Pulsar Astrophysics the Next Fifty Years
Proceedings IAU Symposium No. 337, 2017
P. Weltevrede, B.B.P. Perera, L.L. Preston
& S. Sanidas, eds.

doi:10.1017/S1743921317008754

On the Radio Emitting Particles of the Crab Nebula: Stochastic Acceleration Model

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Abstract. The standard shock acceleration model of pulsar wind nebulae (PWNe) does not account for the hard spectrum in radio wavelengths. The origin of the radio-emitting particles is also important to determine the pair production efficiency in the pulsar magnetosphere. Here, we propose a possible resolution for the particle energy distribution in PWNe; the radio-emitting particles are not accelerated at the pulsar wind termination shock but are stochastically accelerated by turbulence inside PWNe. We upgrade our past one-zone spectral evolution model including the energy diffusion, i.e., the stochastic acceleration, and apply to the Crab Nebula. For a particle injection to the stochastic acceleration process, we consider the continuous injection from the supernova ejecta or the impulsive injection associated with supernova explosion. The observed broadband spectrum and the decay of the radio flux are reproduced by tuning the amount of the particle injected to the stochastic acceleration process. Our results imply that some unveiled mechanisms, such as back reaction to the turbulence, are required to make the energies of stochastically and shock accelerated particles comparable.

Keywords. ISM: individual (Crab Nebula), pulsars: individual (PSR B0531+21), acceleration of particles, radiation mechanisms: non-thermal

1. Introduction

Pulsar wind nebulae (PWNe) are formed by the plasmas supplied by their central pulsars (c.f., Rees & Gunn (1974), Kennel & Coroniti (1984a)). We observe non-thermal radiation in radio through γ-rays from the particles accelerated at the termination shock, while the standard magnetohydrodynamic model of PWNe does not account for the characteristic flat radio spectrum (see §Vd of Kennel & Coroniti (1984b)). Recent spectral evolution models of PWNe have simply assumed a broken power-law distribution of particles at injection without theoretical background (e.g, Tanaka & Takahara 2010, 2011, 2013b, Tanaka 2016).

Not only the observed very hard spectrum in radio but also the particle number of the radio-emitting particles are long-standing problems in the origin of the radio emission. The particle number flux of the pulsar wind is estimated from the pair cascade process in the pulsar magnetosphere (e.g., Timokhin & Harding 2015). However, any theoretical predictions of the pair amount are at least one to two orders of magnitudes less than the number estimated from the radio observation (c.f., Arons 2012, Tanaka & Takahara 2013a).

Here, we consider physically motivated model of the origin of the radio-emitting particles: stochastic or second-order Fermi acceleration (Tanaka & Asano 2017). The importance of the stochastic acceleration via turbulences inside PWNe has been already mentioned in some previous studies (Komissarov 2013, Olmi *et al.* 2014). Multi-dimensional numerical studies have showed that PWNe are at a highly turbulent state Porth *et al.* (2014). In addition, polarization observations (e.g., Bietenholz & Kronberg 1991) infer that the Crab Nebula has a significantly turbulent magnetic field structure (c.f., Shibata *et al.* 2003). It is natural to consider the effect of the momentum diffusion inside PWNe, since the spatial and momentum diffusions are related to each other (c.f., Schlickeiser 1989).

2. Model

We constructed a stochastic acceleration model based our past studies (Tanaka & Takahara 2010, hereafter, TT10. We consider a PWN as a uniform sphere expanding at a constant velocity $v_{\rm PWN}$ and then the radius of the PWN is $R_{\rm PWN}(t) = v_{\rm PWN}t$. The emission processes of the PWN are synchrotron radiation and inverse Compton scattering off synchrotron radiation (SSC) and off the cosmic microwave background (CMB). Introducing the magnetic fraction parameter $\eta_{\rm B}$, we assume that the magnetic field inside the PWN $B(t)$ evolves according to

$$\frac{4\pi}{3}R^3_{\rm PWN}(t)\frac{B^2(t)}{8\pi} = \eta_{\rm B}\int_0^t L_{\rm spin}(t) \equiv \eta_{\rm B}E_{\rm rot}(t), \qquad (2.1)$$

where $L_{\rm spin}(t) = L_0(1+t/\tau_0)^{-(n+1)/(n-1)}$ is the spin-down power of the central pulsar, n is braking index, and τ_0 is spin-down time.

The evolution of the particle energy distribution in the PWN $N(\gamma, t)$ including the stochastic acceleration is described by the Fokker-Planck equation

$$\begin{aligned}\frac{\partial}{\partial t}N(\gamma,t) &+ \frac{\partial}{\partial\gamma}\left[\left(\dot\gamma_{\rm cool}(\gamma,t) - \gamma^2 D_{\gamma\gamma}(\gamma,t)\frac{\partial}{\partial\gamma}\frac{1}{\gamma^2}\right)N(\gamma,t)\right] \\ &= Q_{\rm PSR}(\gamma,t) + Q_{\rm ext}(\gamma,t), \qquad (2.2)\end{aligned}$$

where γ is the Lorentz factor of electrons/positrons. For the cooling term $\dot\gamma_{\rm cool}(\gamma,t)$, we use the same expression as our past studies, which include adiabatic cooling $\dot\gamma_{\rm ad}(\gamma,t)$, synchrotron radiation $\dot\gamma_{\rm syn}(\gamma,t)$, and inverse Compton scattering $\dot\gamma_{\rm IC}(\gamma)$.

We introduce the two injection terms $Q_{\rm PSR}(\gamma,t)$ and $Q_{\rm ext}(\gamma,t)$. The term $Q_{\rm PSR}(\gamma,t)$ represents the injection from the central pulsar and set to a 'single' power-law injection. The normalization $\dot n_{\rm PSR}(t)$ is set to satisfy $\int d\gamma Q_{\rm PSR}(\gamma,t)\gamma m_{\rm e}c^2 d\gamma = \eta_{\rm e}L_{\rm spin}(t)$ with $\eta_{\rm e} \lesssim 1$. The synchrotron radiation from optical through X-rays (c.f., Kennel & Coroniti 1984b) is attributed to the particles originated from $Q_{\rm PSR}$. We call those particles injected from the pulsar the 'X-ray-emitting particles' below. The extra particle injection $Q_{\rm ext}(\gamma,t)$ is the source of the radio-emitting particles that have another origin, for example, mixing of line-emitting plasmas from the SN ejecta filaments (e.g., Lyutikov 2003). We consider two forms of time-dependence of $Q_{\rm ext}(\gamma,t)$, the impulsive and the continuous injections, and we express them

$$Q_{\rm ext}(\gamma,t) = \begin{cases} Q_{\rm imp}(\gamma,t) \equiv N_{\rm imp}\delta(t)\delta(\gamma-\gamma_{\rm inj}), \\ Q_{\rm cont}(\gamma,t) \equiv \dot n_{\rm cont}\,(t/t_{\rm age})^s\,\delta(\gamma-\gamma_{\rm inj}), \end{cases} \qquad (2.3)$$

respectively. The injection energy $\gamma_{\rm inj} = 2$ is adopted. Although only the results for the continuous injection case with $s = 2$ is shown in this paper, the results for the impulsive injection case are found in Tanaka & Asanoa (2017).

The particles are stochastically accelerated by the term expressed with the diffusion coefficient in energy space $D_{\gamma\gamma}$. We phenomenologically describe the functional form of the diffusion coefficient as

$$D_{\gamma\gamma}(\gamma,t) \equiv \frac{\gamma^2_{\rm min}}{2\tau_{\rm acc,m}}\left(\frac{\gamma}{\gamma_{\rm min}}\right)^q \exp\left(-\frac{t}{\tau_{\rm turb}}\right)\exp\left(-\frac{\gamma}{\gamma_{\rm cut}}\right), \qquad (2.4)$$

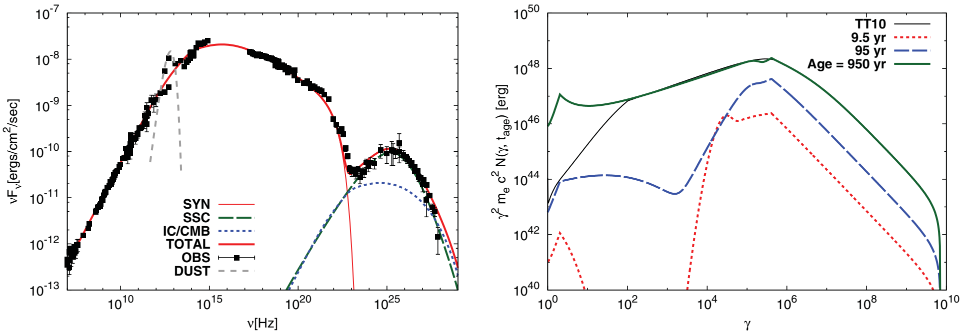

Figure 1. Left: the current photon spectrum with the observational data (Macías-Pérez 2010 for radio, Temim *et al.* 2006 for IR, Kuiper *et al.* 2001 for X-rays, and Abdo *et al.* 2010 for γ-rays). Right: evolution of the particle spectra from $10^{-2}t_{\rm age}$ =9.5 yr (dotted red) to $t_{\rm age}$ =950 yr (thick green) with the result of TT10 at $t_{\rm age}$ (thin black). Model 1 adopts $Q_{\rm ext} = Q_{\rm cont}$ and finite values of $\tau_{\rm turb}$ and $\gamma_{\rm cut}$ in Equation (2.4).

where the parameters are the acceleration time-scale $\tau_{\rm acc,m}$ at $\gamma = \gamma_{\rm min}$. (note that $\gamma_{\rm min} \neq \gamma_{\rm inj}$), the duration time-scale of the turbulence acceleration $\tau_{\rm turb}$, and the cut-off Lorentz factor $\gamma_{\rm cut}$, which is artificially introduced to avoid the acceleration of X-ray-emitting particles in our one-zone treatment. The acceleration time becomes longer at higher energy for $q < 2$ as $t_{\rm acc} = \gamma^2/(2D_{\gamma\gamma}) \propto \gamma^{2-q}$. The duration $\tau_{\rm turb}$ is interpreted as a time-scale at which the feedback process starts to decay the turbulence, or the interaction between the SN ejecta and pulsar wind becomes insufficient to induce the turbulence as the PWN and ejecta expand. Here, for simplicity, we assume that $D_{\gamma\gamma}$ is almost constant for $t < \tau_{\rm turb}$.

3. Results

Here, the model is applied to the Crab Nebula. The same observational properties of the Crab Nebula as TT10 is adopted; the period $P = 33.1$ ms, its derivative $\dot{P} = 4.21 \times 10^{-13}$ s s^{-1}, the braking index $n = 2.51$, the age $t_{\rm age} = 950$ yr, and the expansion velocity $v_{\rm PWN} = 1800$ km s^{-1}. Assuming the moment of inertia of the pulsar as 10^{45} g cm^2, those values imply $L_0 \simeq 3.4 \times 10^{39}$ erg s^{-1} and $\tau_0 \simeq 700$ yr. In addition, we fix the magnetic fraction $\eta_{\rm B} = 5 \times 10^{-3}$ and then the present magnetic field strength of 85 μG is also the same as TT10.

Fig. 1 shows one of the results which reproduce the current photon spectrum of the Crab Nebula. We consider the case of $Q_{\rm ext} = Q_{\rm cont}(\gamma, t)$. The parameters used to reproduce the spectrum are $\dot{n}_{\rm cont} = 6 \times 10^{42}$ s^{-1}, $\tau_{\rm acc,m} = 25$ yr, $\tau_{\rm turb} = 250$ yr, $\gamma_{\rm cut} = 10^5$ and $q = 2$. We also found that the flux evolution in radio and optical wavelengths is similar to the observed ones.

4. Discussion and Conclusions

There are some other parameter sets reproducing the current spectrum (Tanaka & Asano 2017). However, $\tau_{\rm turb} \sim$ a few decades and $\tau_{\rm acc,m} \sim$ a few hundreds years are required in any cases. Because the radio emission from the Crab Nebula is fading with time, we expect that the radio-emitting particles are not accelerating at present. Although the main reason for the radio flux decreasing is the decreasing magnetic field strength inside the nebula, we require no stochastic acceleration to reproduce the rate of the radio flux decreasing. On the other hand, too small $\tau_{\rm turb}$ is violates the energy budget

https://doi.org/10.1017/S1743921317008754 Published online by Cambridge University Press

restricted by the released energy from the central pulsar. It is important to note that the radio-emitting particles have a significant amount of energy released from the pulsar so far. For a given value of $\tau_{\rm turb}$, the value of $\tau_{\rm acc,m}$ is tuned to reproduce the flat radio spectrum of the Crab Nebula.

The spectral component due to the stochastically accelerated particles is required to smoothly match the component from the pulsar wind. The radio flux is exactly proportional to the normalization $\dot{n}_{\rm cont}$. This seems accidental, and the parameter tuning is required in the present model. However, some self-regulated feedback mechanism to adjust the energy budgets of the two components may exist.

References

Abdo, A. A., Ackermann, M., & Ajello, M., *et al.* 2010, *ApJ*, 708, 1254
Arons, J. 2012, *Space Sci. Revs* 173, 341
Bietenholz, M. F. & Kronberg, P. P. 1991, *ApJ*, 368, 231
Kennel, C. F. & Coroniti, F. V. 1984a, *ApJ*, 283, 694
Kennel, C. F. & Coroniti, F. V. 1984b, *ApJ*, 283, 710
Komissarov, S. S. 2013, *MNRAS*, 428, 2459
Kuiper, L., Hermsen, W., & Cusumano, G., *et al.* 2001, *A&A*, 378, 918
Lyutikov, M. 2003, *MNRAS* 339, 623
Macías-Pérez, J. F., Mayet, F., Aumont, J., & Désert, F.-X., 2010, *ApJ*, 711, 417
Olmi, B., Del Zanna, L., Amato, E., Bandiera, R., & Bucciantini, N. 2014, *MNRAS*, 438, 1518
Porth, O., Komissarov, S. S., & Keppens, R. 2014, *MNRAS*, 443, 547
Rees, M. J. & Gunn, J. E. 1874, *MNRAS*, 167, 1
Schlickeiser, R. 1989, *ApJ*, 336, 243
Shibata, S., Tomatsuri, H., Shimanuki, M., Saito, K., & Mori, K. 2003, *MNRAS*, 346, 841
Tanaka, S. J. 2016, *ApJ*, 827, 135
Tanaka, S. J. & Asano, K. 2017, *ApJ*, 841, 78
Tanaka, S. J. & Takahara, F. 2010, *ApJ*, 715, 1248
Tanaka, S. J. & Takahara, F. 2011, *ApJ*, 741, 40
Tanaka, S. J. & Takahara, F. 2013, *Prog. Theor. Exp. Phys.*, 123E01
Tanaka, S. J. & Takahara, F. 2013, *MNRAS*, 429, 2945
Temim, T., Gehrz, R. D., & Woodward, C. E., *et al.* 2006, *ApJ*, 132, 1610
Timokhin, A. N. & Harding, A. K. 2015, *ApJ*, 810, 144