentence can still be analyzed in terms of model sets. This apparent contradiction is explained by the fact that IF-logic is not closed under negation. Hodges 1997 gave IF-logic a compositional semantics in terms of sets of assignments, and Cameron and Hodges [3] proved that it has no compositional semantics in terms of assignments only, which Hintikka considered a vindication of his claim that IF-logic does not have a compositional semantics. However, the compositional semantics of Hodges has turned out to be closely connected to the theory of database dependency (e.g., [19]), and IF-logic has thereby given rise, perhaps unexpectedly, to a new lively research area, not in philosophy, but in computer science logic.

Hintikka had high hopes for IF-logic. He increasingly turned to the idea that we have to rethink everything in terms of IF-logic, including mathematics and physics. According to him, this would lead to the resolution of paradoxes, incompleteness results, and famous unsolved problems, in particular the Continuum Hypothesis. This is testimony to his tremendous optimism about the power of radical new ideas to lead to breakthroughs, not only in logic and philosophy, but in science and humanities in general. For 60 years he inspired generations of logicians and philosophers who shared his optimism, even if they could not always agree with the conclusions.

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