Ensembles Parfaits et Séries Trigonométriques, by J.-P. Kahane and R. Salem. Hermann, Paris, 1963. Paperback, 188 pages. 27 F.

As the authors say in their introduction, this book is not meant to be read straight through. After three introductory chapters, each of the remaining nine deals with some property of perfect sets suggested by a problem in trigonometric series. Concepts not defined in the first three chapters but used later, as well as certain results from functional analysis and number theory, are included in a series of six short appendices.

In chapter 1 various types of perfect sets are described and the Fourier-Stieltjes coefficients of certain measures, defined by monotonic functions constant on the contiguous intervals of such sets, are calculated. Chapter 2 considers Hausdorff measures, one way of measuring sets of Lebesgue measure zero. Another method, capacity, is dealt with in chapter 3, which also has a brief account of a potential theory.

The contents of chapter 4 are best described by its main theorem. This states that there exists a trigonometric series $\Sigma(a_n \cos nx + b_n \sin nx)$, satisfying the hypothesis that $\Sigma(a_n^2 + b_n^2)n^b$ converges, $0 < b < 1$, divergent in a given closed set F if and only if the $(1-b)$-capacity of F is zero.

Chapters 5 and 6 discuss sets of multiplicity of trigonometric series. A criterion is obtained for determining if a set is a set of multiplicity and is then used to determine completely all the perfect sets of constant ratio of dissection that are sets of multiplicity. Whether or not such a perfect set is a set of multiplicity depends on the arithmetical properties of the ratio of dissection.

In chapter 7 sets of absolute convergence are investigated. In particular, it is shown that the Fourier-Stieltjes' coefficients of any positive measure supported by such a set do not tend to zero; in fact their upper limit is the total mass of the set.

Probabilistic methods are used in chapter 8 to obtain new results and to extend some of the earlier ones. Elements of a class of perfect sets are shown to be almost surely sets of multiplicity. The existence of independent perfect sets is demonstrated and Kronecker's theorem on enumerable independent sets is shown not to hold for them.

The problems of the next three chapters all use the properties of absolutely convergent series and, as a result, ideas drawn from functional analysis. Chapter 9 discusses spectral synthesis, chapter 10 the restrictions on the modulus of continuity of a function that imply the absolute convergence of its Fourier series, and chapter 11 proves the identity of the Carleson and Helson classes of sets.

In the last chapter properties of rare and lacunary series are considered. The same class of perfect sets of constant ratio of dissection considered in chapter 6 reappears here to play an important role in this apparently unrelated subject.

Most of these results are appearing in book form for the first time. Much of the material is the original work of the authors and some of the results are in fact new. Although most of the problems are very special, the crucial counterexamples and restrictions of more general theories are often explained by just such situations as are discussed here. The authors' reasons for the study of these sets are ably put forward in a very short preface.

The book is one of a newer series by this press and is well bound and printed. It contains a useful index of definitions and notations and a full bibliography.

P.S. Bullen, Paris

The application of continued fractions and their generalizations to problems in approximation theory, by Alexey Nikolaevitch Khovanskii. Translated by Peter Wynn. P. Noordhoff N.V., Groningen, 1963. xii + 212 pages. Dfl. 28.-

This book on continued fractions is devoted to certain selected topics in the analytic theory, with particular emphasis on those aspects that deal with rational approximations to functions and with numerical applications and computations It is a translation into English of the Russian work written in 1956 by A.N. Khovanskii.

The first chapter is concerned with an exposition of important recurrence relations and analytic theory of continued fractions, in particular, with transformations of continued fractions, transformations of series into equivalent and corresponding continued fractions, and considerations of convergence theory. Chapter II is devoted to continued fraction expansions of certain functions. Here is derived a solution of a certain Riccati equation with the help of continued fractions, from which are found continued fraction expansions of binomial functions, $\sqrt[x]{x}$, $\ln x$, e^x, trigonometric, inverse trigonometric, and hyperbolic functions, and $\int_0^x dx/(1 + x^k)$. Also derived are continued fraction expansions for the ratios of Bessel functions, the ratios of hypergeometric functions, Prym's function, and the incomplete gamma

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