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BOOK REVIEWS

BANCHOFF, T., GAFFNEY, T. and MCCRORY, C. *Cusps of Gauss mappings* (Research Notes in Mathematics 55, Pitman, 1981), 89 pp. £7.95.

These notes are about certain applications of the theory of singularities to problems in the differential geometry of surfaces in $\mathbb{R}^3$. The results that are proved are quite elementary to state but many of them are new. The work is part of a new activity in differential geometry that has resulted from the remarkable work on singularities pioneered by Whitney and Thom in the 1950's and continued by Arnold and others. A new development which should have an important impact on the teaching of geometry is computer graphics. There are accurate and clear computer produced diagrams throughout these notes; they were done by Tom Banchoff and Charles Strauss who have also made a film "The Gauss map, a dynamic approach" that covers related material. Banchoff has played a leading rôle in the development of films and computer graphics as aids in understanding geometry; a brief account of various examples is given in his article in *Proc. I.C.M. Helsinki* (1978).

The Gauss map N of a non-singular surface in $\mathbb{R}^3$ assigns to a point x of the surface its unit normal vector $N(x)$. The Jacobian of N determines much of the local geometry of the surface, in particular the Jacobian determinant is the curvature. At a point where the curvature does not vanish the topology of the map N is very simple because it is locally invertible. However at parabolic points the map N will have singularities. These notes study how the local geometry of the surface is related to the nature of the singularity of N. A theorem of Whitney (*Annals of Mathematics*, 1955) shows that the only stable local singularities that can occur for maps between surfaces are folds and cusps (locally the map is $(x,y)\to(x^2,y)$ or $(x,y)\to(xy-x^3,y)$. When the map N is stable, that is, every small perturbation is of the same topological type, the parabolic points form a one-dimensional set and a finite number of these are cusps of N. It is not surprising therefore that the cusp points have very special geometric properties and the main result of these notes is to give ten purely geometric characterisations of cusp points. An example of such a characterisation is that if P is a cusp point then every neighbourhood of P has bitangent planes to the surface.

A preliminary chapter deals with the analogous problem for plane curves and another chapter deals with important examples of surfaces and it is here that the value of the diagrams is most apparent. The ten properties of the cusps of a Gauss map are studied in detail for many examples before general proofs are given. The chapters giving the proofs are of necessity more technical than the remainder of the notes. They rely on general results on singularities. These results are quoted from other works but they are explained carefully and the reader who wants to see the proofs can read the references.

The notes are a model of exposition. Non-trivial examples are handled in depth and easier cases are discussed before proofs are given. Everything is well motivated and illustrated. Some of the material is useful for honours courses but the notes are probably more appropriate for a postgraduate course illustrating the applications of singularity theory to concrete problems. Although the applications given are purely mathematical there is some mention (mainly through references) of the applications to the visual mapping in biology and to physical problems related to Lagrange and Legendre singularities as well as to catastrophe theory.

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