

RESEARCH ARTICLE

# On the extraction of common-pool resources: impacts of population, amenity values and income inequality

Jussi Uusivuori<sup>1</sup>, Janne Rämö<sup>1</sup>  and Olli-Pekka Kuusela<sup>2</sup> 

<sup>1</sup>Natural Resources Institute Finland, Helsinki, Finland and <sup>2</sup>UNU-WIDER, Helsinki, Finland

**Corresponding author:** Janne Rämö; Email: [janne.ramo@luke.fi](mailto:janne.ramo@luke.fi)

(Received 19 March 2024; revised 6 March 2025; accepted 8 April 2025)

## Abstract

This paper studies the dynamic extraction problem of an exhaustible common-pool resource. We build on classical closed-economy growth models with intertemporally maximizing, infinitely lived dynasties exhibiting a constant population growth rate. Utility is obtained from periodic consumption based on the fixed-rate capital and the extraction of the resource, and from the amenity values derived from the standing resource stock. The resource contributes to both consumptive and amenity utilities, while different generations are interconnected by intergenerational altruism. Dynamic allocation of the natural resource is determined by a benevolent social planner. This allows us to examine intra-generational inequity issues in combination with the intergenerational concerns. We demonstrate how the optimal allocation of the resource depends on the population growth, wealth level, inequality, ecological vulnerability of the resource and rivalry on the amenity value. Our results highlight the trade-offs between reducing the degree of inequality and preserving the ecological values of the resource.

**Keywords:** dynamic optimization; intra-generational altruism; public goods; resource vulnerability; social planner; tropical forests

**JEL classification:** D63; H41; O13; Q23; Q32

## 1. Introduction

There has been a long-standing tension in many emerging and developing countries between protecting primary rainforests on the one hand and using the land for economic development on the other. While there are several factors that contribute to the dynamic relationship between these two competing land uses, some typically identified challenges are associated with inequality, population growth, and ownership arrangements (e.g., Leblois *et al.*, 2017; Sant'Anna, 2017; Ceddia, 2019; Balboni *et al.*, 2023).

The reason for concern, and the motivation for a large body of research, is the high rates of deforestation observed in the tropics and the associated risks imposed on biodiversity and the climate. Building economic models capable of capturing these critical elements, which contribute to deforestation processes, helps us to better understand the mechanisms behind these drivers and to design better policies to mitigate deforestation problems while at the same time enabling the improvement of livelihoods.

This paper makes a contribution by presenting an analytical model of the depletion of non-renewable resources with *in-situ* amenity values, such as primary tropical rainforests. The model integrates the effects of ownership structures, population growth and income inequality with the use of the resource. The model is based on utility maximization over an infinite time horizon, utility being derived both from the consumption as well as from *in-situ* externalities of the resource. We illustrate a common-pool resource with community ownership. This ownership class is fairly commonly applied for forestland worldwide.

The structure of the model presented here draws on some classical models of economic growth. Ramsey (1928), Cass (1965) and Koopmans (1963) analysed closed-economy growth models with intertemporally maximizing, infinitely lived dynasties exhibiting an assumed population growth rate. These dynasties were egalitarian, accounting for the utilities of each generation according to their size. With such growth models, it is possible to study, for example, the intergenerational aspects of the consumption and savings problem. In the present study, we also assume egalitarian dynasties and that the generations are interconnected by intergenerational altruism. Furthermore, we incorporate in our model intra-generational wealth inequality together with the decision to extract the common-pool resource and divide the revenues among the members. The social planner's solution to the model enables us to study both intra-generational inequity and inter-generational properties of the common-pool extraction problem.

Koopmans (1963) introduced the endogeneity of resource flows to the problem of best allocation of exhaustible resources over time by operating with an exogenous initial stock of resources. In his continuous-time model, the total utility flow is obtained from a flow of consumption of a good produced with the use of capital stock along the lines of the Ramsey (1928) model, and a flow arising from the rate of depletion of an exhaustible stock. The model structure used here is related to Koopmans model combining capital and an exhaustible resource, but with a constant interest rate. In the discrete-time model presented here, the resource contributes to both consumptive and amenity utilities.

We examine the case where a natural resource, such as a tropical forest with externality values, is a common-pool resource owned by a regional or local government or by a communal group. While a private actor is concerned about her own and her offspring's utilities and excludes the utility of other community members, we introduce a social planner's problem, where there is a concern over the utility of all community members. The members of the community are thus the dynasties whose combined utilities the benevolent social planner is maximizing.

Our research also contributes to the literature modelling the dynamics of deforestation in tropical countries (e.g., Amacher *et al.*, 2009; Wolfersberger *et al.*, 2022). Our research differs from these in that we focus on modelling the common property

aspects of the resource together with trying to better understand the impacts of wealth inequalities in the communities on optimal resource extraction plans as defined by a benevolent planner. Community lands, such as forests, as an ownership class are found both in the developed and in the developing countries. According to Agrawal and Angelsen (2009), communities have the control rights on nearly 10 per cent of the global forest cover, and they provide livelihood to more than a billion people worldwide. In developing countries, their significance is even greater, as community management covers 25 per cent of the forest area (Bluffstone *et al.*, 2020).<sup>1</sup>

Introducing both inter- and intra-generational preferences will result in the depletion of the resource to depend on the population size and its growth rate as well as on intra-generational inequity rate. Furthermore, depletion will be dependent on the population-driven ecological impacts on the common-pool resources. For example, empirical literature on deforestation dynamics has found that population pressures have contributed to deforestation in the tropics (e.g., Barbier, 2004; Leblois *et al.*, 2017).

Like public good resources, common-pool resources are non-exclusive among the community members. However, unlike public good resources, common-pool resources exhibit rivalry. We show that the optimal allocation of the resource becomes dependent on population growth, wealth level, wealth inequality, ecological vulnerability of the resource and rivalry on the amenity value of the resource. We further illustrate the dynamic path of the resource extraction using several numerical scenarios.

The rest of the study is structured as follows. In section 2, we first introduce the amenity valuation function of the resources and show how the population size and rivalry enter this valuation. In sections 3 and 4, we then present the social planner's problem of the extraction of the forest resource. The optimal consumption and extraction rules are derived using logarithmic utilities. In section 5, we introduce a measure for income inequality within the community. This is then used for the first-order conditions. In section 6, numerical examples are used to illustrate the model solutions. Finally, in the last section, we discuss our results and provide conclusions.

## 2. Rivalry over the common-pool resources and the amenity function

Unlike pure public goods, common-pool resources face problems of congestion or overuse, because they are subtractable. This is typical, for example, in the case of timber and many non-timber benefits generated by common-pool forests. Additionally, the non-tangible externality values of a forest resource may face rivalry, and this needs to be taken into account in the model. However, rivalry over a resource may start from a certain resource-specific threshold level of population, below which level there is no rivalry or, on the contrary, population growth may induce improvements in the quality of the resource. This phenomenon is closely related to the concept of local public goods with congestion, discussed by, e.g., Greenberg (1978) and Buchanan (1965).

<sup>1</sup> Community lands are sometimes also referred to as communal forests, collective forests, joint forests or social forests. Management of common-pool resources is also found in the case of grazing lands (Wiersum, 2004). In other contexts, amenity values within resource context have been incorporated into the models by, e.g., Krautkraemer (1985), Gerlagh and Keyzer (2004), and D'Autume and Schubert (2008).

For numerical purposes, we will specify the amenity utility function in such a way that it accounts for the impact of population pressure on the resource externalities. We do this through a rivalry function  $\sigma(N_t/\bar{N})$  which measures the rivalry over the amenity values of the common-pool resource, as a function of the population pressure  $N_t/\bar{N}$  (with population  $N_t > 1$  and a reference or saturation population size  $\bar{N}$ ). The rivalry function is defined so that when  $N < \bar{N} \Rightarrow \sigma(N_t/\bar{N}) < 1$  (co-operation exists), when  $N = \bar{N} \Rightarrow \sigma(N_t/\bar{N}) = 1$  (neither co-operation nor rivalry exists), and when  $N > \bar{N} \Rightarrow \sigma(N_t/\bar{N}) > 1$  (rivalry exists). Therefore, a higher value of  $\sigma$  means greater rivalry.

The forest amenity value,  $A^i$ , for an individual member  $i$  is then specified as follows:

$$A^i_t(Q_t, N_t/\bar{N}) = \frac{\gamma \log(Q_t)}{\sigma(N_t/\bar{N})}, \quad (1)$$

where  $Q_t$  is the stock of the resource and  $\gamma$  is a scaling parameter. The amenity function is thus a combination of a concave function of the resource quantity and the rivalry function. We furthermore choose the following quadratic specification for  $\sigma(N_t/\bar{N})$ :

$$\sigma(N_t/\bar{N}) = 1 - \frac{N_t}{\bar{N}} + \left(\frac{N_t}{\bar{N}}\right)^2. \quad (2)$$

By combining (1) and (2), it can be seen that the amenity value experienced by an individual member for a given level of the resource stock improves with population size (i.e.,  $\sigma$  becomes smaller) until  $N = \bar{N}/2$ , at which level the amenity function reaches its maximum and decreases thereafter since rivalry over amenity values is becoming more intense. We will use these in the social planner's problem.

Figure 1 illustrates the effect of the community's population size on the amenity utility experienced by an individual. With the non-linearity of expressions (1) and (2), the amenity utility first increases and is at a higher level up to a point determined by the reference population level  $\bar{N}$  (which can be understood as an ecological vulnerability point of the resource), and then begins to decrease relative to the starting point, thus exhibiting the crowding impact on the amenity value of the forests.

### 3. Common-pool resources: the problem

Common-pool forestry is practiced in such a way that the private agents, families, keep their financial assets, or capital, under individual private ownership, while the natural resources form a jointly-owned asset. Financial assets can be broadly considered to consist of farmland, financial wealth, or the capital value of labour skills. The income from the common-pool resource is divided evenly among the community members.<sup>2</sup> Thus, nobody is excluded from the monetary benefits. However, the initial financial wealth does not need to be evenly distributed among the community families/dynasties

<sup>2</sup>Local fisheries in Apesteguia and Maier-Rigaud (2006) and Gaspart and Seki (2005) provide an example of this.

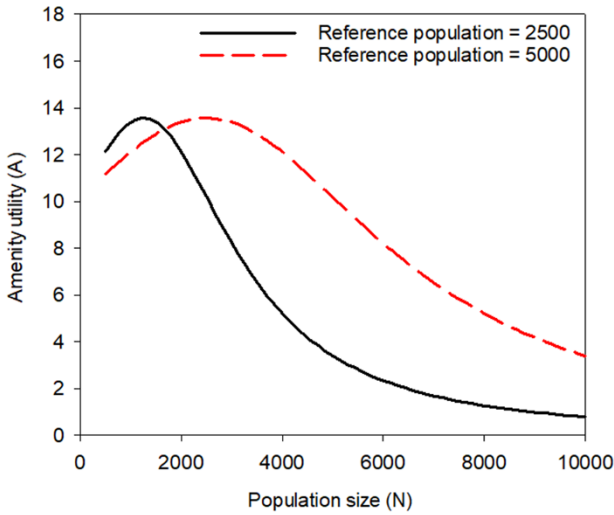


Figure 1. The crowding effect on the amenity value of forest resources.

and their proceeds are kept under private ownership. The community takes the production function of the financial assets as given; thus the interest rate is given to the community. We apply Benthamian utility structure: the joint social welfare of the community is sought to be maximized assuming that the total communal utility is the sum of the utilities of community members (Bentham, 1948).

We assume an altruistic social planner to maximize the utility of current and future population (Ramsey, 1928; Koopmans, 1963). The current population at time  $t$  is  $N_t$ , and the given population growth rate is  $n$ . In each period, the planner decides the extent of resource extraction measured in land area units, or as the share of the current land area that is harvested. The deforested area is allocated to alternative land uses, such as agriculture. The problem can be written as follows:

$$\max_{\{a_s, w_{s+1}^1, \dots, w_{s+1}^{N_t}\}_t} \sum_{s=t}^{\infty} \left( \frac{1}{1+\rho} \right)^{s-t} (1+n)^{s-t} \sum_{i=1}^{N_t} \left[ u(c_s^i) + A_s^i(Q_s, N_s/\bar{N}) \right], \quad (3a)$$

s.t.

$$\sum_{s=t}^{\infty} \left( \frac{1}{1+r} \right)^{s-t} c_s^i \leq w_t^i + LV_t^i(x_t) \quad \text{for } i = 1, \dots, N_t \quad (3b)$$

$$w_{s+1}^i = (1+r)(w_s^i - c_s^i) \quad \text{for } i = 1, \dots, N_t \quad (3c)$$

$$Q_s = qx_s \quad (3d)$$

$$Q_{s+1} = (1-a_s)qx_s \quad (3e)$$

$$LV_t^i(x) = \sum_{s=t}^{\infty} \left( \frac{1}{1+r} \right)^{s-t} \frac{1}{N_t} \left[ \left( pq + \frac{rent}{r} - k \right) a_s + m(1 - a_s) \right] x_s \quad (3f)$$

$$N_s = N_t(1+n)^{s-t} \quad (3g)$$

$$0 \leq a_s \leq 1, \quad (3h)$$

where  $u(\cdot)$  is a strictly concave utility function of the period  $s$  consumption  $c_s^i$  of family (dynasty)  $i$ . Function  $A(\cdot)$  is the amenity utility function of the periodic volume of the forest resources,  $Q_s$ , and the population pressure is measured by  $N_s/\bar{N}$ . The periodic extraction is measured by  $a_s q x_s$  where  $a$  is the percentage share  $[0,1]$  of cutover area of the current available land area of the forests,  $x_s$ , and  $q$  is the per hectare volume of the resource, such as forests, assumed constant over time. Parameter  $\rho$  is the time preference rate of the social planner. Variable  $w_s^i$  is the non-forest assets owned by family  $i$  and  $r$  is the interest rate.  $LV$  denotes the land value measured in present value terms where  $m$  is an *in-situ* rent generated by the existing resource which represents, for example, a periodic value of the non-timber product flows obtainable from standing forest (extractable fringe units, e.g., fruit crops). Parameter *rent* is an annualized land rent from alternative land uses such as agriculture and it is assumed to flow perpetually after deforestation. Finally,  $p$  is the unit price of the resource and  $k$  is a possible harvest cost of the resource per area.

Equation (3b) is the lifetime consumption constraint, whereas equation (3c) describes how the external, non-forest assets evolve over time. Equation (3d) represents the standing volume of forests at the beginning of period  $s$ , whereas equation (3e) describes the equation of motion of the land area allocated to standing forests. Equation (3f) gives the land value measured in present value terms, and equation (3g) describes the population growth. Observe also that the boundedness of (3a) is determined by  $r$ ,  $\rho$  and  $n$ .

The problem thus describes the behaviour of an intertemporally maximizing social planner concerned about the utility of infinitely-lived members of the community. The social planner is assumed to be egalitarian, giving weight to the consumption and amenity utilities of each generation in proportion to its size. In this respect, the model is related to Ramsey, Cass, and Koopmans (cf. Obstfeld and Rogoff, 1996). In Koopman's model, the dynasties are identical. The difference here, besides the aspect of the natural resource, is that in the current model, there are  $N_t$  non-identical families (or 'dynasties'), whereas in their model the dynasties are identical. This property in fact makes it possible to study in the present context the intra-generational equity issues, besides the inter-generational aspects. In this sense, the social planner in the present model plays the role of an emperor over the dynasties, using the terminology of the relevant literature. Note, however, that we are normalizing the  $s = t$  size of each dynasty to be unity. Relaxing this would not change the analysis.

The monetary capital structure represents the simplest possible production structure, namely a pure capital model, in which output is a fixed proportion of wealth, through the fixed constant interest rate (Dasgupta, 2008). The natural resource can be freely transformed into monetary capital, but the latter cannot be transformed into

the natural resource. In this respect, it is the natural resource capital, the forests in the present model, that represents the ‘cake’ in the cake-eating characterization of Gale (1967) and Romer (1986). Since we have the  $A$  utility function along with the  $u$  utility function, the case here would be a combined ‘cake-eater and cake-preserver’.

The flow of non-timber products is an indication that the forest can generate resource rents which are an important aspect of forest use in tropical countries. For example, Vedeld *et al.* (2004) carried out a meta-analysis of 54 studies undertaken across the tropical world and found ‘forest environmental income’ (largely fuelwood, wild foods and fodder for animals) made an average contribution to rural household incomes of 22 per cent.

#### 4. Optimal consumption and extraction

Under logarithmic utilities, the optimal path for the periodic consumption of the families is given by the following rule:

$$\frac{c_s^i}{c_t^i} = \left( \frac{(1+r)(1+n)}{1+\rho} \right)^{s-t}. \quad (4)$$

The above rule is the standard consumption solution found in dynamic contexts (Obstfeld and Rogoff, 1996) augmented with the population growth rate  $n$ . This consumption rule incorporates the social planner’s altruism. The population growth term tilts the consumption pattern from current consumption toward future consumption. The ruler of the dynasties (emperor) is forward-looking and weighs the utility of future generations in making today’s consumption and resource use decisions. When the dynasty emperor expects more progeny (higher  $n$ ), it provides for the future generations by reducing current consumption today (cf. Obstfeld and Rogoff, 1996). When the impatience rate,  $\rho$ , equals the capital growth rate,  $r$ , the consumption develops with the population growth rate  $n$ . Thus, in this case, the per capita consumption remains constant over time.

Utilizing the optimal consumption rule recursively and assuming an interior solution where a part of the resources is used (and using logarithmic utilities) yields the following optimal rule for the use of the resources for  $s = t$  (see the appendix):

$$\begin{aligned} & - \sum_{s=t}^{\infty} \left( \frac{1+n}{1+\rho} \right)^{s-t} \frac{\gamma q N_t}{\sigma_s Q_{t+1}} + \sum_{i=1}^{N_t} \lambda_t^i \left[ \frac{1}{N_t} \left( pq + \frac{rent}{r} - k - m \right) \right. \\ & \left. - \left( \frac{1}{1+r} \right) \frac{LV_{t+1}^i}{x_{t+1}} \right] = 0, \end{aligned} \quad (5)$$

where  $\lambda_t^i$  are the Lagrange multipliers. For convergence,  $n$  needs to be smaller than  $\rho$  in the first term on the left-hand side.

To untangle the role of income inequality in determining the extent of the resource extraction, we use in the next section the fact that, in equation (5), the following condition holds:

$$\sum_{i=1}^{N_t} \lambda_t^i = \sum_{i=1}^{N_t} \frac{1}{c_t^i}.$$

## 5. Characterizing the solution and the role of inequality

Next, we introduce our concept of income inequality used in the model. This will help in clarifying the role of population in the first-order condition for the resource (forest) clearance in [equation \(5\)](#). We also compare our inequality measure to the commonly used Gini coefficient.

Denote the average consumption within the community with  $\bar{C}_t$ . In the case of perfectly equal income distribution, the communal marginal utility of consumption  $\sum_{i=1}^{N_t} \frac{1}{c_t^i}$  will be given by  $\frac{N_t}{\bar{C}_t}$ . Then, define the following expression as a measure for the communal marginal utility of consumption relative to the ‘perfect equality’ case:

$$\mu_t = \frac{\sum_{i=1}^{N_t} \frac{1}{c_t^i}}{\frac{N_t}{\bar{C}_t}}. \quad (6)$$

Note that  $\mu$  takes its minimum value 1 with the perfect equality case. Therefore, it can be considered to measure the stringency of the communal budget constraint (shadow prices  $\lambda_t^i$  in [equation \(5\)](#)) relative to the case where wealth and thus income is evenly distributed among the community members. We use this measure of inequality in the model because of its analytical consistency in the context of the optimal solution of the utility maximization problem. In other words, it has a utility driven interpretation.

When compared to the Gini index, the most commonly used measure of unequal income distribution, it can be shown that while the Gini index reacts linearly when inequality increases, our measure in [\(6\)](#) increases non-linearly. Thus,  $\mu$  increases at a greater rate the higher the prevailing inequality. In this sense, our measure reflects the social planner’s increasing aversion towards exceedingly unequal income distributions, whereas the Gini index can be viewed as an arithmetic measure characterizing the income distribution.

Now, using  $\mu$ , the communal marginal utility of consumption can be expressed as follows:

$$\sum_{i=1}^{N_t} \frac{1}{c_t^i} = \mu_t \frac{N_t}{\bar{C}_t}. \quad (7)$$

When [\(7\)](#) is used in [\(5\)](#), the optimal resource use at  $s = t$  will follow (see [appendix](#)):

$$\sum_{s=t}^{\infty} \left( \frac{1+n}{1+\rho} \right)^{s-t} \frac{\gamma q N_t \bar{C}_t}{\sigma_s \mu_t Q_{t+1}} + m = pq + \frac{rent}{r} - k - N_t \left( \frac{1}{1+r} \right) \frac{LV_{t+1}^i}{x_{t+1}}. \quad (8)$$

In the case of forest resources, the left-hand side of [\(8\)](#) represents the marginal costs of deforestation, whereas the marginal benefits of deforestation are on the right-hand side of the equation. The marginal benefits consist of net revenues from timber cutting and of the rental value of cleared forest, both being exogenously given to the landowner. The marginal cost of deforestation on the left-hand side of [\(8\)](#) can also be thought of as the marginal benefits of maintaining the forest resources. These consist of two parts, the monetary value of *in-situ* forest benefits  $m$ , and the second part with the discounted periodic flow of the relative amenity values of the standing forest,

$$\sum_{s=t}^{\infty} \left( \frac{1+n}{1+\rho} \right)^{s-t} \frac{\gamma q N_t \bar{C}_t}{\sigma_s \mu_t Q_{t+1}}.$$



The implications of the non-monetary externalities for the use of the resource can now be seen more easily. By introducing a positive term for the marginal benefits of standing forests, the valuation of the externalities tends to lead to greater preservation of forests, as does an increase in the non-timber rents  $m$  of the forest. The relative amenity valuation part now links the monetary wealth of the landowner with the deforestation decision. The larger the consumption level, the smaller the extent of deforestation. However, higher inequality leads to increased resource extraction. In particular, the higher the term  $\mu_t$ , the smaller the amenity valuation of the standing forest. This impact results from the concave (logarithmic) utilities, as the social planner benefits from using the income from forest extraction to increase the consumption utility of the poorest households in particular. In other words, while all households receive an equal share of the deforestation income, that income will generate a greater increase in utility for the poorest, which then ultimately contributes more towards increasing the total utility.

From [equation \(8\)](#), it can also be observed that the population size has two effects on the optimal deforestation decision. First, there is a direct positive effect on the amenity value of the forest resources, as the social planner counts each member's amenity utility. Second, there is the indirect effect through the rivalry function which tends to decrease the amenity value for all members. We will leave the investigation of these effects for the numerical illustrations.

Finally, we note that expression [\(8\)](#) assumes an interior solution, meaning that some of the existing resources are deforested while a part of them is left standing. However, an optimal depletion could also exhibit corner solutions, meaning no deforestation or total deforestation in the current period. More precisely, marginal benefits of deforestation could be larger than the marginal costs of deforestation for all feasible choices, thus leading to a total cutover of the existing forests. Or alternatively, the marginal benefit of deforestation could be smaller than the marginal cost of deforestation for all feasible choices, thus leading to full preservation of the forests, at least in the current period. The decision to preserve all standing forests could change in the subsequent periods.

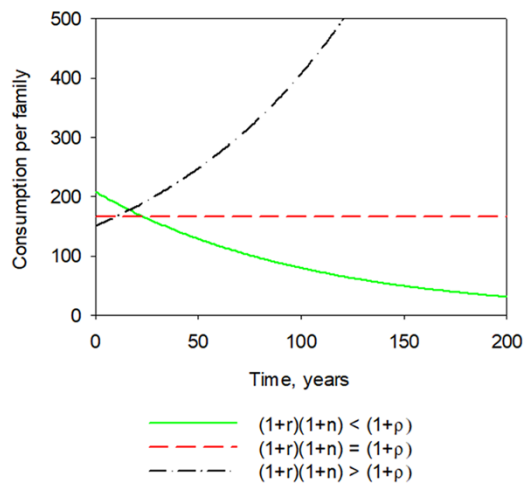
## 6. Numerical illustrations

In this section, we use numerical methods to solve the model and to illustrate the impacts of the key variables and parameters on deforestation. Numerical analysis is based on the Karush–Kuhn–Tucker theorem and is solved using AMPL and Knitro optimization software. Parameter values are given in [table 1](#). We conduct sensitivity analysis with respect to the interest rate, initial wealth level, reference population level, and the extent of wealth inequality. Due to potential non-convexities, we use 30 starting points to increase the probability of finding the global optimum.

[Figure 2](#) illustrates the three possible paths of consumption based on [equation \(4\)](#). When the numerator and the denominator are equal, the optimal consumption path is such that the consumption of the family is constant, as exhibited by the red dashed line in [figure 2](#). The green solid line represents a decreasing consumption path, whereas the dash-dot black line represents an increasing consumption path. The former occurs when the planner has a high degree of impatience (high  $\rho$ ) relative to the population

**Table 1.** Parameter values for numerical analysis unless otherwise stated ( $t = 0$ )

Parameter	Value	Definition
$N_0$	500	Initial population
$\bar{N}$	1500	Reference population level
$r$	0.02	Interest rate
$\rho$	0.05	Time preference rate
$\bar{w}_0$	5000	Community's average initial wealth
$x_0$	100	Initial forest area
$pq \cdot k$	17500	Net timber income per area unit
$rent$	300	Land rent level from cleared land
$n$	0.01	Population growth rate
$m$	30	In-situ rent generated by existing resource
$\gamma$	0.3	Amenity scaling parameter



**Figure 2.** Optimal consumption paths.  
*Notes:* For green (solid), red (dashed) and black (dash-dot) lines,  $r$  is respectively set to 0.01, 0.02, and 0.03, while  $n = 0.01$  and  $\rho = 0.03$ .

growth and the return on wealth (i.e., the interest rate), and the latter occurs in the opposite case.

In what follows, we focus on investigating the case with a decreasing consumption path. In figure 3, the impact of discounting on the forest resource extraction is demonstrated by varying the interest rate while holding other parameters constant (note  $\rho = 0.05$ ). Interestingly, the interest rate has a dual effect on the optimal extraction path. Initially, a higher interest rate leads to a higher amount of forest land to be cleared. This is the *substitution effect*. Later in time, a higher interest rate indicates a slower pace of deforestation as compared to a lower interest rate. This is the *income effect*: with a higher interest rate, the community eventually has a higher

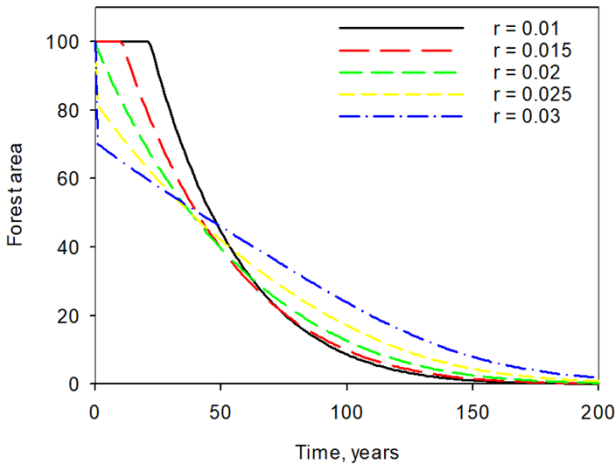


Figure 3. Effects of the interest rate on the optimal extraction of forest resources.

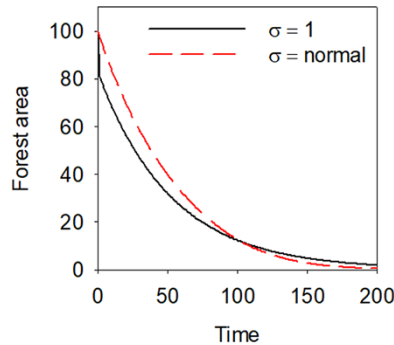
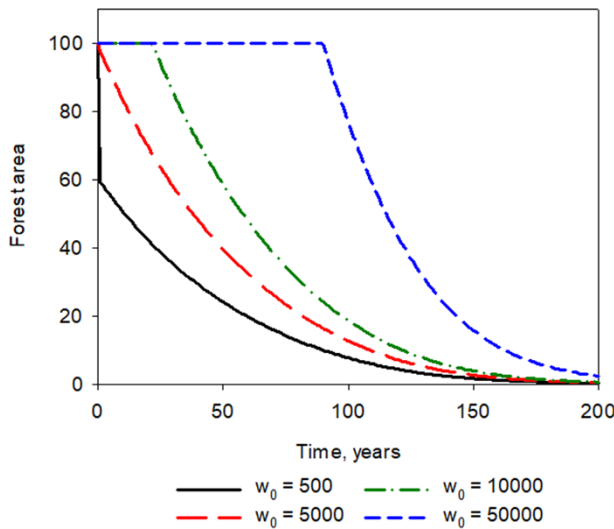


Figure 4. The impact of rivalry in amenity values on the optimal extraction path.  
Note:  $r = 0.02$ ,  $\rho = 0.05$ .

income level, implying that it also values the amenity utilities of the forests more than a poorer community (when assuming otherwise identical communities), as indicated by equation (8).

As discussed earlier, population growth influences forest extraction in two different ways. To explore and compare these effects, we solve the optimal extraction path when assuming that there is no rivalry and hence no congestion in the amenity values generated by the forests. In other words, we let  $\sigma = 1$  in the amenity function presented in equation (1).

Figure 4 illustrates the effects together with the benchmark extraction path. As can be seen from the figure, when rivalry is present (red dashed line), the area of the forest cover starts from a slightly higher level than in the case with no rivalry (black line). However, during the planning horizon, the optimal extraction path with rivalry depletes the forest stock more quickly than without rivalry. This result highlights the fact that as population grows, congestion reduces the *in-situ* values of the forests and

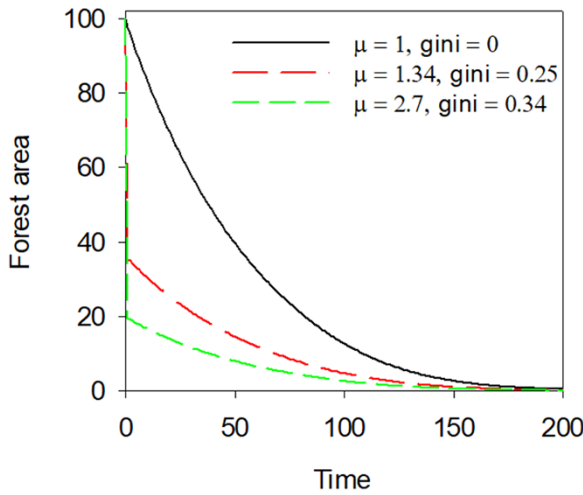


**Figure 5.** The effects of initial wealth on optimal forest clearing.  
*Note:*  $r = 0.02$ ,  $\rho = 0.05$ .

hence reduces the benefit of postponing harvesting. The planner responds to this by accelerating forest extraction. But initially, when the population size is still relatively small, population growth contributes positively to amenity valuation (equation (2)). This explains why the starting point of the forest cover is slightly higher when the rivalry function is included. Conversely, when  $\sigma = 1$ , both the positive and the negative effects from population growth on amenity valuations are absent.

Figure 5 illustrates the direct impacts of the wealth level on the extraction of the forest resources. As expected, a higher income level implies a higher forest area being protected from clearing. But given the decreasing consumption path, there is a point in time after which deforestation begins, as can be seen from the figure. The wealthier the community, the later in time this transition point occurs. Note that if the consumption path were increasing, then it might be optimal to never deforest after the initial period. However, congestion and rivalry associated with the common-pool resource could still deteriorate the amenity values enough so that deforestation commences at a later point in time even with an increasing consumption path.

In Figure 6, we illustrate the impact of income distribution on forest extraction. The three cases we study vary with how initial wealth is distributed in the population. In the first case the distribution is equal, with all families starting with 5,000 units of wealth. In the second case, the richer half of the population starts with 14,000 units, and the poorer half with 1,000 units of initial wealth. And in the final case, the richer half starts with 50,000 units, and the poorer with 500 units of wealth. These cases correspond to Gini index values of 0, 0.25 and 0.34, respectively, when the Gini index is computed using the resulting consumption levels (recall that revenue from forest clearing is equally divided among members). Hence, the first case reflects a community with perfect equality, whereas the second can be considered as a modest case of inequality, and the last one as a more severe case of inequality. Note that the underlying wealth



**Figure 6.** Effect of inequality on forest area development.

Notes:  $\mu$  values 1, 1.34, and 2.7, correspond to Gini-index values 0, 0.25, and 0.34, respectively. Respective initial wealth levels for richer and poorer halves of population are (5000;5000), (14000;1000), and (50000;500).

inequality in these scenarios is even more severe than the resulting consumption-based Gini index values tend to suggest.

Figure 6 shows that the higher the inequality within the community, the more imminent the extraction of the forest resources. This results from the social planner utilizing the revenues from forest extraction to reduce the inequalities in consumption utility in the community. This occurs even when the average wealth in the community is higher in the more unequal scenarios. Thus, from an ecological perspective, the actions of the benevolent planner might not be as desirable as they might seem from the social and economic perspective. The planner's goal of reducing the differences in consumption utilities comes at the expense of greater deforestation. There are naturally several ways in which the outcome could be improved upon also from the ecological perspective. We provide a discussion of some of these possibilities next in the final section.

## 7. Discussion and conclusions

In this paper, we have contributed to the literature studying the dynamic problem of natural resource extraction with common-pool features by integrating ecological, demographic and social features into the model. We demonstrated how population pressure on the amenity value of the resource, and intra-generational inequity between community members, can have considerable impacts on the dynamic path of the extraction of the resource. Furthermore, we contributed to the deforestation modelling literature by explicitly operating with a common-pool forestland ownership which is commonly observed in many parts of the world.

The numerical results show that higher economic growth rates can be associated with higher initial losses in amenities generated by forests but lower losses in the long run when compared to the case with lower economic growth rates. Thus, countries

with weak economic performance might initially perform better in terms of ecological integrity, but in the long run the gradual deterioration of the amenity producing resources leads to an even lower level of the resource base. This highlights the challenges and contradictions associated with economic development, especially when short-term losses in ecological assets are exchanged for better long-run economic and ecological performance. However, the degree of inequality and the planner's commitment to reducing relative inequalities in the community could still result in an ecologically unsustainable outcome, as illustrated by our numerical results.

Our results also highlight several important observations relevant to tropical countries attempting to balance social, economic and ecological goals. The social and economic objectives of reducing relative income inequalities through resource extraction could lead to deleterious impacts on the ecosystems. Hence, using other means of reducing inequalities associated with income and wealth, such as progressive taxation, could contribute to reducing pressures to convert forest land to alternative uses, thus alleviating ecological deterioration of primary forest resources. Also, under certain conditions, ensuring that the economy can grow at a pace that sustains increasing consumption may help to reduce the pressures to convert primary forests to alternative uses. However, this type of sustainable outcome necessitates that the community assigns a high enough utility value to the amenities generated by standing forests. It also requires, among other things, that the planner is patient enough and hence values the utility of the future members enough to ensure that consumption increases in time. Moreover, this comes at the expense of current consumption, which might not be a popular policy outcome among the current members of the community or society.

There are some important limitations with our model when it comes to capturing complex ecological, social, and political economy aspects associated with common-pool resources. Our model does not incorporate or assess the role of amenities generated by secondary forests, as is done for example in Wolfersberger *et al.* (2022). Our model assumes that there is no potential for substitution of primary forests with secondary forests, since amenity values are lost once the forest is cleared for alternative uses. Furthermore, the role of international trade in agricultural goods has been found to be a driver of forests losses in the tropics (e.g., Abman and Lundberg, 2020). In our model, increasing the relative value of agricultural goods would be captured by the increasing land rent parameter which would also increase the rate of deforestation in our model.

Furthermore, institutional environments in many tropical countries could also challenge any plans to protect primary forest areas. Such impediments include the culture of corruption, insecure property rights, and special interest groups (e.g., Kuusela and Amacher, 2016). However, it is also possible for communities to overcome the congestion and rivalry problems associated with common-pool resources by devising norms and institutions to govern the use of the resources (e.g., Ostrom, 2008). Additionally, there is evidence showing that institutions and individual behaviours interact in the context of community resource management. For example, Bluffstone *et al.* (2020) find that more cooperative individuals are likelier to contribute to the management of common forests in Nepal. Our analysis abstracts from behavioural and game-theoretic considerations. Future research could explore how altruistic and strategic behaviour could affect the use of common-pool resources in our dynamic extraction model (e.g.,

Dragicevic, 2019). Finally, in future work, the model presented in this paper could also be extended to include ecosystem dynamics and natural hazards, hence allowing a more comprehensive analysis of the issues surrounding the use of exhaustible common-pool resources, such as primary tropical forests.

**Data availability statement.** The data that support the findings of this study are available from the corresponding author upon reasonable request.

**Acknowledgements.** This study was funded by Academy of Finland project SuCCESs, decision number 341313. Acknowledgments are extended to the participants of the following conferences: 6th Faustmann Symposium 2019, 7–9 October 2019, Darmstadt (Germany), 30th European Conference on Operational Research, 23–26 June 2019, Dublin (Ireland).

**Competing interests.** The authors declare none.

## References

- Abman R and Lundberg C** (2020) Does free trade increase deforestation? The effects of regional trade agreements. *Journal of the Association of Environmental and Resource Economists* 7, 35–72.
- Agrawal A and Angelsen A** (2009) Using community forest management to achieve REDD+ goals. In Angelsen A with Brockhaus M, Kanninen M, Sills E, Sunderlin WD and Wertz-Kanounnikoff S (eds), *Realising REDD+: national Strategy and Policy Options*. Bogor, Indonesia: Center for International Forestry Research (CIFOR), 201–212.
- Amacher GS, Koskela E and Ollikainen M** (2009) Deforestation and land use under insecure property rights. *Environment and Development Economics* 14, 281–303.
- Apesteigua J and Maier-Rigaud FP** (2006) The role of rivalry: Public goods versus common-pool resources. *Journal of Conflict Resolution* 50, 646–663.
- Balboni C, Berman A, Burgess R and Olken BA** (2023) The economics of tropical deforestation. *Annual Review of Economics* 15, 723–754.
- Barbier EB** (2004) Explaining agricultural land expansion and deforestation in developing countries. *American Journal of Agricultural Economics* 86, 1347–1353.
- Bentham J** (1948) *An Introduction to the Principle of Morals and Legislation* Rev. ed. of 1823. Oxford: Blackwell.
- Bluffstone R, Dannenberg A, Martinsson P, Jha P and Bista R** (2020) Cooperative behavior and common pool resources: Experimental evidence from community forest user groups in Nepal. *World Development* 129, 104889.
- Buchanan JM** (1965) An economic theory of clubs. *Economica* 32, 1–14.
- Cass D** (1965) Optimum growth in an aggregative model of capital accumulation. *Review of Economic Studies* 32, 233–240.
- Ceddia MG** (2019) The impact of income, land, and wealth inequality on agricultural expansion in Latin America. *Proceedings of the National Academy of Sciences* 116, 2527–2532.
- Dasgupta P** (2008) Discounting climate change. *Journal of Risk and Uncertainty* 37, 141–169.
- D'Auume A and Schubert K** (2008) Hartwick's rule and maxmin paths when the exhaustible resource has an amenity value. *Journal of Environmental Economics and Management* 56, 260–274.
- Dragicevic A** (2019) Conditional rehabilitation of cooperation under strategic uncertainty. *Journal of Mathematical Biology* 79, 1973–2003.
- Gale D** (1967) On optimal development in a multi-sector economy. *Review of Economic Studies* 34, 1–18.
- Gaspard F and Seki E** (2005) Cooperation, status seeking and competitive behavior: Theory and evidence. *Journal of Economic Behavior & Organization* 51, 51–77.
- Gerlagh R and Keyzer MA** (2004) Path-dependence in a Ramsey model with resource amenities and limited regeneration. *Journal of Economic Dynamics and Control* 28, 1159–1184.
- Greenberg J** (1978) Pure and local public goods: A game-theoretic approach. In Sandmo A (ed), *Essays in Public Economics*. Lexington, MA: Heath and Co, 49–78.

- Koopmans TC** (1963) On the concept of optimal economic growth. *Cowles Foundation Discussion Paper* 372. Available at: <https://elischolar.library.yale.edu/cowles-discussion-paper-series/392>
- Krautkraemer JA** (1985) Resource amenities and the preservation of natural environments. *Review of Economic Studies* **LII**, 153–170.
- Kuusela OP and Amacher GS** (2016) Changing political regimes and tropical deforestation. *Environmental and Resource Economics* **64**, 445–463.
- Leblois A, Damette O and Wolfersberger J** (2017) What has driven deforestation in developing countries since the 2000s? Evidence from new remote-sensing data. *World Development* **92**, 82–102.
- Obstfeld M and Rogoff K** (1996) *Foundations of International Macroeconomics*. Cambridge, MA: The MIT Press.
- Ostrom E** (2008) Institutions and the environment. *Economic Affairs* **28**, 24–31.
- Ramsey F** (1928) A mathematical theory of saving. *Economic Journal* **38**, 543–559.
- Romer P** (1986) Cake eating, chattering, and jumps: Existence results for variational problems. *Econometrica* **54**, 897–908.
- Sant'Anna AA** (2017) Land inequality and deforestation in the Brazilian Amazon. *Environment and Development Economics* **22**, 1–25.
- Vedeld P, Angelsen A, Sjaastad E and Kobugabe B** (2004) Counting on the environment: Forest incomes and the rural poor. Environmental Economics Series Paper 98. World Bank.
- Wiersum KF** (2004) Social and community forestry. In Burley J, Evans J and Youngquist JA (eds), *Encyclopedia of Forest Sciences*. Oxford, UK: Elsevier, 1136–1143.
- Wolfersberger J, Amacher GS, Delacote P and Dragicevic A** (2022) The dynamics of deforestation and reforestation in a developing economy. *Environment and Development Economics* **27**, 272–293.

## Appendix. Deriving the optimal conditions for the consumption and extraction paths (interior solutions)

$$\max_{\{a_s, w_{s+1}^1, \dots, w_{s+1}^{N_t}\}_t} \sum_{s=t}^{\infty} \left( \frac{1}{1+\rho} \right)^{s-t} (1+n)^{s-t} \sum_{i=1}^{N_t} [u(c_s^i) + A_s^i(Q_s, N/\bar{N})]$$

s.t.

$$\sum_{s=t}^{\infty} \left( \frac{1}{1+r} \right)^{s-t} c_s^i \leq w_t^i + LV_t^i(x_t) \quad \text{for } i = 1, \dots, N_t$$

$$w_{s+1}^i = (1+r)(w_s^i - c_s^i) \quad \text{for } i = 1, \dots, N_t$$

$$x_{s+1} = (1-a_s)x_s$$

$$Q_{s+1} = (1-a_s)qx_s$$

$$LV_t^i(x) = \sum_{s=t}^{\infty} \left( \frac{1}{1+r} \right)^{s-t} \frac{1}{N_t} \left[ \left( pq + \frac{\text{rent}}{r} - k \right) a_s + m(1-a_s) \right] x_s$$

$$N_s = N_t(1+n)^{s-t}$$

Lagrangian:

$$L_t = \sum_{s=t}^{\infty} \left[ \frac{1+n}{1+\rho} \right]^{s-t} \sum_{i=1}^{N_t} \left[ \log(c_s^i) + \frac{\gamma \log((1-a_s)qx_s)}{\sigma_s} \right] - \sum_{s=t}^{N_t} \lambda_t^i \left\{ \sum_{s=t}^{\infty} \left( \frac{1}{1+r} \right)^{s-t} c_s^i - w_t^i \right. \\ \left. - \sum_{s=t}^{\infty} \left( \frac{1}{1+r} \right)^{s-t} \frac{1}{N_t} \left[ \left( pq + \frac{\text{rent}}{r} - k \right) a_s + m(1-a_s) \right] x_s \right\}$$



Consumption path:

$$\begin{aligned}\frac{\partial L_t^i}{\partial c_s^i} &= \left(\frac{1+n}{1+\rho}\right)^{s-t} \frac{1}{c_s^i} - \lambda_t \left(\frac{1}{1+r}\right)^{s-t} \Rightarrow \lambda_t^i = \left(\frac{(1+r)(1+n)}{1+\rho}\right)^{s-t} \frac{1}{c_s^i} \\ &\Rightarrow \frac{c_s^i}{c_t^i} = \left(\frac{(1+r)(1+n)}{1+\rho}\right)^{s-t}\end{aligned}$$

Extraction path when  $s = t$ :

$$\begin{aligned}\frac{\partial L_t}{\partial a_t} &= \frac{\partial}{\partial a_t} \left\{ \sum_{s=t}^{\infty} \left[ \frac{1+n}{1+\rho} \right]^{s-t} \sum_{i=1}^{N_t} \left[ \log(c_s^i) + \frac{\gamma \log((1-a_s)qx_s)}{\sigma_s} \right] \right\} \\ &\quad - \frac{\partial}{\partial a_t} \left\{ \sum_{i=1}^{N_t} \lambda_t^i \left\{ \sum_{s=t}^{\infty} \left( \frac{1}{1+r} \right)^{s-t} c_s^i - w_t^i \right. \right. \\ &\quad \left. \left. - \sum_{s=t}^{\infty} \left( \frac{1}{1+r} \right)^{s-t} \frac{1}{N_t} \left[ \left( pq + \frac{rent}{r} - k \right) a_s + m(1-a_s) \right] x_s \right\} \right\} = 0,\end{aligned}$$

where the first term can be written as:

$$\begin{aligned}\frac{\partial}{\partial a_t} \left\{ \sum_{s=t}^{\infty} \left[ \frac{1+n}{1+\rho} \right]^{s-t} \sum_{i=1}^{N_t} \left[ \log(c_s^i) + \frac{\gamma \log((1-a_s)qx_s)}{\sigma_s} \right] \right\} \\ = - \sum_{s=t}^{\infty} \left[ \frac{1+n}{1+\rho} \right]^{s-t} \sum_{i=1}^{N_t} \left[ \frac{\gamma qx_t}{\sigma_s Q_{t+1}} \right] = - \sum_{s=t}^{\infty} \left[ \frac{1+n}{1+\rho} \right]^{s-t} \frac{\gamma q N_t x_t}{\sigma_s Q_{t+1}}\end{aligned}$$

and the second term can be written as:

$$\begin{aligned}\frac{\partial}{\partial a_t} \left\{ \sum_{i=1}^{N_t} \lambda_t^i \left\{ \sum_{s=t}^{\infty} \left( \frac{1}{1+r} \right)^{s-t} c_s^i - w_t^i - \sum_{s=t}^{\infty} \left( \frac{1}{1+r} \right)^{s-t} \frac{1}{N_t} \left[ \left( pq + \frac{rent}{r} - k \right) a_s + m(1-a_s) \right] x_s \right\} \right\} \\ = - \sum_{i=1}^{N_t} \lambda_t^i x_t \left[ \frac{1}{N_t} \left( pq + \frac{rent}{r} - k - m \right) - \left( \frac{1}{1+r} \right) \frac{LV_{t+1}^i}{x_{t+1}} \right].\end{aligned}$$

Hence, combining the above two expressions, we get:

$$\frac{\partial L_t}{\partial a_t} = - \sum_{s=t}^{\infty} \left( \frac{1+n}{1+\rho} \right)^{s-t} \frac{\gamma q N}{\sigma_s Q_{t+1}} + \sum_{i=1}^{N_t} \lambda_t^i \left[ \frac{1}{N_t} \left( pq + \frac{rent}{r} - k - m \right) - \left( \frac{1}{1+r} \right) \frac{LV_{t+1}^i}{x_{t+1}} \right] = 0.$$

Using  $\sum_{i=1}^{N_t} \lambda_t^i = \sum_{i=1}^{N_t} \frac{1}{c_t^i} = \mu_t \frac{N_t}{C_t}$  and that  $LV_t^i$  is the same for all families:

$$\begin{aligned}\Rightarrow \sum_{s=t}^{\infty} \left( \frac{1+n}{1+\rho} \right)^{s-t} \frac{\gamma q N_t}{\sigma_s Q_{t+1}} &= \mu_t \frac{N_t}{C_t} \left[ \frac{1}{N_t} \left( pq + \frac{rent}{r} - k - m \right) - \left( \frac{1}{1+r} \right) \frac{LV_{t+1}^i}{x_{t+1}} \right] \\ \Rightarrow \sum_{s=t}^{\infty} \left( \frac{1+n}{1+\rho} \right)^{s-t} \frac{\gamma q N_t}{\sigma_s Q_{t+1}} &= \frac{\mu_t}{C_t} \left[ pq + \frac{rent}{r} - k - m - N_t \left( \frac{1}{1+r} \right) \frac{LV_{t+1}^i}{x_{t+1}} \right].\end{aligned}$$