Introduction

Geodynamics by Turcotte and Schubert (2014) provides a deterministic, physics-based exposition of solid-Earth processes at a mathematical level assessable to most students. This classic textbook begins with a clear and concise overview of plate tectonics, followed by stress and strain in solids, elasticity and flexure, heat transfer, gravity, fluid mechanics, rock rheology, faulting, flows in porous media, and chemical geodynamics; the latest edition has sections on numerical modeling. I have used this textbook, including earlier editions, in a graduate level class for the past 28 years to prepare students in quantitative modeling of Earth processes. The book uses a minimum of mathematical complexity, so it can be understood by a wide range of students in a variety of fields. However, this more limited mathematical approach does not provide the graduate student with the tools to develop more advanced models having three-dimensional geometries and time dependence.

This new book, *Advanced Geodynamics*, was developed to augment *Geodynamics* with more complex and foundational mathematical methods and approaches. The main new tool is multi-dimensional Fourier analysis for solving linear partial differential equations. Each chapter has a set of homework problems that make use of the higher-level mathematical and numerical methods. These are intended to augment the already excellent homework problems provided in *Geodynamics*. Detailed solutions are available from the author on request.

Chapter 1 – Observations Related to Plate Tectonics
This chapter reviews the global observations that were used to develop and refine the theory of plate tectonics. These include the latest maps of topography, marine gravity, seismicity, seafloor age, crustal thickness, and lithospheric thickness.

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This chapter also provides the global grids as overlays to Google Earth for exploration and interaction by students. In addition, all the data and tools needed to prepare the global maps using Generic Mapping Tools (GMT) are provided at the Cambridge web site.

Chapter 2 – Fourier Transform Methods in Geophysics

This chapter provides a brief overview of Fourier transforms and their properties including: similarity, shift, derivative, and convolution as well as the Cauchy residue theorem for calculating inverse transforms. These tools are used throughout the book to solve multi-dimensional linear partial differential equations (PDE). Some examples include: Poisson's equation for problems in gravity and magnetics; the biharmonic equation for problems in linear viscoelasticity, flexure, and post-glacial rebound; and the diffusion equation for problems in heat conduction. There are two approaches to solving this class of problem. In some cases, one can derive a fully analytic solution, or Green's function, to the point-source problem. Then a more general model is constructed by convolution using the actual distribution of sources. We focus on the second semi-analytic approach since it can be used to solve more complicated problems where the development of a fully analytic Green's function is impossible. This involves using the derivative property of the Fourier transform to reduce the PDE and boundary conditions to algebraic equations that can be solved in the transform domain. A more general model can then be constructed by taking the Fourier transform of the source, multiplying by the transform domain solution, and performing the inverse transform numerically. When dealing with spatially complex models, the second approach can be orders of magnitude more computationally efficient, because of the efficiency of the fast Fourier transform algorithm.

Chapter 3 – Plate Kinematics

This chapter is focused on the basics of plate kinematics and relative plate motions. Students are encouraged to learn the names of the major plates, the plate boundaries, and triple junctions. We then review the rules governing the relative motions across the three types of plate boundaries, spreading ridges, transform faults, and subduction zones and use these rules for triple junction closure of the relative velocity vectors. The remainder of the chapter is concerned with plate motions on a sphere using vector calculus. The exercises involve calculations of plate motions and plate circuit closure using published rotation poles.

Chapter 4 – Marine Magnetic Anomalies

This chapter uses the Fourier transform tools developed in Chapter 2 to compute the scalar magnetic field that is recorded by a magnetometer towed behind a ship, given a magnetic timescale, a spreading rate, and a skewness. We first review the origin of natural remnant magnetism, to illustrate that the magnetized layer is thin compared with its horizontal dimension. Then the relevant differential equations are developed and solved under the ideal case of seafloor spreading at the north magnetic pole. Anomalies that formed at lower latitudes have a skewness that causes a wavelength-dependent phase shift. The exercises include the calculation of the magnetic anomalies associated with seafloor magnetic stripes and comparisons with shipboard magnetic data to establish the seafloor spreading rate and skewness.

Chapter 5 – Cooling of the Oceanic Lithosphere
This chapter uses the Fourier transform method to solve for the temperature in the cooling oceanic lithosphere for half space and plate cooling models. For researchers in the areas of marine geology and geophysics, this is the essence of geodynamics since it explains the age variations of marine heat flow, seafloor depth, elastic thickness, and geoid height. The cooling models are also used to calculate the driving forces of plate motions including ridge push and slab pull. We focus on the buoyancy of the lithosphere as a function of crustal thickness to explain the conditions when subduction is possible. A highlight of this chapter are nine challenging heat flow exercises based on publications including thermal evolution of an oceanic fracture zone, lithospheric reheating from a mantle plume, and frictional heating during an earthquake.

Chapter 6 – A Brief Review of Elasticity
This chapter reviews stress, strain, and elasticity in three dimensions using tensors. There is a brief presentation of tensor rotations, principal stress, and stress invariants. The principal stress vs. strain is inverted using the symbolic algebra in MATLAB. This is used to translate the Lamé elastic constants to Poisson's ratio and Young's modulus. The plane stress formulation is used to develop the moment versus curvature relationship for a thin elastic plate. This chapter is intended as a review and reminds some students that they need to master this material.

Chapter 7 – Crustal Structure, Isostasy, Swell Push Force, and Rheology
This chapter covers four topics. The first is the basic structure of the oceanic and continental crust. The second and third topics are the vertical and horizontal force balances due to variations in crustal thickness. The vertical force balance, isostasy, provides a remarkably accurate description of variations in crustal thickness based on a knowledge of the topography. The horizontal force balance provides a lower bound on the force needed to maintain topographic variations on the Earth. The fourth topic is the rheology of the lithosphere. How does the lithosphere strain in response to applied deviatoric stress? The uppermost part of the lithosphere is

cold, so frictional sliding along optimally oriented, pre-existing faults governs the strength. At greater depth, the rocks can yield by non-linear flow mechanisms. The overall strength-versus-depth profile is called the yield-strength envelope (YSE). The integrated yield strength transmits the global plate tectonic stress. Moreover, the driving forces of plate tectonics cannot exceed the integrated lithospheric strength. This provides an important constraint on the geodynamics of oceans and continents.

Chapter 8 – Flexure of the Lithosphere
This chapter covers lithospheric flexure theory for an arbitrary vertical load. The approach is similar to the solutions of the marine magnetic anomaly problem, the lithospheric heat conduction problem, the strike-slip fault problem, and the flat-Earth gravity problem. In all these cases, we use the Cauchy residue theorem to perform the inverse Fourier transform. In a later chapter we combine this flexure solution with the gravity solution to develop the gravity-to-topography transfer function. Moreover, one can take this approach further, to develop a Green's function relating temperature, heat flow, topography, and gravity to a point heat source. In addition to the constant flexural rigidity solution found in the literature, we develop an iterative solution to flexure with spatially variable rigidity.

Chapter 9 – Flexure Examples
This chapter provides practical examples of flexural models applied to structures in the lithospheres of Earth and Venus. The models are all solutions to the thin and thick-plate flexure equation, with a variety of surface loads, sub-surface loads, and boundary conditions. Both gravity and topography data are used to constrain the models. We provide a numerical example that takes arbitrary topography and gravity anywhere on the Earth and uses Generic Mapping Tools to find the best elastic thickness and densities. A unique feature of this chapter is a comprehensive discussion of the non-linear relationship between plate bending moment and curvature that dominates at all subduction zones. This chapter includes eight challenging flexure exercises based on publications including: ice shelf flexure, seamount flexure, fracture zone flexure, trench and outer rise yield strength and fracturing, and flexure on Venus.

Chapter 10 – Elastic Solutions for Strike-Slip Faulting
This chapter provides the mathematical development for the deformation and strain pattern due to a strike-slip fault in an elastic half space. We develop the solution from first principles using the Fourier transform approach. This approach

does not explicitly use dislocations but simulates dislocations using body force couples following Steketee (1958) and Burridge and Knopoff (1964). The main advantage of this method is that it is easily extended to three dimensions as well as complicated fault geometries. We also demonstrate the inherent non-uniqueness of inverting for slip versus depth from surface geodetic data yet show that the overall seismic moment is well resolved by surface data. The exercises at the end of the chapter illustrate the use of the 3-D Fourier transform, the Cauchy residue theorem, and computer algebra to solve for the response of an elastic half space to 3-D vector body forces.

Chapter 11 – Heat Flow Paradox

This chapter is a quantitative investigation of the heat flow paradox that relates the expected frictional heating on a fault to the measurements of surface heat flow above the fault (e.g., (Lachenbruch and Sass, 1980)). A straightforward calculation, using a reasonable coefficient of friction, predicts measurably high heat flow above the fault that is not observed. We also investigate the maximum tectonic moment that could be sustained by a fault and show that it is at least an order of magnitude greater than what is observed. Finally, we discuss the implications in terms of fault strength and earthquake predictability.

Chapters 12 – The Gravity Field of the Earth, Part 1

This chapter provides a brief introduction to physical geodesy that describes the size and shape of the Earth and its gravity field. We decompose the Earth's reference gravity field into a spherical term and terms related to hydrostatic flattening by rotation. Superimposed on this reference model are anomalies discussed in later chapters. This chapter also describes how the reference Earth model has been developed and defined using satellite observations.

Chapters 13 – Reference Earth Model: WGS84

This chapter is a summary of the reference shape and gravity field of the Earth as defined by the WGS84 parameters. Deviations from this reference model are defined in terms of geoid height, gravity anomaly, and deflections of the vertical.

Chapter 14 – Laplace's Equation on Spherical Coordinates

This chapter introduces spherical harmonics and their properties for representing planetary gravity fields. We explain how the harmonic decomposition of a function on a sphere is analogous to the Fourier series decomposition of a 2-D function in Cartesian coordinates. We then use this spherical harmonic formulation to solve

Laplace's equation and discuss upward continuation. Finally, we describe how the Earth's gravity field is represented as spherical harmonic coefficients and their time variation.

Chapter 15 – Laplace's Equation in Cartesian Coordinates and Satellite Altimetry
This chapter is focused on shorter wavelength components of the gravity field that are best represented in Cartesian coordinates using Fourier series. The Fourier transform of Laplace's equation is used to illustrate upward continuation as well as the connection between the anomalous potential (geoid height), its first derivatives (gravity anomaly and deflections of the vertical), and the second derivatives (gravity gradient tensor). This chapter also contains a quite complete discussion of satellite radar altimetry and how it is used to recover short wavelength variations in gravity which provides an important tool for investigating plate tectonics.

Chapter 16 – Poisson's Equation in Cartesian Coordinates
This chapter is focused on solving Poisson's equation using Fourier transforms. This solution is used to generate models of the disturbing potential and its derivatives from a 3-D density model. One approach is to perform a convolution of the Green's function with the 3-D density model. However, this approach, which appears in most textbooks, is error prone, computationally inefficient, and almost never used in modern publications. Instead, we illustrate the Fourier transform approach where the model is divided into layers and the density of each layer is Fourier transformed, upward continued, and summed to generate a surface model. A uniform density leads to the Bouguer slab correction. Finally, we develop Parker's exact formula for computing the gravity model for a layer with non-uniform topography.

Chapter 17 – Gravity/Topography Transfer Function and Isostatic Geoid Anomalies
This chapter combines thin-elastic plate flexure theory with the solution to Poisson's equation, to develop a linear relationship between gravity and topography. We discuss three uses of this relationship: (1) If both the topography and gravity are measured over an area that is several times greater than the flexural wavelength, then the gravity/topography relationship (in the wavenumber domain) can be used to estimate the elastic thickness of the lithosphere and/or the crustal thickness. (2) At wavelengths greater than the flexural wavelength, where features are isostatically compensated, the geoid/topography ratio can be used to estimate the depth of compensation of crustal plateaus and hot-spot swells. (3) If the gravity

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field is known over a large area, but there is rather sparse ship-track coverage, the topography/gravity transfer function can be used to interpolate the seafloor depth between the sparse ship soundings. Finally, we show that the geoid height for isostatically compensated topography is proportional to the swell push or ridge push force so under ideal conditions, one component of the plate driving force can be measured from the geoid height.

Chapter 18 – Postglacial Rebound

This chapter considers the classic postglacial rebound problem using the Fourier transform approach. The chapter relies on Chapter 6 in *Geodynamics* where the differential equations for viscous flow of an incompressible fluid are developed. We then solve for the response of a viscous half space to an arbitrary initial topography to illustrate the effect of load wavelength on the relaxation time. In addition, an elastic lithosphere is added to the viscous half space to simulate the present-day collapse of the flexural forebulge on the perimeter of the major Laurentide and Fennoscandia ice loads.

Chapter 19 – Driving Forces of Plate Tectonics

This chapter discusses the three major driving forces of plate motion — ridge push, slab pull, and viscous drag. In previous chapters we showed that the ridge-push force is proportional to the age of the cooling ocean lithosphere. In this chapter we focus on the slab pull force which depends on age of the subducted lithosphere as well as the depth that the slab remains coupled to the surface. We use results from recent publications to calculate the positive and negative buoyancy of three major phase changes. (1) The basalt crust and depleted layer undergo a phase change to the higher density eclogite. (2) Endothermic phase changes (positive Clapeyron slope) produce a zone of increased density in the cold lithosphere for a large part of the transition zone between depths of 310 and 660 km. (3) Exothermic phase changes (negative Clapeyron slope) below 660 km result in a zone of decreased density between depths of 660 and 720 km. Finally, we discuss the magnitudes of the forces for subduction of a small young plate as well as a large old plate to illustrate that the slab pull force (thermal plus phase changes) dominates ridge push and the difference must be attributed to the drag force.

The mathematical developments refer back to the section or equation in *Geodynamics* where the solutions are provided. Note that *Geodynamics* contains much more information than is provided in this new book *Advanced Geodynamics*, so both will be needed for a graduate-level geodynamics course.

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